PHYS 234
Third Tutorial

2. An object of mass $m_1 = 9.00$ kg is in equilibrium while connected to a light spring of constant $k = 100$ N/m that is fastened to a wall as shown in Figure P15.52a. A second object, $m_2 = 7.00$ kg, is slowly pushed up against m_1, compressing the spring by the amount $A = 0.200$ m, (see Figure P15.52b). The system is then released, and both objects start moving to the right on the frictionless surface. (a) When m_1 reaches the equilibrium point, m_2 loses contact with m_1 (see Fig. P15.5c) and moves to the right with speed v. Determine the value of v. (b) How far apart are the objects when the spring is fully stretched for the first time (D in Fig. P15.52d)? (*Suggestion:* First determine the period of oscillation and the amplitude of the $m1$–spring system after m_2 loses contact with m_1.)

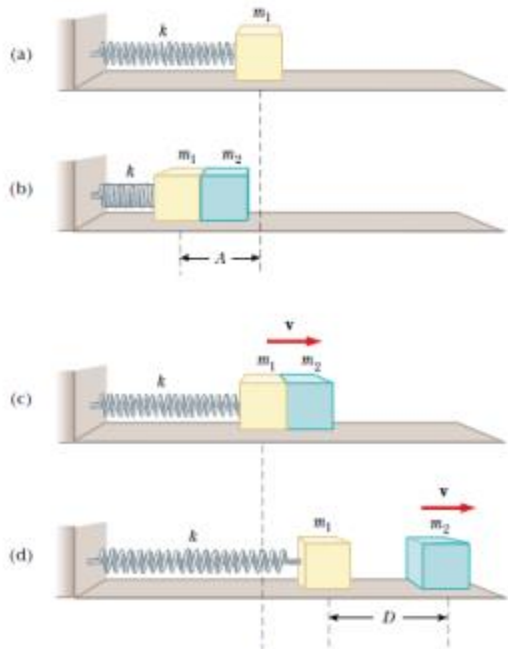

Figure P15.52

(a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = 2.00\ \text{J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0\ \text{kg})v^2 = (8.00\ \text{kg})v^2.$$

Therefore,

$$(8.00\ \text{kg})v^2 = 2.00\ \text{J},\ \text{and}\ v = \boxed{0.500\ \text{m/s}}.$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\ \text{kg})(0.500\ \text{m/s})^2 = 1.125\ \text{J}.$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system. Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125\ \text{or}\ A' = 0.150\ \text{m}.$$

The period of the m_1-spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\ \text{s}$

and it takes $\frac{1}{4}T = 0.471\ \text{s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500\ \text{m/s}(0.471\ \text{s}) - 0.150\ \text{m} = 0.085\,6 = \boxed{8.56\ \text{cm}}.$$

3.

A block of mass m is connected to two springs of force constants k_1 and k_2 as shown in Figures P15.71a and P15.71b. In each case, the block moves on a frictionless table after it is displaced from equilibrium and released. Show that in the two cases the block exhibits simple harmonic motion with periods

(a) $$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

(b) $$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

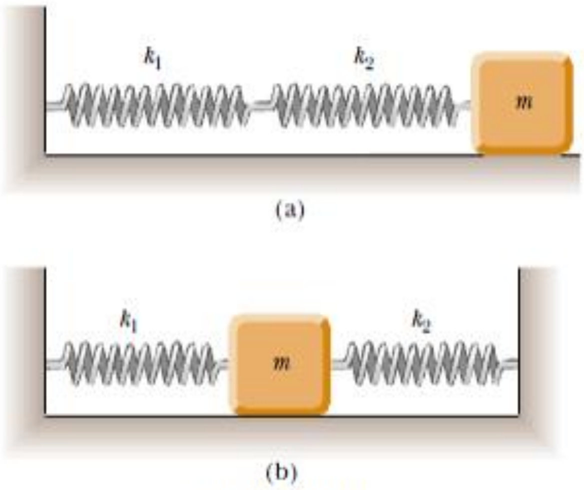

Figure P15.71

(a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.

By Newton's third law, we expect

$$k_1 x_1 = k_2 x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

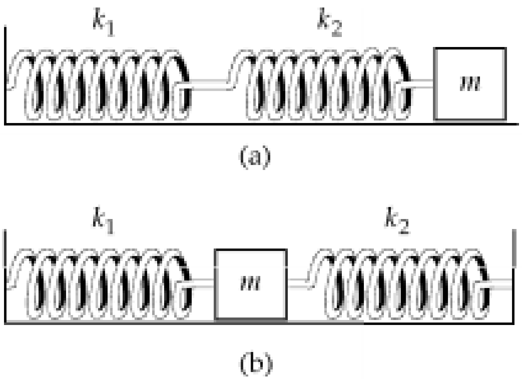

FIG. P15.71

we find $$x_1 = \left[\frac{k_2}{k_1 + k_2}\right]x$$

The force on either spring is given by $$F_1 = \left[\frac{k_1 k_2}{k_1 + k_2}\right]x = ma$$

where a is the acceleration of the mass m.

This is in the form $$F = k_{eff}\,x = ma$$

and $$T = 2\pi\sqrt{\frac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$$

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1 + k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $$T = \boxed{2\pi\sqrt{\frac{m}{(k_1 + k_2)}}}.$$

4. A light spring of relaxed length a_0 is suspended from a point. It carries a mass m at its lower free end which stretches it through a distance l. Show that the vertical oscillations of the system are simple harmonic in nature and have time period, $T = 2\pi\sqrt{l/g}$.

Solution

The spring is elongated through a distance l due to the weight mg. Thus we have

$$kl = mg$$

where k is the spring constant. Now the mass is further pulled through a small distance from its equilibrium position and released. When it is at a distance x from the mean position (Fig. 1.3), the net upward force on the mass m is

$$k(l + x) - mg = kx = mgx/l.$$

Upward acceleration $= gx/l = \omega^2 x$, which is proportional to x and directed opposite to the direction of increasing x. Hence the motion is simple harmonic and its time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{l/g}.$$

Note: Young's modulus of the material of the wire is given by

$$Y = \frac{mg}{A}/(l/L) = \frac{mgL}{Al},$$

where L is the length of the wire and A is the cross-sectional area of the wire.

Thus, $\frac{mg}{l} = \frac{AY}{L} = k =$ spring constant of the wire.

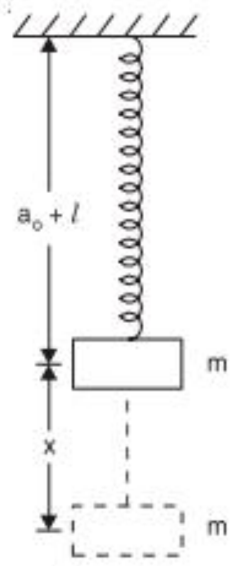

Fig. 1.3

5. Two masses m1 and m2 are suspended together by a massless spring of spring constant k as shown in Fig. When the masses are in equilibrium, m1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation.

Solution

When only the mass m_2 is suspended let the elongation of the spring be x_1. When both the masses $(m_2 + m_1)$ together are suspended, the elongation of the spring is $(x_1 + x_2)$.

Thus, we have

$$m_2 g = kx_1$$

$$(m_1 + m_2)g = k(x_1 + x_2)$$

where k is the spring constant.

Hence $$m_1 g = kx_2.$$

Thus, x_2 is the elongation of the spring due to the mass m_1 only. When the mass m_1 is removed the mass m_2 executes SHM with the amplitude x_2.

Amplitude of vibration = $x_2 = m_1 g/k$

Angular frequency $\omega = \sqrt{k/m_2}$.

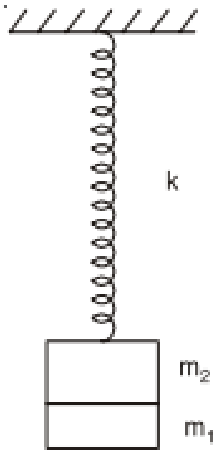

Fig. 1.4

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6. Two blocks (m = 1.0 kg and M = 11 kg) and a spring (k = 300 N/m) are arranged on a horizontal, frictionless surface as shown in Fig. 1.8. The coefficient of static friction between the two blocks is 0.40. What is the maximum possible amplitude of the simple harmonic motion if no slippage is to occur between the blocks?

Solution

Angular frequency of SHM = $\omega = \sqrt{300/12}$

Maximum force on the smaller body without any slippage is $m\omega^2 A = \mu mg$

Thus, $$A = \frac{\mu g}{\omega^2} = \frac{0.4 \times 9.8 \times 12}{300}\text{m} = 15.68 \text{ cm}.$$

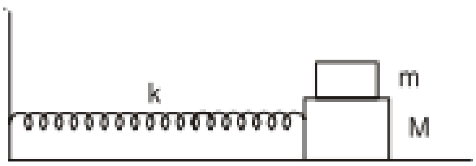
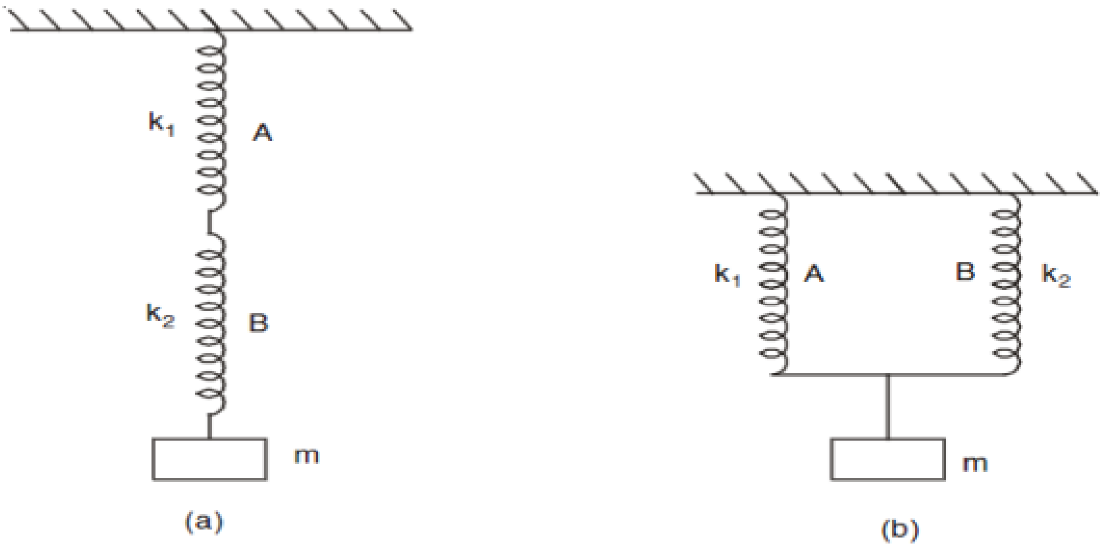


Fig. 1.8

7. Two massless springs A and B each of length a0 have spring constants k1 and k2. Find the equivalent spring constant when they are connected in (a) series and (b) parallel as shown in Fig. and a mass m is suspended from them.



Solution

(*a*) Let x_1 and x_2 be the elongations in springs *A* and *B* respectively. Total elongation $= x_1 + x_2$.

$$mg = k_1x_1 \text{ and } mg = k_2x_2$$

Thus,
$$x_1 + x_2 = mg\left(\frac{1}{k_1}+\frac{1}{k_2}\right).$$

If *k* is the equivalent spring constant of the combination (*a*), we have

$$x_1 + x_2 = mg/k$$

or
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad or,\ k = \frac{k_1k_2}{k_1+k_2}.$$

(*b*) Let *x* be the elongation in each spring.

$$mg = (k_1 + k_2)x$$

If *k* is the equivalent spring constant of the combination (*b*), we have

$$mg = kx$$

Thus,
$$k = k_1 + k_2.$$

8. **The cone of a loudspeaker oscillates in SHM at a frequency of 262 Hz ("middle C"). The amplitude at the center of the cone is $A = 1.5 \times 10^{-4}$ m, and at $t = 0$, $x = A$. (a) What equation describes the motion of the center of the cone? (b) What are the velocity and acceleration as a function of time? (c) What is the position of the cone at $t = 1.00$ ms ($= 1.00 \times 10^{-3}$ s)?**

Solution:

a. $\omega = 2\pi f = 2\pi \cdot 262\text{Hz} = 1650\text{rad/s},$

$$x(t) = A\sin(\omega t + \varphi),$$

$$x(0) = A\sin\varphi = A \Rightarrow \varphi = \pi/2,$$

$$x(t) = A\sin(\omega t + \pi/2)$$

$$= (1.5\times10^{-4}\text{m})\cos(1650t).$$

b. $v(t) = \dfrac{dx}{dt} = (-0.25\text{m/s})\sin(1650t);$

$$a(t) = \frac{dv}{dt} = (-410\text{m/s}^2)\cos(1650t)$$

c. at $t = 1.00\text{ms},$

$$x = (1.5\times10^{-4}\text{m})\cos(1650\text{rad/s}\cdot1.00\times10^{-3}\text{s})$$

$$= -1.2\times10^{-5}\text{m}.$$

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9. **A spring stretches 0.150 m when a 0.300-kg mass is gently attached to it. The spring is then set up horizontally with the 0.300-kg mass resting on a frictionless table. The mass is pushed so that the spring is compressed 0.100 m from the equilibrium point, and released from rest. Determine: (a) the spring stiffness constant k and angular frequency ω; (b) the amplitude of the horizontal oscillation A; (c) the magnitude of the maximum velocity v_{max}; (d) the magnitude of the maximum acceleration a_{max} of the mass; (e) the period T and frequency f; (f) the displacement x as a function of time; and (g) the velocity at $t = 0.150$ s.**

Solution:

a. $k = \dfrac{F}{x} = \dfrac{0.300\text{kg}\cdot9.80\text{m/s}^2}{0.150\text{m}} = 19.6\text{N/m},$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{19.6\text{N/m}}{0.300\text{kg}}} = 8.08\text{rad/s}.$$

b. $A = 0.100\text{m}.$

c. $v_{\max} = \omega A = 0.808\text{m/s}.$

d. $a_{\max} = \omega^2 A = 6.53\text{m/s}^2.$

10. **Suppose the spring of the former example (where $\omega = 8.08\ s^{-1}$) is compressed 0.100 m from equilibrium (x_0 = -0.100 m) but is given a shove to create a velocity in the +x direction of v_0 = 0.400 m/s. Determine (a) the phase angle φ, (b) the amplitude A, and (c) the displacement x as a function of time, $x(t)$.**

Solution:

a. $x = A\cos(\omega t + \phi),\ \ x_0 = A\cos\phi,$

$v = -\omega A\sin(\omega t + \phi),\ \ v_0 = -\omega A\sin\phi,$

$\frac{v_0}{x_0} = -\omega\tan\phi,\ \ \phi = \arctan\frac{v_0}{-\omega x_0} = 3.60\text{rad}.$

b. $A = \frac{x_0}{\cos\phi} = 0.112\text{m}.$

c. $x = (0.112\text{m})\cos(8.08t + 3.60)$

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11. **For the simple harmonic oscillation where k = 19.6 N/m, A = 0.100 m, x = -(0.100 m) cos 8.08t, and v = (0.808 m/s) sin 8.08t, determine (a) the total energy, (b) the kinetic and potential energies as a function of time, (c) the velocity when the mass is 0.050 m from equilibrium, (d) the kinetic and potential energies at half amplitude ($x = \pm A/2$).**

Solution:

a. $E = \frac{1}{2}kA^2 = \frac{1}{2}\cdot 19.6\text{N/m}\cdot(0.100\text{m})^2 = 9.80\times10^{-2}\,\text{J}.$

b. $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2\omega t = (9.80\times10^{-2}\,\text{J})\cos^2 8.08t,$

$K = E - U = (9.80\times10^{-2}\,\text{J})\sin^2 8.08t.$