PHYS 234
First Tutorial

In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x = (5.00 \text{ cm})\cos(2t + \pi/6)$$

where x is in centimeters and t is in seconds. At $t = 0$, find (a) the position of the piston, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion.

Solution:

(a) $x = (5.00 \text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00 \text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0 \text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) $A = \boxed{5.00 \text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14 \text{ s}}$

The position of a particle is given by the expression $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$, where x is in meters and t is in seconds. Determine (a) the frequency and period of the motion, (b) the amplitude of the motion, (c) the phase constant, and (d) the position of the particle at $t = 0.250$ s.

Solution:

$x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

(a) A hanging spring stretches by 35.0 cm when an object of mass 450 g is hung on it at rest. In this situation, we define its position as $x = 0$. The object is pulled down an additional 18.0 cm and released from rest to oscillate without friction. What is its position x at a time 84.4 s later? (b) **What If?** A hanging spring stretches by 35.5 cm when an object of mass 440 g is hung on it at rest. We define this new position as $x = 0$. This object is also pulled down an additional 18.0 cm and released from rest to oscillate without friction. Find its position 84.4 s later. (c) Why are the answers to (a) and (b) different by such a large percentage when the data are so similar? Does this circumstance reveal a fundamental difficulty in calculating the future? (d) Find the distance traveled by the vibrating object in part (a). (e) Find the distance traveled by the object in part (b).

Solution:

(a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45\ \text{kg}\ 9.8\ \text{m/s}^2}{0.35\ \text{m}} = 12.6\ \text{N/m}$$

we take the x-axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0\ \text{cm}\cos\sqrt{\frac{12.6\ \text{kg}}{0.45\ \text{kg}\cdot\text{s}^2}}84.4\ \text{s} = 18.0\ \text{cm}\cos 446.6\ \text{rad} = \boxed{15.8\ \text{cm}}$$

(d) Now $446.6\ \text{rad} = 71\times 2\pi + 0.497\ \text{rad}$. In each cycle the object moves $4(18) = 72\ \text{cm}$, so it has moved $71(72\ \text{cm}) + (18 - 15.8)\ \text{cm} = \boxed{51.1\ \text{m}}$.

(b) By the same steps, $k = \dfrac{0.44\ \text{kg}\ 9.8\ \text{m/s}^2}{0.355\ \text{m}} = 12.1\ \text{N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0\ \text{cm}\cos\sqrt{\frac{12.1}{0.44}}84.4 = 18.0\ \text{cm}\ \cos 443.5\ \text{rad} = \boxed{-15.9\ \text{cm}}$$

(e) $443.5\ \text{rad} = 70(2\pi) + 3.62\ \text{rad}$

$$\text{Distance moved} = 70(72\ \text{cm}) + 18 + 15.9\ \text{cm} = \boxed{50.7\ \text{m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

A particle moving along the x axis in simple harmonic motion starts from its equilibrium position, the origin, at $t = 0$ and moves to the right. The amplitude of its motion is 2.00 cm, and the frequency is 1.50 Hz. (a) Show that the position of the particle is given by

$$x = (2.00 \text{ cm})\sin(3.00\pi t)$$

Determine (b) the maximum speed and the earliest time ($t > 0$) at which the particle has this speed, (c) the maximum acceleration and the earliest time ($t > 0$) at which the particle has this acceleration, and (d) the total distance traveled between $t = 0$ and $t = 1.00$ s.

Solution:

(a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $\quad v = v_i \cos\omega t$

Since $f = 1.50$ Hz, $\quad \omega = 2\pi f = 3.00\pi$

Also, $A = 2.00$ cm, so that $\quad \boxed{x = (2.00 \text{ cm})\sin 3.00\pi t}$

(b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi \text{ cm/s} = \boxed{18.8 \text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $\quad t = \frac{T}{2} = \boxed{\frac{1}{3} \text{ s}}$

(c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2 \text{ cm/s}^2 = \boxed{178 \text{ cm/s}^2}$

This positive value of acceleration first occurs at $\quad t = \frac{3}{4}T = \boxed{0.500 \text{ s}}$

(d) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(= \frac{3}{2}T\right)$, the particle will travel $\quad 8.00 \text{ cm} + 4.00 \text{ cm} = \boxed{12.0 \text{ cm}}$.

A simple harmonic oscillator takes 12.0 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency in radians per second.

Solution:

(a) $T = \frac{12.0 \text{ s}}{5} = \boxed{2.40 \text{ s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417 \text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62 \text{ rad/s}}$

A 7.00-kg object is hung from the bottom end of a vertical spring fastened to an overhead beam. The object is set into vertical oscillations having a period of 2.60 s. Find the force constant of the spring.

Solution:

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

Solving for k, $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(7.00 \text{ kg})}{(2.60 \text{ s})^2} = \boxed{40.9 \text{ N/m}}$.

A 200-g block is attached to a horizontal spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 2.00 J, find (a) the force constant of the spring and (b) the amplitude of the motion.

Solution:

$m = 200 \text{ g}, T = 0.250 \text{ s}, E = 2.00 \text{ J}; \omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1 \text{ rad/s}$

(a) $k = m\omega^2 = 0.200 \text{ kg}(25.1 \text{ rad/s})^2 = \boxed{126 \text{ N/m}}$

(b) $E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00)}{126}} = \boxed{0.178 \text{ m}}$

A vibration sensor, used in testing a washing machine, consists of a cube of aluminum 1.50 cm on edge mounted on one end of a strip of spring steel (like a hacksaw blade) that lies in a vertical plane. The mass of the strip is small compared to that of the cube, but the length of the strip is large compared to the size of the cube. The other end of the strip is clamped to the frame of the washing machine, which is not operating. A horizontal force of 1.43 N applied to the cube is required to hold it 2.75 cm away from its equilibrium position. If the cube is released, what is its frequency of vibration?

Solution:

The mass of the cube is

$$m = \rho V = \left(2.7 \times 10^{3}\ \text{kg/m}^{3}\right)(0.015\ \text{m})^{3} = 9.11 \times 10^{-3}\ \text{kg}$$

The spring constant of the strip of steel is

$$k = \frac{F}{x} = \frac{14.3\ \text{N}}{0.027\,5\ \text{m}} = 52.0\ \text{N/m}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{52\ \text{kg}}{\text{s}^{2} 9.11 \times 10^{-3}\ \text{kg}}} = \boxed{12.0\ \text{Hz}}$$

A particle that hangs from a spring oscillates with an angular frequency ω. The spring is suspended from the ceiling of an elevator car and hangs motionless (relative to the elevator car) as the car descends at a constant speed v. The car then stops suddenly. (a) With what amplitude does the particle oscillate? (b) What is the equation of motion for the particle? (Choose the upward direction to be positive.)

Solution:

(a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \boxed{\frac{v}{\omega}}$$

(b) $x = -A \sin \omega t = \boxed{-\left(\frac{v}{\omega}\right)\sin \omega t}$

A 0.500-kg object attached to a spring with a force constant of 8.00 N/m vibrates in simple harmonic motion with an amplitude of 10.0 cm. Calculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the object is 6.00 cm from the equilibrium position, and (c) the time interval required for the object to move from $x = 0$ to $x = 8.00$ cm.

Solution:

(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8.00\ \text{N/m}}{0.500\ \text{kg}}} = 4.00\ \text{s}^{-1}$ so position is given by $x = 10.0\sin(4.00t)\ \text{cm}$.

From this we find that $v = 40.0\cos(4.00t)\ \text{cm/s}$ $v_{max} = \boxed{40.0\ \text{cm/s}}$

$a = -160\sin(4.00t)\ \text{cm/s}^2$ $a_{max} = \boxed{160\ \text{cm/s}^2}$.

(b) $t = \left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$ and when $x = 6.00\ \text{cm}$, $t = 0.161\ \text{s}$.

We find $v = 40.0\cos[4.00(0.161)] = \boxed{32.0\ \text{cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\ \text{cm/s}^2}$.

(c) Using $t = \left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00\ \text{cm}$, $t = 0.232\ \text{s}$.

Therefore, $\Delta t = \boxed{0.232\ \text{s}}$.

A 1.00-kg glider attached to a spring with a force constant of 25.0 N/m oscillates on a horizontal, frictionless air track. At $t = 0$ the glider is released from rest at $x = -3.00$ cm. (That is, the spring is compressed by 3.00 cm.) Find (a) the period of its motion, (b) the maximum values of its speed and acceleration, and (c) the position, velocity, and acceleration as functions of time.

Solution:

$m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm. At $t = 0$, $x = -3.00$ cm

(a) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00 \text{ rad/s}$$

so that, $$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = \boxed{1.26 \text{ s}}$$

(b) $$v_{max} = A\omega = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s}) = \boxed{0.150 \text{ m/s}}$$

$$a_{max} = A\omega^2 = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s})^2 = \boxed{0.750 \text{ m/s}^2}$$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $$\boxed{x = -3.00\cos(5.00t) \text{ cm}}$$

$$v = \frac{dx}{dt} = \boxed{15.0\sin(5.00t) \text{ cm/s}}$$

$$a = \frac{dv}{dt} = \boxed{75.0\cos(5.00t) \text{ cm/s}^2}$$

A block–spring system oscillates with an amplitude of 3.50 cm. If the spring constant is 250 N/m and the mass of the block is 0.500 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the block, and (c) the maximum acceleration.

Solution:

(a) $$E = \frac{kA^2}{2} = \frac{250 \text{ N/m}\left(3.50 \times 10^{-2} \text{ m}\right)^2}{2} = \boxed{0.153 \text{ J}}$$

(b) $v_{max} = A\omega$ where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.500}} = 22.4 \text{ s}^{-1}$ $\quad v_{max} = \boxed{0.784 \text{ m/s}}$

(c) $$a_{max} = A\omega^2 = 3.50 \times 10^{-2} \text{ m}\left(22.4 \text{ s}^{-1}\right)^2 = \boxed{17.5 \text{ m/s}^2}$$

A 2.00-kg object is attached to a spring and placed on a horizontal, smooth surface. A horizontal force of 20.0 N is required to hold the object at rest when it is pulled 0.200 m from its equilibrium position (the origin of the x axis). The object is now released from rest with an initial position of $x_i = 0.200$ m, and it subsequently undergoes simple harmonic oscillations. Find (a) the force constant of the spring, (b) the frequency of the oscillations, and (c) the maximum speed of the object. Where does this maximum speed occur? (d) Find the maximum acceleration of the object. Where does it occur? (e) Find the total energy of the oscillating system. Find (f) the speed and (g) the acceleration of the object when its position is equal to one third of the maximum value.

Solution:

(a) $$k = \frac{|F|}{x} = \frac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$$

(b) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\text{ rad/s} \quad \text{so} \quad f = \frac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$$

(c) $$v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}} \text{ at } x = 0$$

(d) $$a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2} \text{ at } x = \pm A$$

(e) $$E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$$

(f) $$|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$$

(g) $$|a| = \omega^2 x = 50.0\left(\frac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$$