27. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 12.0 s. (a) How tall is the tower? (b) **What If?** If this pendulum is taken to the Moon, where the free-fall acceleration is 1.67 m/s^2, what is its period there?

Solution:

(a) $$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(12.0 \text{ s})^2}{4\pi^2} = \boxed{35.7 \text{ m}}$$

(b) $$T_{\text{moon}} = 2\pi\sqrt{\frac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\frac{35.7 \text{ m}}{1.67 \text{ m/s}^2}} = \boxed{29.1 \text{ s}}$$

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28. A "seconds pendulum" is one that moves through its equilibrium position once each second. (The period of the pendulum is precisely 2 s.) The length of a seconds pendulum is 0.992 7 m at Tokyo, Japan and 0.994 2 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

Solution:

The period in Tokyo is $$T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$$

and the period in Cambridge is $$T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$$

We know $$T_T = T_C = 2.00 \text{ s}$$

For which, we see $$\frac{L_T}{g_T} = \frac{L_C}{g_C}$$

or $$\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$$

30. The angular position of a pendulum is represented by the equation $\theta = (0.320 \text{ rad})\cos \omega t$, where θ is in radians and $\omega = 4.43$ rad/s. Determine the period and length of the pendulum.

Solution:

$$\omega = \frac{2\pi}{T}: \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$$

$$\omega = \sqrt{\frac{g}{L}}: \qquad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$$

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33. A particle of mass m slides without friction inside a hemispherical bowl of radius R. Show that, if it starts from rest with a small displacement from equilibrium, the particle moves in simple harmonic motion with an angular frequency equal to that of a simple pendulum of length R. That is, $\omega = \sqrt{g/R}$.

Referring to the sketch we have

$F = -mg\sin\theta$ and $\tan\theta = \dfrac{x}{R}$

For small displacements, $\tan\theta \approx \sin\theta$

and $F = -\dfrac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}}$.

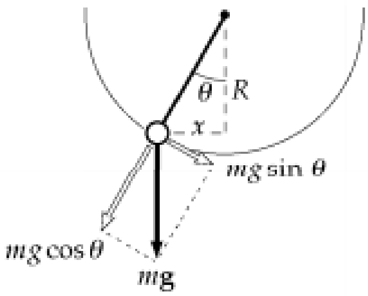

FIG. P15.33

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34. A small object is attached to the end of a string to form a simple pendulum. The period of its harmonic motion is measured for small angular displacements and three lengths, each time clocking the motion with a stopwatch for 50 oscillations. For lengths of 1.000 m, 0.750 m, and 0.500 m, total times of 99.8 s, 86.6 s, and 71.1 s are measured for 50 oscillations. (a) Determine the period of motion for each length. (b) Determine the mean value of g obtained from these three independent measurements, and compare it with the accepted value. (c) Plot T^2 versus L, and obtain a value for g from the slope of your best-fit straight-line graph. Compare this value with that obtained in part (b).

Solution:

(a) $T = \dfrac{\text{total measured time}}{50}$

The measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
$g\ (\mathrm{m/s^2})$	9.91	9.87	9.76

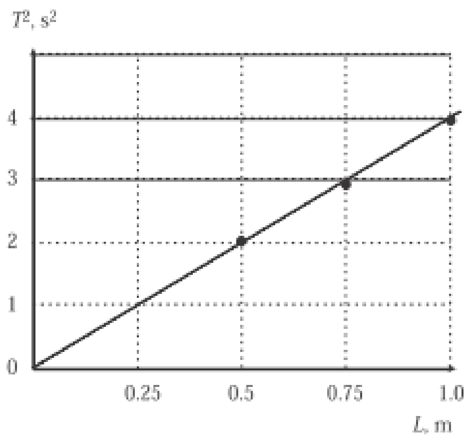

FIG. P15.34

Thus, $g_{\text{ave}} = \boxed{9.85\ \mathrm{m/s^2}}$ this agrees with the accepted value of $g = 9.80\ \mathrm{m/s^2}$ within 0.5%.

(c) From $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph $= \dfrac{4\pi^2}{g} = 4.01\ \mathrm{s^2/m}$.

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = \boxed{9.85\ \mathrm{m/s^2}}$. This is the same as the value in (b).

38. A torsional pendulum is formed by taking a meter stick of mass 2.00 kg, and attaching to its center a wire. With its upper end clamped, the vertical wire supports the stick as the stick turns in a horizontal plane. If the resulting period is 3.00 minutes, what is the torsion constant for the wire?

Solution:

We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}mL^2 = \frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 = 0.167\text{ kg}\cdot\text{m}^2$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2\left(0.167\text{ kg}\cdot\text{m}^2\right)}{(180\text{ s})^2} = \boxed{203\ \mu\text{N}\cdot\text{m}}$$

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39. A clock balance wheel (Fig. P15.39) has a period of oscillation of 0.250 s. The wheel is constructed so that its mass of 20.0 g is concentrated around a rim of radius 0.500 cm. What are (a) the wheel's moment of inertia and (b) the torsion constant of the attached spring?

Solution:

$$T = 0.250\text{ s},\ I = mr^2 = \left(20.0\times10^{-3}\text{ kg}\right)\left(5.00\times10^{-3}\text{ m}\right)^2$$

(a) $I = \boxed{5.00\times10^{-7}\text{ kg}\cdot\text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\ \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = \left(5.00\times10^{-7}\right)\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

FIG. P15.39