PHYS 301
HANDOUT 1

1. Prove the trigonometric identity for two complex numbers z_1 and z_2.

2. Show that for a complex number $z = x + iy$ ($z \neq 0$) we have

$$z^{-1} = \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right).$$

3. For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ ($z_2 \neq 0$) find the number z_1 / z_2.

4. Verify that the numbers $z = 1 \pm i$ satisfy the equation $z^2 - 2z + 2 = 0$.

5. Use the method of mathematical induction to prove the relation:

$$(z_1 + z_2)^n = z_1^n + \frac{n}{1!} z_1^{n-1} z_2 + \frac{n(n-1)}{2!} z_1^{n-2} z_2^2 + \dots + \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} z_1^{n-k} z_2^k + \dots + z_2^n$$

where z_1, z_2 are any complex numbers and $n = 1, 2, 3, \dots$.

6. What geometrical object is represented by the equation $|z - 1 + 3i| = 2$?

7. What geometrical object is represented by the equation $|z - 4i| + |z + 4i| = 10$?

8. What geometrical object is represented by the equation $|z - 1| = |z + i|$

9. Show that $(iz)^* = -iz^*$.

10. Show that $\sqrt{2}|z| \geq |\mathrm{Re}\, z| + |\mathrm{Im}\, z|$.

11. Show that: $(z_1 z_2)^* = z_1^* z_2^*$.

12. Show that $(z^4)^* = (z^*)^4$

13. Express the following complex numbers in exponential form (Sch, 14)
$2 + 2\sqrt{3}i$, $-5 + 5i$, $1 - i$, $-3i$

14. Represent graphically on the x-y plane the following complex numbers

$6\left(\cos 240^0 + i\sin 240^0\right)$, $2e^{-i\pi/4}$. (Sch, 14)

15. Calculate the quantity: $\left[3\left(\cos 60^0 + i\sin 60^0\right)\right]\cdot\left[4\left(\cos 30^0 + i\sin 30^0\right)\right]$

16. Calculate the quantity: $\left(\dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10}$

(Ans: $-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$)

17. If $z_1 = r_1\left(\cos\theta_1 + i\sin\theta_1\right)$ and $z_2 = r_2\left(\cos\theta_2 + i\sin\theta_2\right)$ calculate $z_1\cdot z_2$ and z_1/z_2. (Sch, 15)

(Ans: $r_1 r_2\left\{\cos\left(\theta_1+\theta_2\right)+i\sin\left(\theta_1+\theta_2\right)\right\}$, $\dfrac{r_1}{r_2}\left\{\cos\left(\theta_1-\theta_2\right)+i\sin\left(\theta_1-\theta_2\right)\right\}$)

18. Show the following relations:

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}, \qquad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}.$$

19. If z is a given complex number represent graphically the number ze^{ia}, where a is a real number (Sch 17).

20. Show that for two complex numbers z_1, z_2 we have $e^{z_1}\cdot e^{z_2} = e^{z_1+z_2}$. (Ver, 6).

21. Study the identity $\arg\left(z_1\cdot z_2\right)=\arg\left(z_1\right)+\arg\left(z_2\right)$.

22. Let $z_1=-1$ and $z_2=i$. Find $\operatorname{Arg}\left(z_1\cdot z_2\right)$ and $\operatorname{Arg}\left(z_1\right)+\operatorname{Arg}\left(z_2\right)$.