Tutorial (5) Chapter 6 & 9

A weight of 40.0 N is suspended from a spring that has a force constant of 200 N/m. The system is undamped and is subjected to a harmonic driving force of frequency 10.0 Hz, resulting in a forced-motion amplitude of 2.00 cm. Determine the maximum value of the driving force.

Since b = 0, $A = \dfrac{F_o}{m\sqrt{\left(\omega^2-\omega_o{}^2\right)^2+\left(\dfrac{b\,\omega}{m}\right)^2}} = \dfrac{F_o/m}{\omega^2-\omega_0^2}$

$\omega = 2\pi f$	$\omega_o{}^2 = k / m$	$A = F/m / (\omega^2 - \omega_o{}^2)$
$\omega = 2\pi 10$	$\omega_o{}^2 = 200/4$ kg	$0.02 = (F/4)/(4000-50)$
$\omega = 20\pi$	$\omega_o{}^2 = 50\ \mathrm{rad}^2/\mathrm{s}^2$	F = 316 N
$\omega^2 = 4000\ \mathrm{s}^{-2}$		

A 2.00-kg object attached to a spring moves without friction and is driven by an external force given by $F = (3.00\ \mathrm{N})\sin(2\pi t)$. If the force constant of the spring is 20.0 N/m, determine (a) the period and (b) the amplitude of the motion.

$$a)\omega' = 2\pi = 6.28/s; T = 1.00s$$

$$b)A = \frac{F_0/m}{\omega_0^2 - \omega'^2} = \frac{3Ns^2/2kg}{100-6.28^2} = 0.248m$$

(6.2) استنتج علاقة الممانعة في المعادلة (5.21) للدارة من نوع LCR؛ وذلك عند:

(أ) ترددات منخفضة جداً $(w \ll w_0 \text{ ؛ } w \ll {}^{1}/_{RC})$

(ب) تردد الرنين $(w = w_0)$

(ج) ترددات عالية جداً $(w \gg w_0 \text{ ؛ } w \gg {}^{R}/_{L})$. (يجب أن يتفق الجواب مع الفكرة المعروفة في نظرية الدارة AC التي لها ممانعة ذات مركبة منفردة).

الحـــــــــل:

$$Z(w) = b + i\left(mw - \frac{s}{w}\right)$$

$$b = R, m = L, s = \frac{1}{C}$$

$$Z(w) = R + i\left(Lw - \frac{1}{Cw}\right)$$

(أ) $Z(w) = R + i\left(Lw - \frac{1}{Cw}\right) \text{؛} w \ll {}^{1}/_{RC}$

$$Z(w) = i\left(-\frac{1}{Cw}\right) = \frac{-i}{Cw}$$

(ب)

$$Z(w) = R + i\left(Lw_0 - \frac{1}{Cw_0}\right)$$

بالتربيع نحصل على:

$$Z^2 = (R + i\left(Lw_0 - \frac{1}{Cw_0}\right))^2$$

$$Z^2 = R + i\left(Lw_0 - \frac{1}{Cw_0}\right).R - i\left(Lw_0 - \frac{1}{Cw_0}\right)$$

$$Z^2 = R^2 - iR\left(Lw_0 - \frac{1}{Cw_0}\right) + iR\left(Lw_0 - \frac{1}{Cw_0}\right) + \left(Lw_0 - \frac{1}{Cw_0}\right)^2$$

$$Z^2 = R^2 + \left(Lw_0 - \frac{1}{Cw_0}\right)^2$$

$$Z^2 = R^2 + \left((Lw_0)^2 - 2\frac{Lw_0}{Cw_0} + \frac{1}{(Cw_0)^2}\right) \therefore w_0 = \frac{1}{\sqrt{LC}}$$

$$Z^2 = R^2 + \left(\frac{L^2}{LC} - 2\frac{L}{C} + \frac{LC}{C^2}\right) = R^2 + \left(\frac{L}{C} - 2\frac{L}{C} + \frac{L}{C}\right)$$

$$Z^2 = R^2 + \left(\frac{L^2}{LC} - 2\frac{L}{C} + \frac{LC}{C}\right)$$

$$Z^2 = R^2 \rightarrow Z = R$$

(ج) $Z(w) = R + i\left(Lw - \frac{1}{Cw}\right), , w \gg {}^{R}/_{L}$

$$Z = iLw$$

(9.2) احسب (أ) التردد (ب) طول الموجة (ج) سرعة الطور، لموجة تعطى بالصورة:

$$\Psi = A\cos[wt - kz + \emptyset]$$

$$\Psi = (0.001\ \mathrm{m})\cos[(15\ s^{-1})t + (7.5\ m^{-1})z]$$

A=0.001 m, w=15 s^-1, k=-7.5 m^-1

$$v = \frac{w}{2\pi} = \frac{15}{2\pi} = 2.38\ HZ$$

$$\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{7.5} = 0.84\ m$$

$$v_{\emptyset} = \frac{w}{k} = \frac{15}{-7.5} = -2\ m/s$$

(د)ماهي المسافة بين نقطتين متقاربتين على الخيط إذا كان لإزاحتيهما فرق طور يساوي 45^0 ؟

$$wt - kz + \emptyset_1 = wt - kz + \emptyset_2$$

$$\emptyset_1 - \emptyset_2 = k\Delta z = \frac{2\pi}{\lambda}\Delta z$$

$$45° = \frac{2\pi}{\lambda}\Delta z$$

$$\frac{45° \times \pi}{180} = \frac{2\pi}{\lambda}\Delta z$$

$$\frac{1}{4} = \frac{2}{\lambda}\Delta z$$

$$\Delta z = \frac{\lambda}{8} = \frac{0.84}{8} = 0.1\ m$$

(9.3) أوجد، للموجة الواردة في المسألة 9.2 (أ) أقصى سرعة مستعرضة تبلغها كل نقطة على الخيط؟ (ب) النسبة المئوية القصوى لاستطالة الخيط عند كل نقاطه من نقاطة.

$$\dot{\Psi} = \frac{\partial\Psi}{\partial t} = -(0.001\ \mathrm{m})(15)sin([(15\ s^{-1})t + (7.5\ m^{-1})z])$$

$$\dot{\Psi}_{max} = (0.001).(15) = 0.015\ m/s$$

A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

Solution:

$$f = \frac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \frac{4}{3} \text{ Hz} \qquad v = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{\frac{4}{3} \text{ Hz}} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$$

When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of waves along the wire.

Solution:

$$v = f\lambda = (4.00 \text{ Hz})(60.0 \text{ cm}) = 240 \text{ cm/s} = \boxed{2.40 \text{ m/s}}$$

A wave is described by $y = (2.00 \text{ cm}) \sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x is in meters, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

Solution:

$$y = (0.020\,0 \text{ m})\sin(2.11x - 3.62t) \text{ in SI units} \qquad A = \boxed{2.00 \text{ cm}}$$

$$k = 2.11 \text{ rad/m} \qquad \lambda = \frac{2\pi}{k} = \boxed{2.98 \text{ m}}$$

$$\omega = 3.62 \text{ rad/s} \qquad f = \frac{\omega}{2\pi} = \boxed{0.576 \text{ Hz}}$$

$$v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{3.62}{2.11} = \boxed{1.72 \text{ m/s}}$$

A sinusoidal wave on a string is described by

$$y = (0.51 \text{ cm}) \sin(kx - \omega t)$$

where $k = 3.10$ rad/cm and $\omega = 9.30$ rad/s. How far does a wave crest move in 10.0 s? Does it move in the positive or negative x direction?

Solution:

$$y = (0.0051 \text{ m})\sin(310x - 9.30t) \text{ SI units}$$

$$v = \frac{\omega}{k} = \frac{9.30}{310} = 0.0300 \text{ m/s}$$

$$s = vt = \boxed{0.300 \text{ m in positive } x\text{-direction}}$$

A sinusoidal wave is described by

$$y = (0.25 \text{ m}) \sin(0.30x - 40t)$$

where x and y are in meters and t is in seconds. Determine for this wave the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

Solution:

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250 \text{ m}}$

(b) $\omega = \boxed{40.0 \text{ rad/s}}$

(c) $k = \boxed{0.300 \text{ rad/m}}$

(d) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.300 \text{ rad/m}} = \boxed{20.9 \text{ m}}$

(e) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda = \left(\frac{40.0 \text{ rad/s}}{2\pi}\right)(20.9 \text{ m}) = \boxed{133 \text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

A telephone cord is 4.00 m long. The cord has a mass of 0.200 kg. A transverse pulse is produced by plucking one end of the taut cord. The pulse makes four trips down and back along the cord in 0.800 s. What is the tension in the cord?

Solution:

The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$.

The speed is then $v = \frac{d_{\text{total}}}{t} = \frac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{\frac{T}{\mu}}$

Now, $\mu = \frac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$

So $T = \mu v^2 = \left(5.00 \times 10^{-2}\ \text{kg/m}\right)(40.0\ \text{m/s})^2 = \boxed{80.0\ \text{N}}$

Transverse waves with a speed of 50.0 m/s are to be produced in a taut string. A 5.00-m length of string with a total mass of 0.060 0 kg is used. What is the required tension?

Solution:

The mass per unit length is: $\mu = \frac{0.060\,0\ \text{kg}}{5.00\ \text{m}} = 1.20 \times 10^{-2}\ \text{kg/m}$.

The required tension is: $T = \mu v^2 = (0.012\,0\ \text{kg/m})(50.0\ \text{m/s})^2 = \boxed{30.0\ \text{N}}$.

A piano string having a mass per unit length equal to 5.00×10^{-3} kg/m is under a tension of 1 350 N. Find the speed of a wave traveling on this string.

Solution:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1\,350\ \text{kg}\cdot\text{m/s}^2}{5.00 \times 10^{-3}\ \text{kg/m}}} = \boxed{520\ \text{m/s}}$$

A transverse traveling wave on a taut wire has an amplitude of 0.200 mm and a frequency of 500 Hz. It travels with a speed of 196 m/s. (a) Write an equation in SI units of the form $y = A\sin(kx - \omega t)$ for this wave. (b) The mass per unit length of this wire is 4.10 g/m. Find the tension in the wire.

Solution:

(a) $\omega = 2\pi f = 2\pi(500) = 3\,140 \text{ rad/s}$, $k = \frac{\omega}{v} = \frac{3\,140}{196} = 16.0 \text{ rad/m}$

$$\boxed{y = \left(2.00\times 10^{-4} \text{ m}\right)\sin(16.0x - 3\,140t)}$$

(b) $v = 196 \text{ m/s} = \sqrt{\frac{T}{4.10\times 10^{-3} \text{ kg/m}}}$

$T = \boxed{158 \text{ N}}$