King Saud University

Part I: Electricity

Chapter 28

Direct Current Circuits

Dr. Saif M. H. Qaid

LECTURE OUTLINE

- **28.2** Resistors in Series and Parallel
- **28.3** Kirchhoff’s Rules

Circuit Analysis

- Simple electric circuits may contain batteries, resistors, and capacitors in various combinations.
- For some circuits, analysis may consist of combining resistors.
- In more complex complicated circuits, Kirchhoff's Rules may be used for analysis.
  - These Rules are based on conservation of energy and conservation of electric charge for isolated systems.
- Circuits may involve direct current or alternating current.

Direct Current

- When the current in a circuit has a constant direction, the current is called ***direct current.***
  - Most of the circuits analyzed will be assumed to be in *steady state*, with constant magnitude and direction.
- Because the potential difference between the terminals of a battery is constant, the battery produces direct current.
- The battery is known as a source of emf.

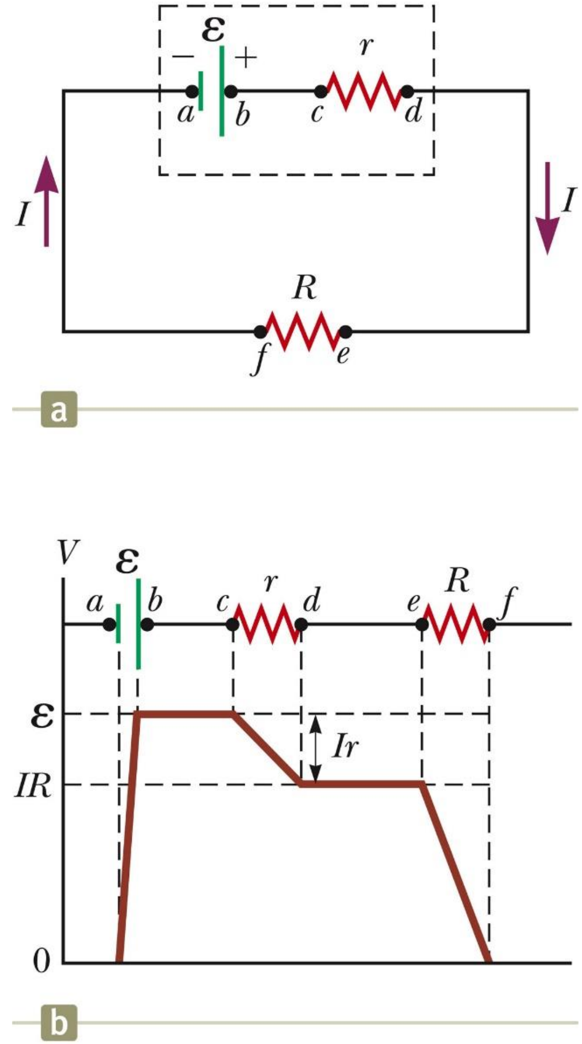
Electromotive Force

- The electromotive force (emf), ε, of a battery is the maximum possible voltage that the battery can provide between its terminals.
  - The emf supplies energy, it does not apply a force.
- The battery will normally be the source of energy in the circuit.
- The positive terminal of the battery is at a higher potential than the negative terminal.
- We consider the wires to have no resistance.

Section 28.1

Internal Battery Resistance

- If the internal resistance is zero, the terminal voltage equals the emf.
- In a real battery, there is internal resistance, r.
- The terminal voltage, $\Delta V = \varepsilon - Ir$
- The emf is equivalent to the *open-circuit* voltage.
  - This is the terminal voltage when no current is in the circuit.
  - This is the voltage labeled on the battery.
- The actual potential difference between the terminals of the battery depends on the current in the circuit.



Section 28.1

Load Resistance

- The terminal voltage also equals the voltage across the external resistance.
  - This external resistor is called the *load resistance.*
  - In the previous circuit, the load resistance is just the external resistor.
  - In general, the load resistance could be any electrical device.
    - These resistances represent *loads* on the battery since it supplies the energy to operate the device containing the resistance.

Power

- The total power output of the battery is

$$P = I\,\Delta V = I\,\varepsilon$$

- This power is delivered to the external resistor ($I^2 R$) and to the internal resistor ($I^2 r$).
- $$P = I^2 R + I^2 r$$
- The battery is a supply of constant emf.
  - The battery does not supply a constant current since the current in the circuit depends on the resistance connected to the battery.
  - The battery does not supply a constant terminal voltage.

Section 28.1

Electromotive Force

Ideally, such a source would have a constant potential difference, $\mathcal{E}$, between its terminals regardless of current

Real sources of emf have an internal resistance which has to be taken into account

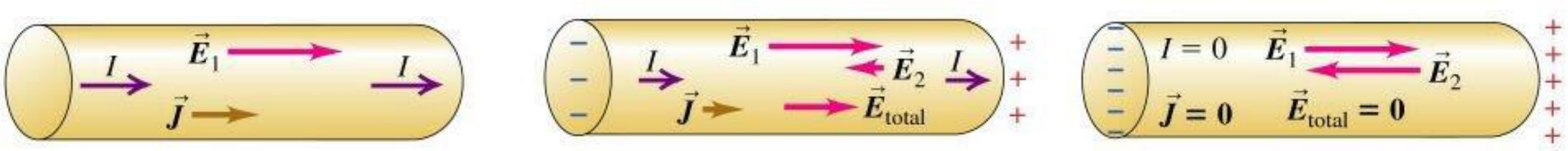
The potential difference across the terminals of the source is then given by

$$V_{ab} = \varepsilon - I\, r_{\text{internal}}$$

Electromotive Force

A steady current will exist in a conductor only if it is part of a complete circuit

For an isolated conductor that has an external field impressed on it



Electric field $\vec{E}_1$ produced inside conductor causes current

Current causes charge to build up at ends, producing opposing field $\vec{E}_2$ and reducing current

After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$: total field $\vec{E}_{total} = 0$ and current stops completely

Electromotive Force

To maintain a steady current in an external circuit we require the use of a source that supplies electrical energy

Whereas in the external circuit the current flows from higher potential to lower potential, in this source the current must flow from lower potential to higher potential, even though the electrostatic force within the source is in fact trying to do the opposite

In order to do this we must have an *electromotive force, emf,* within such a source

The unit for emf is also Volt

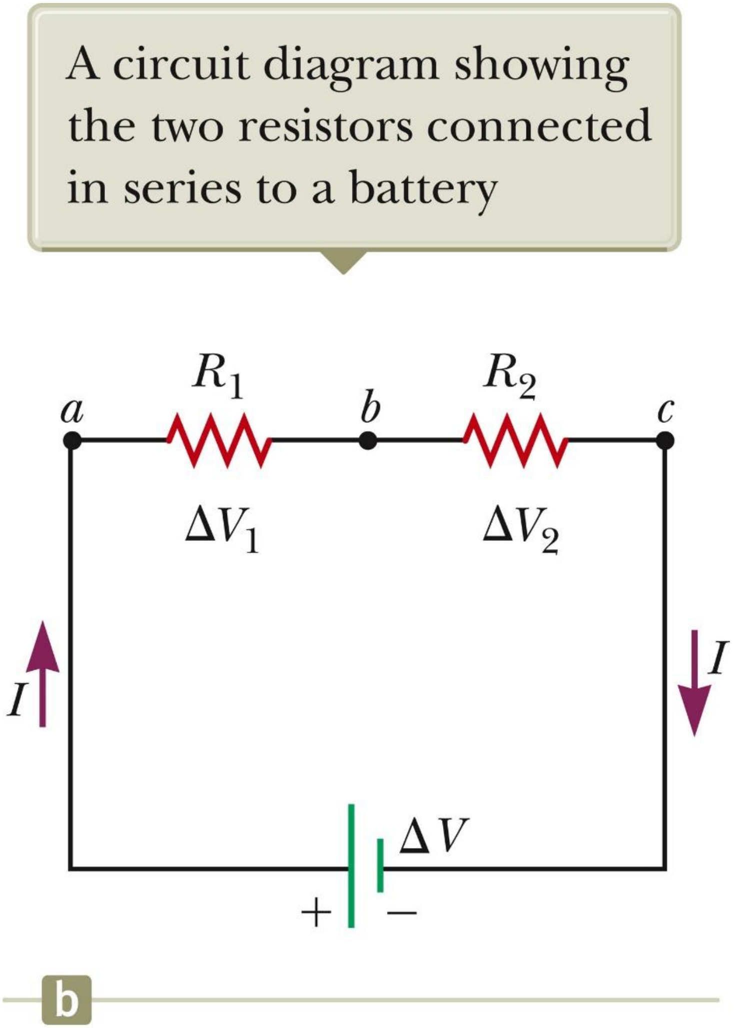
Resistors in Series

- When two or more resistors are connected end-to-end, they are said to be in series.
- For a series combination of resistors, the currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval.
- The potential difference will divide among the resistors such that the sum of the potential differences across the resistors is equal to the total potential difference across the combination.

Section 28.2

Resistors in Series, cont

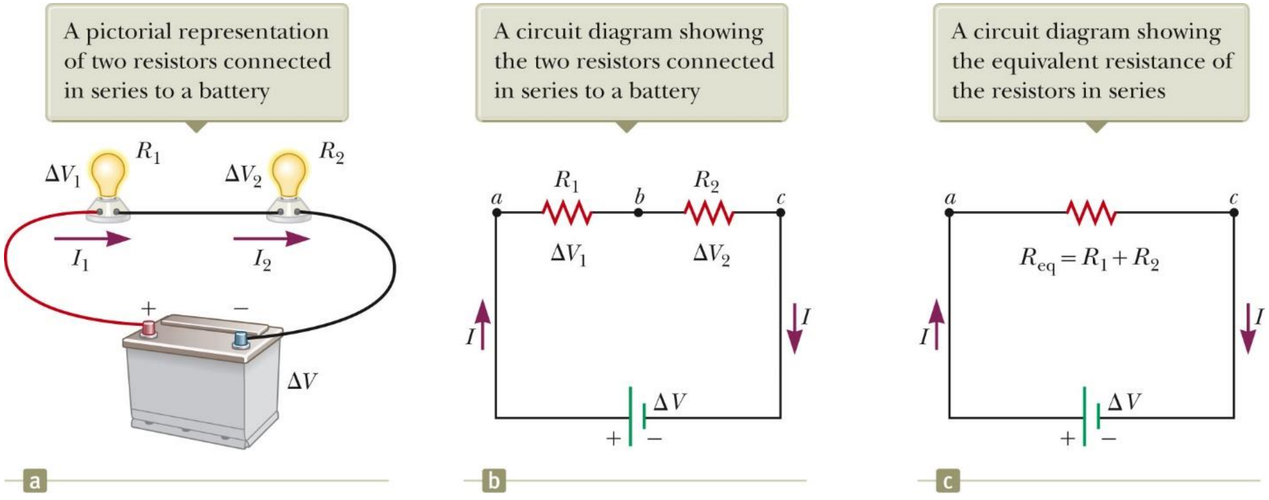
- Currents are the same
  - $I = I_1 = I_2$
- Potentials add
  - $\Delta V = V_1 + V_2 = IR_1 + IR_2$
    $= I(R_1 + R_2)$
  - Consequence of Conservation of Energy
- The equivalent resistance has the same effect on the circuit as the original combination of resistors.



Equivalent Resistance – Series

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance.
- If one device in the series circuit creates an open circuit, all devices are inoperative.

Equivalent Resistance – Series – An Example



- All three representations are equivalent.
- Two resistors are replaced with their equivalent resistance.

Some Circuit Notes

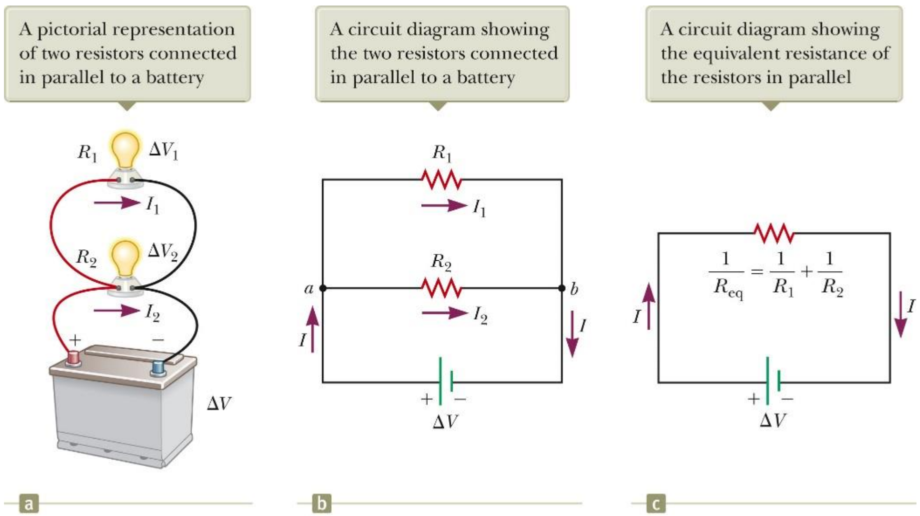
- A local change in one part of a circuit may result in a global change throughout the circuit.
  - For example, changing one resistor will affect the currents and voltages in all the other resistors and the terminal voltage of the battery.
- In a series circuit, there is one path for the current to take.
- In a parallel circuit, there are multiple paths for the current to take.

Section 28.2

Resistors in Parallel

- The potential difference across each resistor is the same because each is connected directly across the battery terminals.
- $\Delta V = \Delta V_1 = \Delta V_2$
- A **junction** is a point where the current can split.
- The current, I, that enters junction must be equal to the total current leaving that junction.
  - $I = I_1 + I_2 = (\Delta V_1 / R_1) + (\Delta V_2 / R_2)$
  - The currents are generally not the same.
  - Consequence of conservation of electric charge

Equivalent Resistance – Parallel, Examples



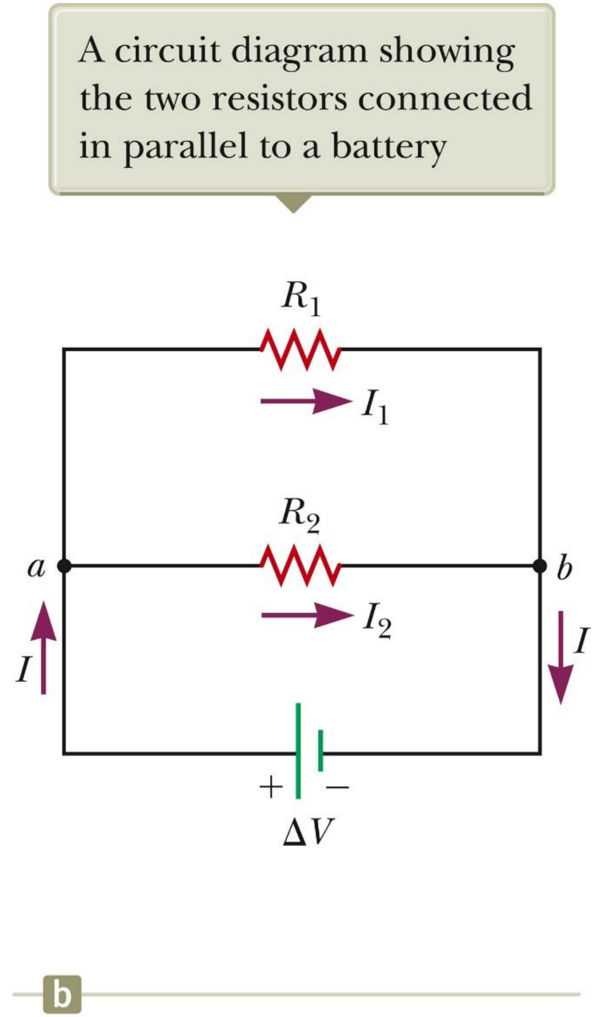
- All three diagrams are equivalent.
- Equivalent resistance replaces the two original resistances.

Equivalent Resistance – Parallel

- Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.
  - The equivalent is always less than the smallest resistor in the group.

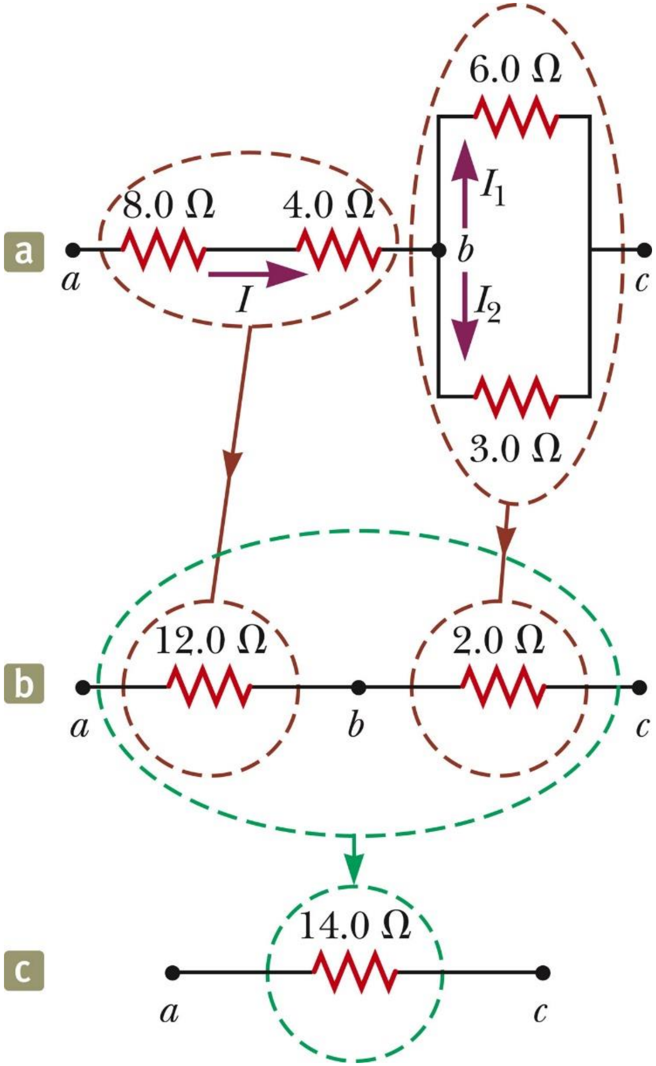


Resistors in Parallel, Final

- In parallel, each device operates independently of the others so that if one is switched off, the others remain on.
- In parallel, all of the devices operate on the same voltage.
- The current takes all the paths.
  - The lower resistance will have higher currents.
  - Even very high resistances will have some currents.
- *Household circuits* are wired so that electrical devices are connected in parallel.

Combinations of Resistors

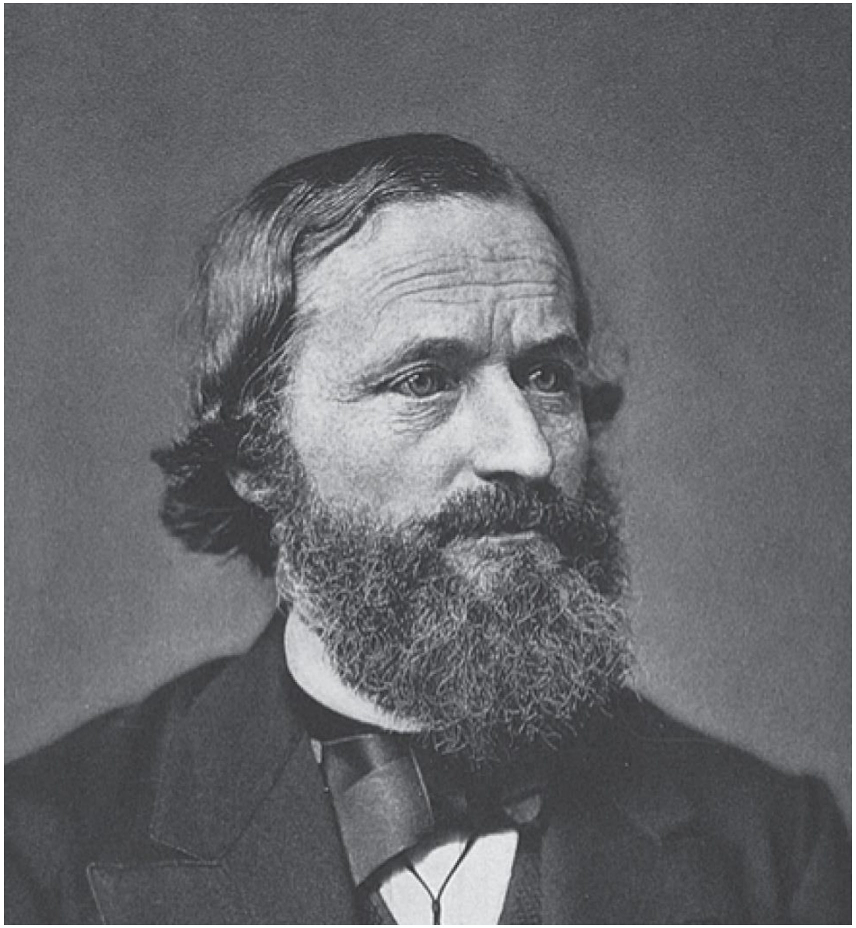
- The 8.0-Ω and 4.0-Ω resistors are in series and can be replaced with their equivalent, 12.0 Ω.
- The 6.0-Ω and 3.0-Ω resistors are in parallel and can be replaced with their equivalent, 2.0 Ω.
- These equivalent resistances are in series and can be replaced with their equivalent resistance, 14.0 Ω.



Section 28.2

Gustav Kirchhoff

- 1824 – 1887
- German physicist
- Worked with Robert Bunsen
- Kirchhoff and Bunsen
  - Invented the spectroscope and founded the science of spectroscopy
  - Discovered the elements cesium and rubidium
  - Invented astronomical spectroscopy



Kirchhoff's Rules

- There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.
- Two rules, called **Kirchhoff's rules**, can be used instead.

Section 28.3

Kirchhoff's Junction Rule

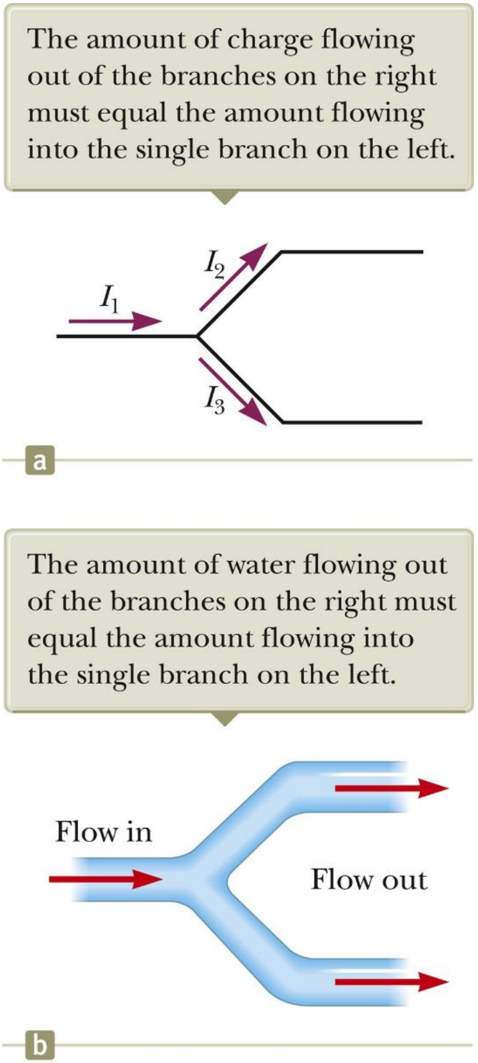
- Junction Rule
  - The sum of the currents at any junction must equal zero.
    - Currents directed into the junction are entered into the equation as +I and those leaving as -I.

$$\sum_{junction} I = 0$$

    - A statement of Conservation of Charge
  - Mathematically,

More about the Junction Rule

- $I_1 - I_2 - I_3 = 0$
- Required by Conservation of Charge
- Diagram (b) shows a mechanical analog



Kirchhoff's Loop Rule

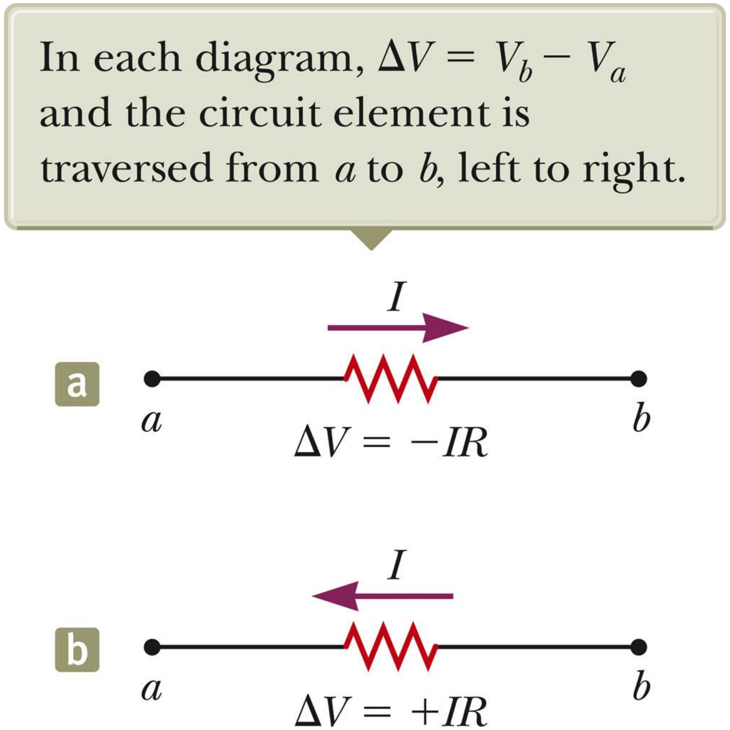
- Loop Rule
  - The sum of the potential differences across all elements around any closed circuit loop must be zero.

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$

    - A statement of Conservation of Energy
- Mathematically,

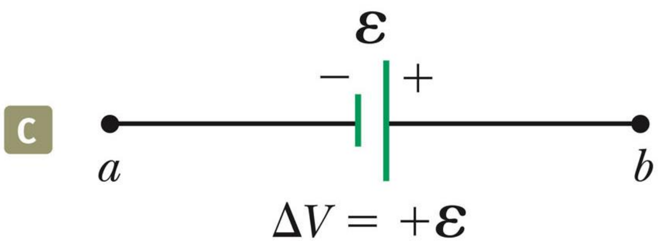
More about the Loop Rule

- Traveling around the loop from *a* to *b*
- In (a), the resistor is traversed in the direction of the current, the potential across the resistor is – *IR.*
- In (b), the resistor is traversed in the direction opposite of the current, the potential across the resistor is is + *IR.*

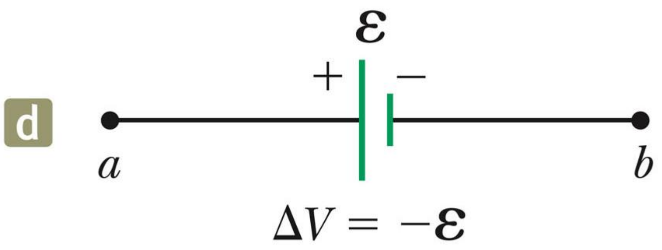


Loop Rule, final

- In (c), the source of emf is traversed in the direction of the emf (from – to +), and the change in the potential difference is +*ε*.



- In (d), the source of emf is traversed in the direction opposite of the emf (from + to -), and the change in the potential difference is -*ε*.



Equations from Kirchhoff's Rules

- Use the junction rule as often as needed, so long as each time you write an equation, you include in it a current that has not been used in a previous junction rule equation.
  - In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.
- The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation.
- In order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.
- Any capacitor acts as an open branch in a circuit.
  - The current in the branch containing the capacitor is zero under steady-state conditions.

Problem-Solving Strategy – Kirchhoff's Rules

- *Conceptualize*
  - Study the circuit diagram and identify all the elements.
  - Identify the polarity of each battery.
  - Imagine the directions of the currents in each battery.
- *Categorize*
  - Determine if the circuit can be reduced by combining series and parallel resistors.
    - If so, proceed with those techniques
    - If not, apply Kirchhoff's Rules

Section 28.3

Problem-Solving Strategy, cont.

- *Analyze*
  - Assign labels and symbols to all known and unknown quantities.
  - Assign directions to the currents.
    - The direction is arbitrary, but you must adhere to the assigned directions when applying Kirchhoff's rules.
  - Apply the junction rule to any junction in the circuit that provides new relationships among the various currents.
  - Apply the loop rule to as many loops as are needed to solve for the unknowns.
    - To apply the loop rule, you must choose a direction in which to travel around the loop.
    - You must also correctly identify the potential difference as you cross various elements.
  - Solve the equations simultaneously for the unknown quantities.

Section 28.3

Problem-Solving Strategy, final

- *Finalize*
  - Check your numerical answers for consistency.
  - If any current value is negative, it means you guessed the direction of that current incorrectly.
    - The magnitude will still be correct.

Summary الخلاصة

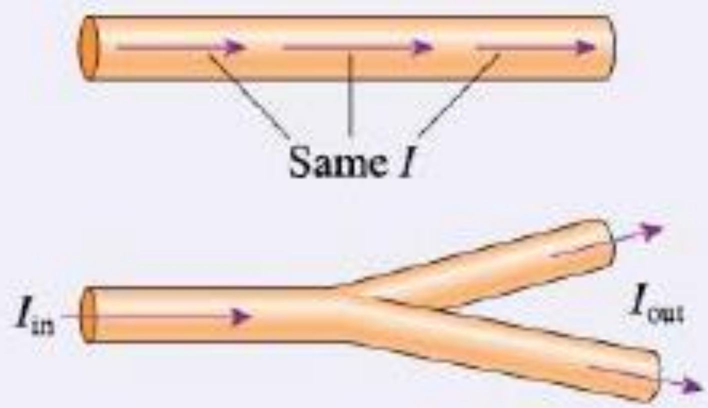
The

Conservation of Charge

The current is the same at any two points in a wire.
At a junction,

$$\sum I_{in} = \sum I_{out}$$

This is **Kirchhoff's junction law.**



Thank You

ACKNOWLEDGEMENTS