Part I: Electricity

Chapter 27

Current and Resistance

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LECTURE OUTLINE

- **27.1** Electric Current
- **27.2** Resistance
- **27.4** Resistance and Temperature
- **27.6** Electrical Power

Electric Current

- Most practical applications of electricity deal with electric currents.
  - The electric charges move through some region of space.
- The *resistor* is a new element added to circuits.
- Energy can be transferred to a device in an electric circuit.
- The energy transfer mechanism is electrical transmission, $T_{ET.}$

Electric Current

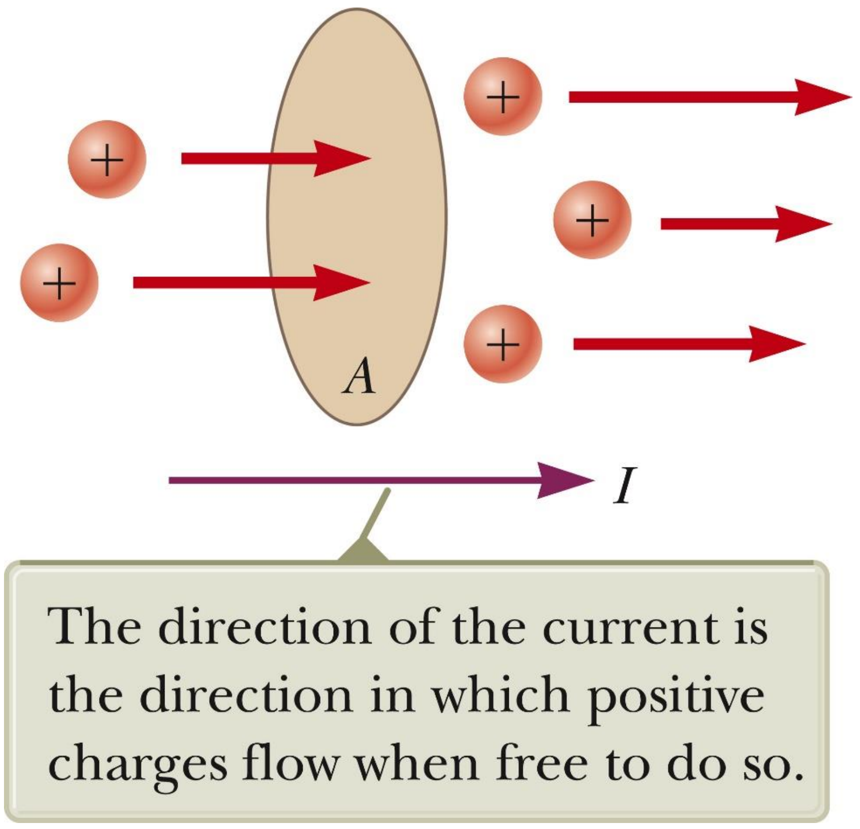
- **Electric current** is the rate of flow of charge through some region of space.
- The SI unit of current is the **ampere** (A).
  - 1 A = 1 C / s
- The symbol for electric current is I.

Current is the motion of any charge, positive or negative, from one point to another

Average Electric Current

- Assume charges are moving perpendicular to a surface of area *A.*
- If ΔQ is the amount of charge that passes through *A* in time Δt, then the average current is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



Instantaneous Electric Current

- If the rate at which the charge flows varies with time, the instantaneous current, I, is defined as the differential limit of average current as $\Delta t \to 0$.

$$I \equiv \frac{dQ}{dt}$$

Direction of Current

- The charged particles passing through the surface could be positive, negative or both.
- It is conventional to assign to the current the same direction as the flow of positive charges.
- In an ordinary conductor, the direction of current flow is opposite the direction of the flow of electrons.
- It is common to refer to any moving charge as a *charge carrier.*

Drift Velocity

- Assume that an external electric field E has been established within a conductor

Then any free charged particle in the conductor will experience a force given by

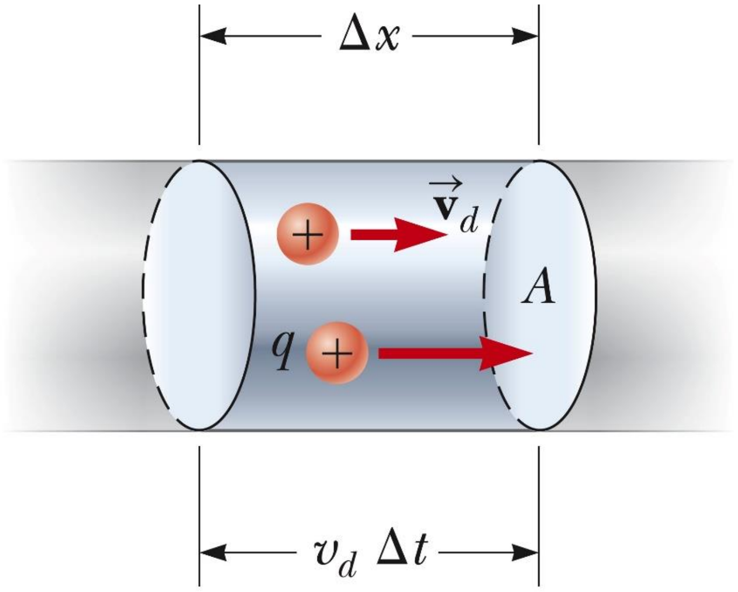
$$\vec{F} = q\vec{E}$$

The charged particle will experience frequent collisions, into random directions, with the particles compromising the bulk of the material

There will however be a net overall motion

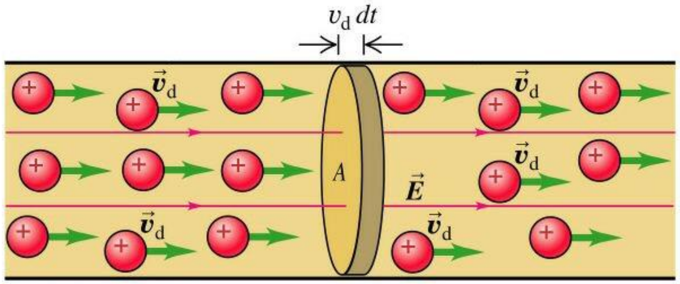
Current and Drift Speed

- Charged particles move through a cylindrical conductor of cross-sectional area *A.*
- *n* is the number of mobile charge carriers per unit volume.
- $nA\Delta x$ is the total number of charge carriers in a segment.



Current and Drift Speed, cont

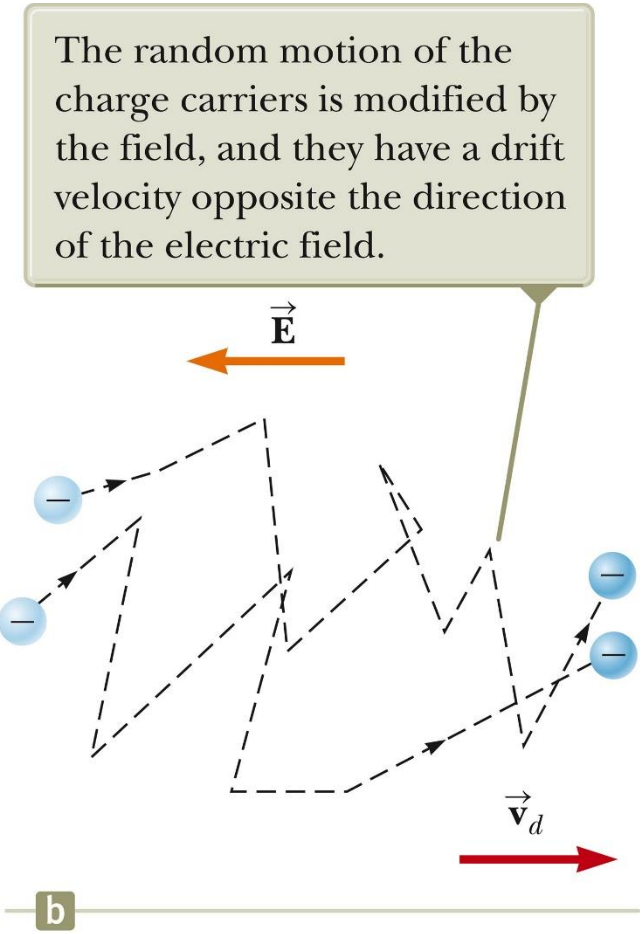
- The total charge is the number of carriers times the charge per carrier, *q.*
  - $\Delta Q = (nA\Delta x)q$
- Assume the carriers move with a velocity parallel to the axis of the cylinder such that they experience a displacement in the x-direction.
- If v_d is the speed at which the carriers move, then
  - $v_d = \Delta x / \Delta t$ and $\Delta x = v_d \Delta t$
- Rewritten: $\Delta Q = (nAv_d\Delta t)q$
- Finally, current, $I_{\text{ave}} = \Delta Q/\Delta t = nqv_dA$
- v_d is an average speed called the **drift speed.**



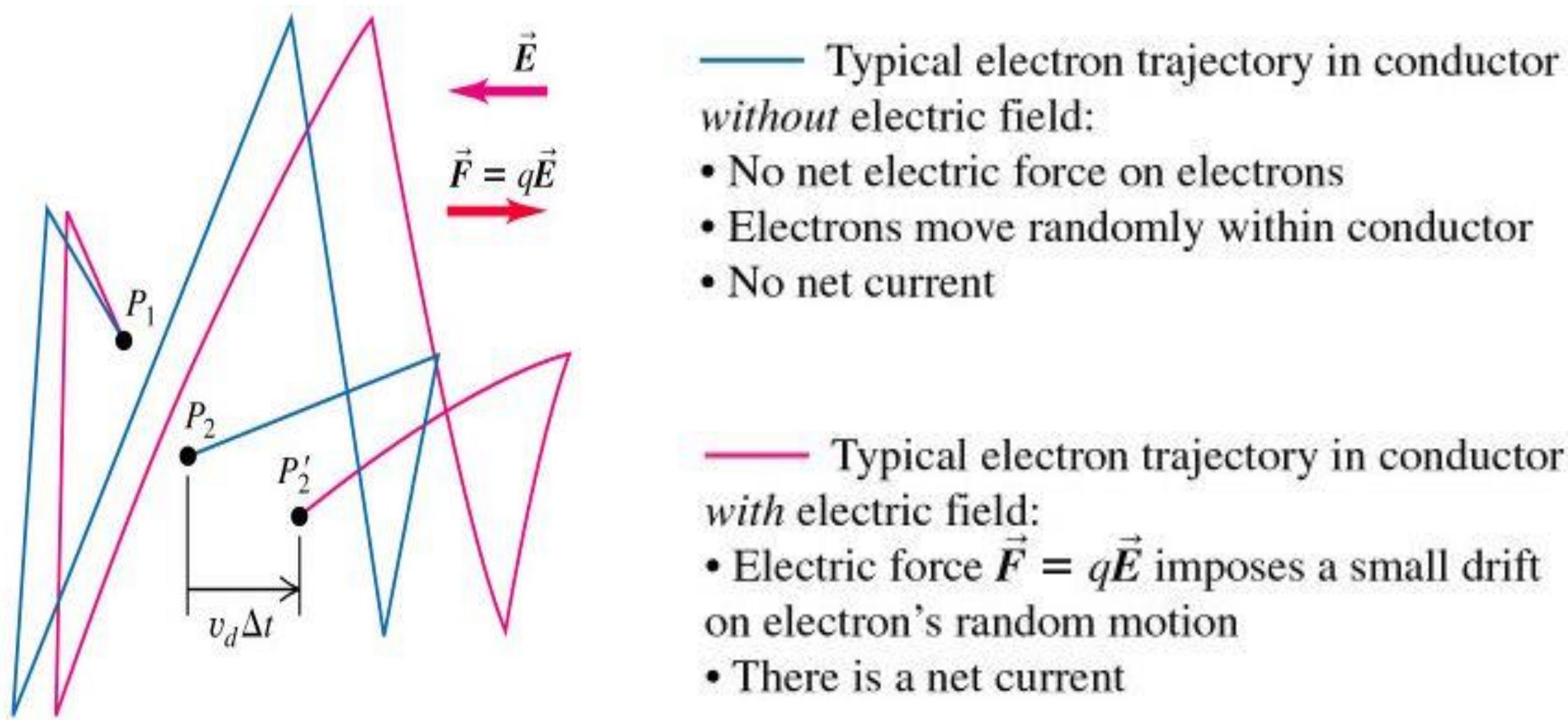
Section 27.1

Charge Carrier Motion in a Conductor

- When a potential difference is applied across the conductor, an electric field is set up in the conductor which exerts an electric force on the electrons.
- The motion of the electrons is no longer random.
- The zigzag black lines represents the motion of a charge carrier in a conductor in the presence of an electric field.
  - The net drift speed is small.
- The sharp changes in direction are due to collisions.
- The net motion of electrons is opposite the direction of the electric field.



Drift Velocity



There is net displacement given by $v_d \Delta t$ where v_d is known as the *drift velocity*

Example Nerve Conduction

- Suppose a large nerve fiber running to a muscle in the leg has a diameter of 0.25 mm.
- When the current in the nerve is 0.05 mA, the drift velocity is 2.0×10^{-6} m/s.
- If we model this problem by assuming free electrons are the charge carriers, what is the density of the free electrons in the nerve fiber?

Solution

- We first calculate the cross-sectional area of the nerve fiber.

- The current density is then:

- We can now calculate the density of the free electrons.

Motion of Charge Carriers, cont.

- In the presence of an electric field, in spite of all the collisions, the charge carriers slowly move along the conductor with a drift velocity, $\vec{\mathbf{v}}_d$

- The electric field exerts forces on the conduction electrons in the wire.
- These forces cause the electrons to move in the wire and create a current.

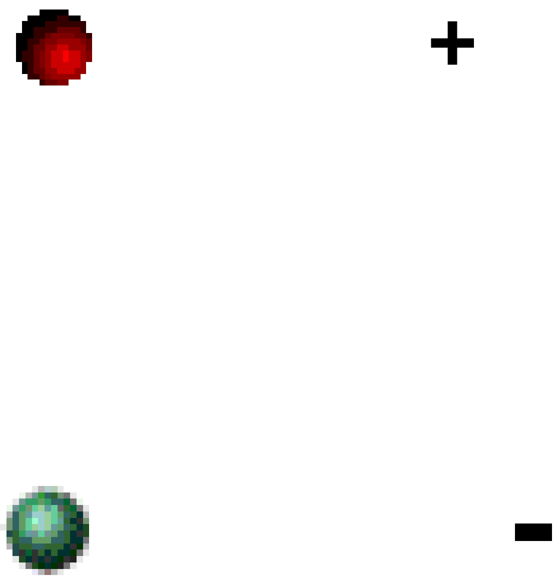
Motion of Charge Carriers, final

- The electrons are already in the wire.
- They respond to the electric field set up by the battery.
- The battery does not supply the electrons, it only establishes the electric field.
- A battery provides a potential energy difference (voltage source).

Remember: Electric Potential Energy Difference-Two Unlike Charges

Higher Potential Energy

Lower Potential Energy



- To cause movement of a charge, there must be a potential difference.

- Direct Current
- DC
- Provided by batteries

- Alternating Current
- AC
- Provided by power companies

Drift Velocity, Example

- Assume a copper wire, with one free electron per atom contributed to the current.
- The drift velocity for a 12-gauge copper wire carrying a current of 10.0 A is

  2.23×10^{-4} m/s

  - This is a typical order of magnitude for drift velocities.

Question:

- If the drift velocity is about 0.01cm/s, why do the lights turn on instantaneously when the circuit switch is closed?

- What is required in order to have an electric current flow in a circuit?

What occurs in a wire when the circuit switch is closed?

- An **electric field is established instantaneously** (at almost the speed of light, $3x10^8$ m/s).

- Free electrons, while still randomly moving, immediately begin drifting due to the electric field, resulting in a net flow of charge.

- Average **drift velocity** is about 0.01cm/s.

Question: Why is the bird on the wire safe?

Question: Why do electricians work with one hand behind their back?

Current Density

- J is the **current density** of a conductor.
- It is defined as the current per unit area.
  - $J \equiv I / A = nq\mathbf{v}_d$
  - This expression is valid only if the current density is uniform and A is perpendicular to the direction of the current.
- J has SI units of A/m^2
- The current density is in the direction of the positive charge carriers.

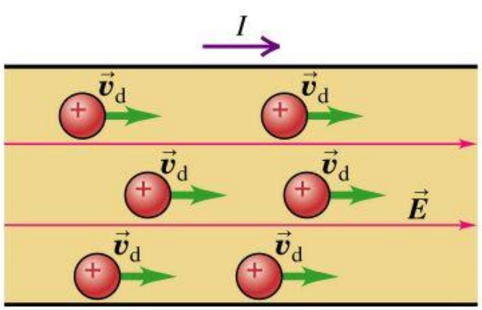
Section 27.2

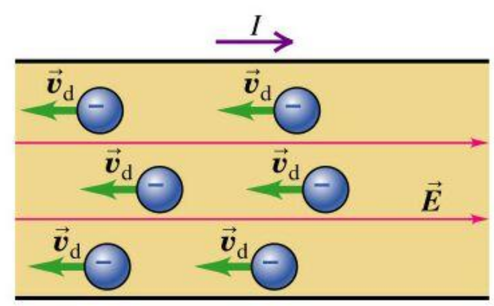
Current Density

Current density can also be defined to be a vector

$$\vec{J} = nq\vec{v}_d$$

Note that this vector definition gives the same direction for the current density whether we are using the positive or negative charges as the current carrier





Example

- The Los Alamos Meson Physics Facility accelerator has a maximum average proton current of 1.0 mA at an energy of 800 Mev.

Example cont.

- a) How many protons per second strike a target exposed to this beam if the beam is of circular cross section with a diameter of 5 mm?

- b) What is the current density?

Solution

- a) The number of protons per second is:

- Here n is the number of protons per second and e is the charge of the proton.

- b) The magnitude of the current density for this problem is just the current divided by the cross sectional area.

Conductivity

- A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.
- For some materials, the current density is directly proportional to the field.
- The constant of proportionality, σ, is called the **conductivity** of the conductor.

Ohm's Law

- **Ohm's law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.
  - Most metals obey Ohm's law
  - Mathematically, $J = \sigma E$
  - Materials that obey Ohm's law are said to be *ohmic*
  - Not all materials follow Ohm's law
    - Materials that do not obey Ohm's law are said to be *nonohmic.*
- Ohm's law is not a fundamental law of nature.
- Ohm's law is an empirical relationship valid only for certain materials.

Section 27.2

Georg Simon Ohm

- 1789 -1854
- German physicist
- Formulated idea of resistance
- Discovered the proportionalities now known as forms of Ohm's Law

Section 27.2

Ohm's Law

Let us take a length of conductor having a certain resistivity

We have that $\vec{E} = \rho \vec{J}$

But *E* and the length of the wire, L, are related to potential difference across the wire by $V = E\,L$

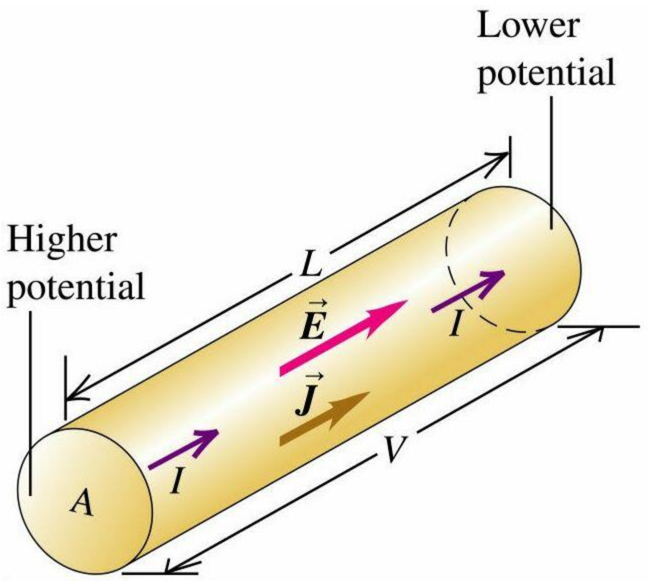
We also have that $J = \dfrac{I}{A}$

Putting this all together, we then have

$$\frac{V}{L} = \frac{\rho I}{A}$$

or

$$V = \frac{\rho L}{A} I$$



Ohm's Law

We take the last equation $$V = \frac{\rho L}{A} I$$

and rewrite it as $$V = I R$$

with $$R = \frac{\rho L}{A}$$ being the resistance

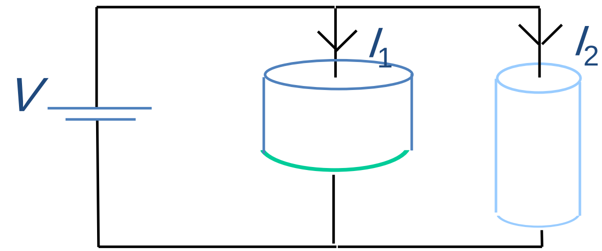
The resistance is proportional to the length of the material and inversely proportional to cross sectional area

$$V = I R$$ is often referred to as Ohm's Law

The unit for R is the *ohm* or *Volt / Ampere*

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:



What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

(a) $I_1 < I_2$ (b) $I_1 = I_2$ (c) $I_1 > I_2$

The resistivity of both resistors is the same (r). Therefore the resistances are related as:

$$R_2 = \rho\frac{L_2}{A_2} = \rho\frac{2L_1}{(A_1/4)} = 8\rho\frac{L_1}{A_1} = 8R_1$$

The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8}I_1$$

Resistance

- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.
- The constant of proportionality is called the **resistance** of the conductor.

$$R \equiv \frac{\Delta V}{I}$$

- SI units of resistance are *ohms* (Ω).
  - $1\ \Omega = 1\ V / A$
- Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.

Section 27.2

Resistance and Current

- If the resistance in a circuit increases, the current will decrease.
- If the resistance in a circuit decreases, the current will increase.
- This is an ***inversely proportional*** relationship.

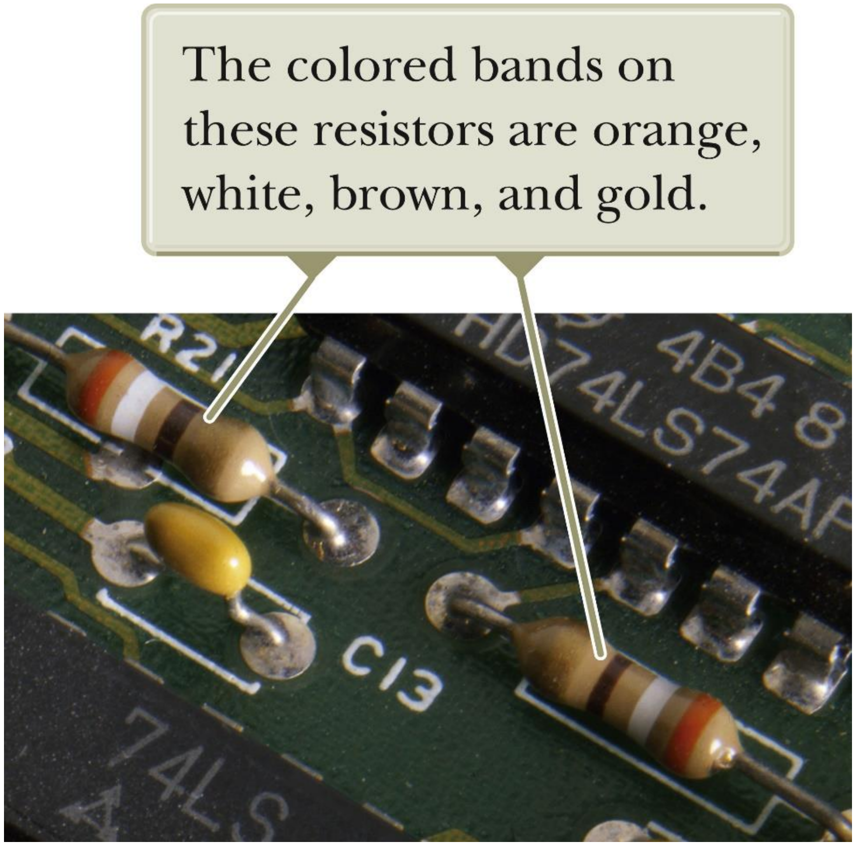
Resistance

- The resistance of a circuit is defined as the potential drop across the circuit divided by the current that pass through the circuit.
- The unit for resistance is the ohm $\Omega = 1$V/A.

- The relationship between resistance and resistivity is:

Resistors

- Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit.
- Stand-alone resistors are widely used.
  - Resistors can be built into integrated circuit chips.
- Values of resistors are normally indicated by colored bands.
  - The first two bands give the first two digits in the resistance value.
  - The third band represents the power of ten for the multiplier band.
  - The last band is the tolerance.



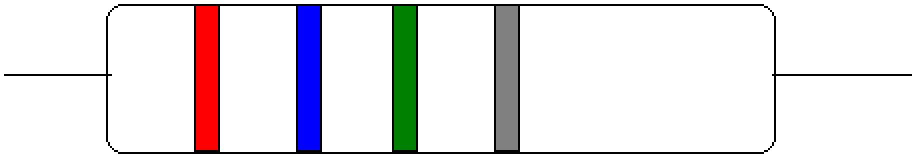
Resistor Color Codes

TABLE 27.1 *Color Coding for Resistors*

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

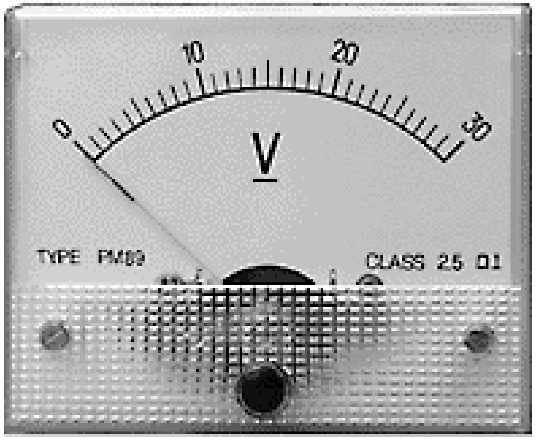
Section 27.2

Resistor Color Code Example



- Red (=2) and blue (=6) give the first two digits: 26
- Green (=5) gives the power of ten in the multiplier: 10^5
- The value of the resistor then is $26 \times 10^5\ \Omega$ (or $2.6\ M\Omega$)
- The tolerance is 10% (silver = 10%) or $2.6 \times 10^5\ \Omega$

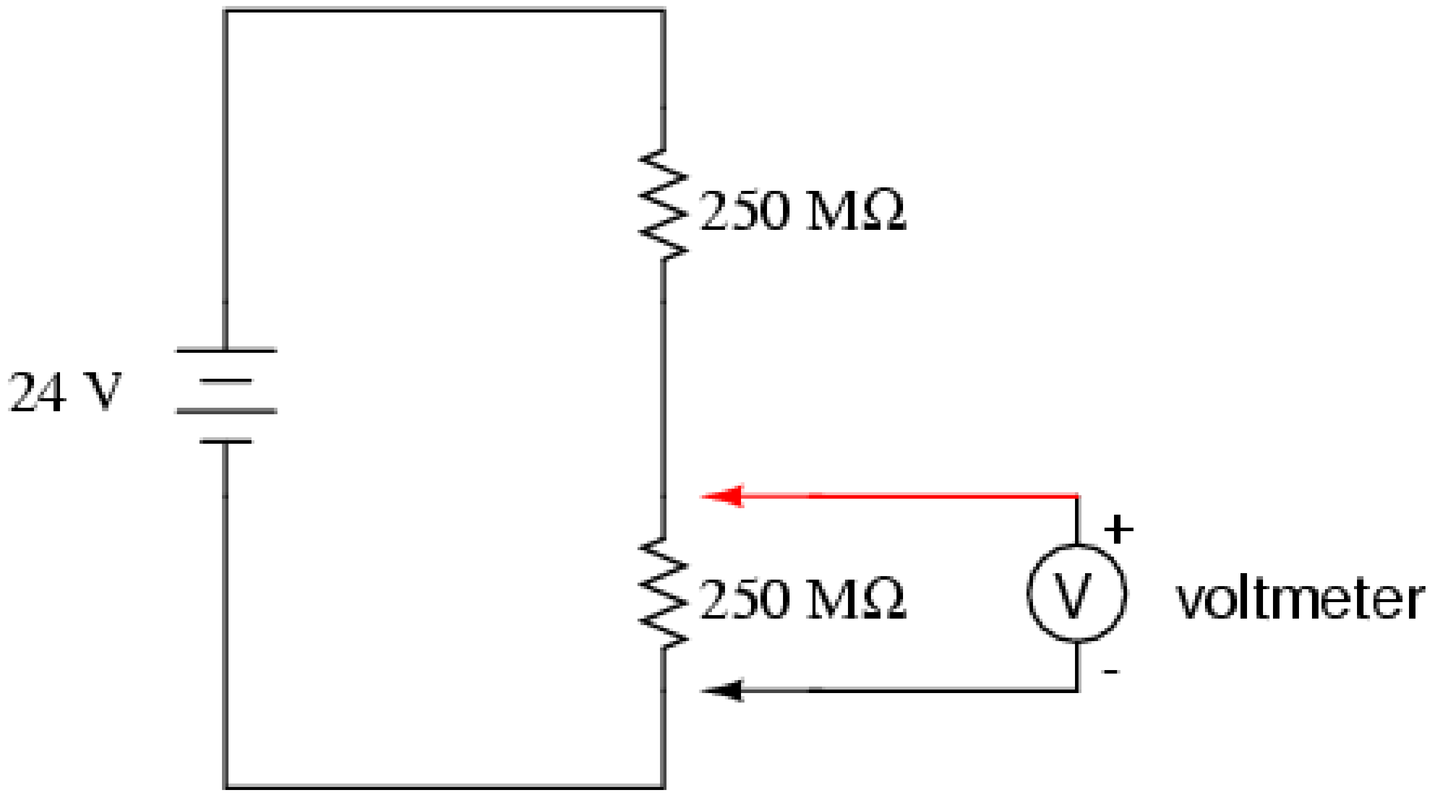
Voltmeter



Measures the voltage between two points in an electric circuit.

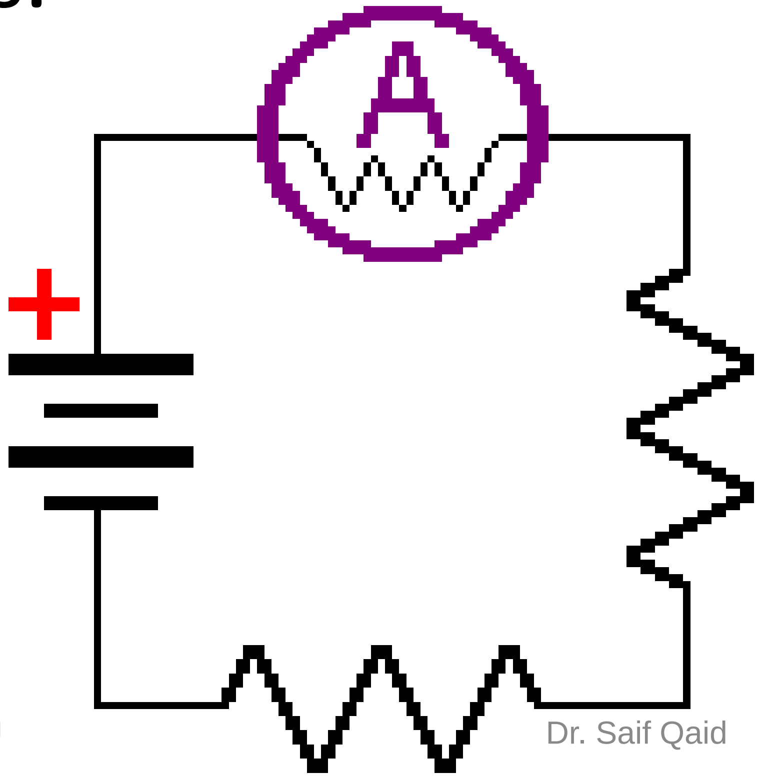
- Must be connected in parallel.

A voltmeter is connected in parallel.



Ammeter

- Measures electric current.
- Must be placed in series.



Example:

- Calculate the current through a 3 Ω resistor when a voltage of 12V is applied across it.

- Answer: 4 A

Example:

- A 6 Ω resistor has a power source of 20V across it. What will happen to the resistance if the voltage doubles?

Ohm’s Law

Voltage is equal to the current multiplied by the resistance.

Voltage, measured in Volts, *V*

Current, measured in Amps, ***A***

$$V=IR$$

Resistance, measured in Ohms, Ω

Ohm's Law Examples…

- **If you want to find <u>Voltage</u> in Volts:**

$$V = IR$$

If I= 2 A and R = 5 Ohms

Then, $V = (2A)(5\Omega) = 10\ V$

Examples…

- **If you want to find <u>Resistance</u> in Ohm’s:**

$$R = V / I$$

If V = 9 Volts and I = 4 A

Then R = 9 V/ 4A = 2.25 Ω

Examples…

- **If you want to find <u>Current</u> in Amps:**

$$I = V / R$$

If V= 140 V and R = 2Ω

Then, I = 140V/ 2Ω = 70 A

Resistivity

- The inverse of the conductivity is the **resistivity**:
  - $\rho = 1 / \sigma$
- Resistivity has SI units of ohm-meters ($\Omega \cdot$ m)
- Resistance is also related to resistivity:

$$R = \rho \frac{\ell}{A}$$

The resistance is a property of the entire object while the resistivity is a property of the material with which the object is made.

Resistivity

The current density in a wire is not only dependent upon the external electric field that is imposed but

It is also dependent upon the material that is being used

Ohm found that J is proportional to E and in an idealized situation it is directly proportional to E

The resistivity is this proportionality constant and is given by

$$\rho = \frac{\mathbf{E}}{\mathbf{J}}$$

The greater the resistivity for a given electric field, the smaller the current density

Resistivity and Conductivity

- The electric field can now be written in terms of the current and resistivity of the circuit.
- The conductivity of a material is the reciprocal of the resistivity.

The inverse of resistivity is defined to be the *conductivity*

Example:

- What happens to the resistance when the length is doubled and the area is quadrupled?

- Answer: It changes by 1/2

Resistivity Values

TABLE 27.2 *Resistivities and Temperature Coefficients of Resistivity for Various Materials*

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and $1.50 \times 10^{-6}\ \Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Section 27.2

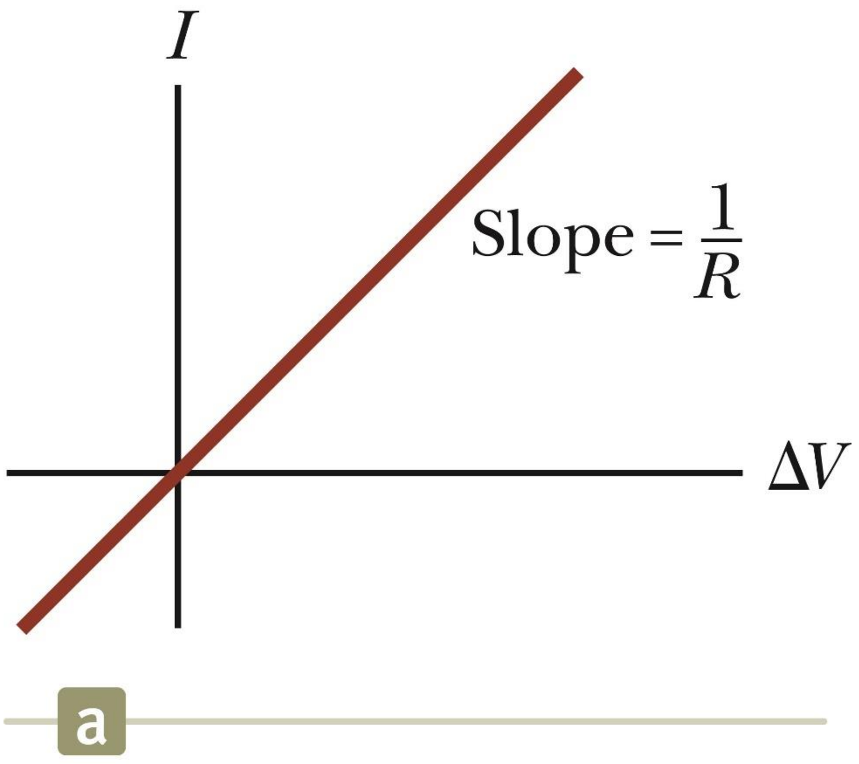
Resistance and Resistivity, Summary

- Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.
  - Resistivity is a property of substances.
- The resistance of a material depends on its geometry and its resistivity.
  - Resistance is a property of an object.
- An ideal conductor would have zero resistivity.
- An ideal insulator would have infinite resistivity.

Section 27.2

Ohmic Material, Graph

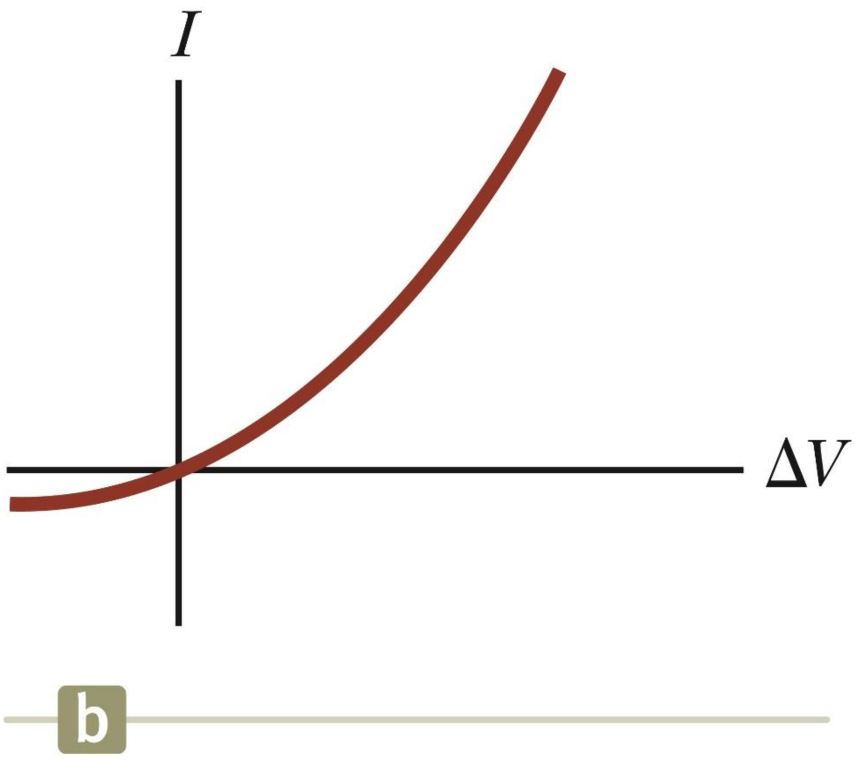
- An ohmic device
- The resistance is constant over a wide range of voltages.
- The relationship between current and voltage is linear.
- The slope is related to the resistance.



Section 27.2

Nonohmic Material, Graph

- Nonohmic materials are those whose resistance changes with voltage or current.
- The current-voltage relationship is nonlinear.
- A junction diode is a common example of a nonohmic device.

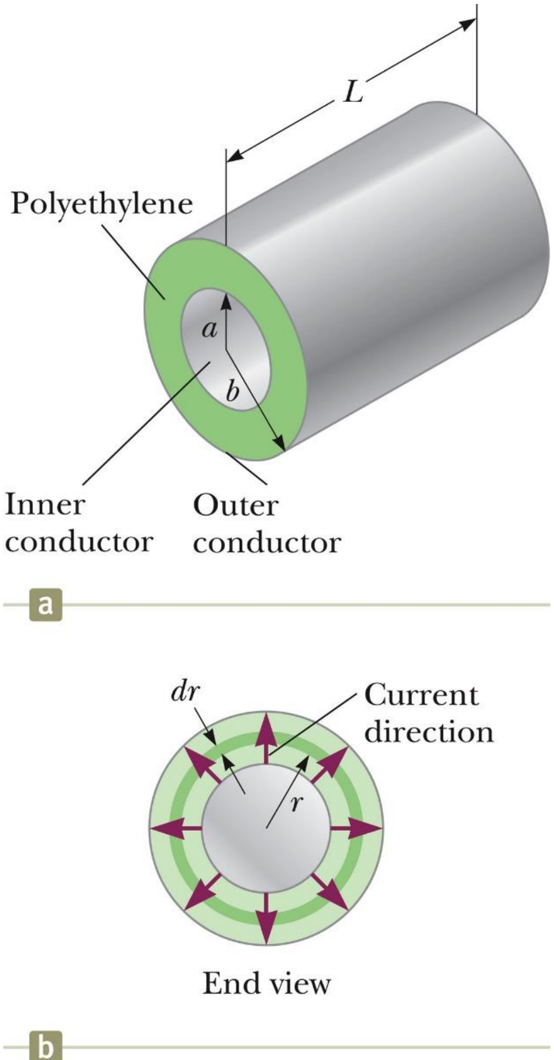


Section 27.2

Resistance of a Cable, Example

- Assume the silicon between the conductors to be concentric elements of thickness *dr.*
- The resistance of the hollow cylinder of silicon is

$$dR = \frac{\rho}{2\pi r L} dr$$



Resistance of a Cable, Example, cont.

- The total resistance across the entire thickness is

$$R = \int_a^b dR = \frac{\rho}{2\pi L} ln\left(\frac{b}{a}\right)$$

- This is the radial resistance of the cable.
- The calculated value is fairly high, which is desirable since you want the current to flow along the cable and not radially out of it.

Resistance and Temperature

- Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature.

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

  - ρ_o is the resistivity at some reference temperature T_o
    - T_o is usually taken to be 20° C
    - α is the **temperature coefficient of resistivity**
      - SI units of α are $^oC^{-1}$

$$\alpha = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta T}$$

- The temperature coefficient of resistivity can be expressed as

Section 27.4

Resistance and Temperature

The resistivity of a material is temperature dependent with the resistivity increasing as the temperature increases

This is due to the increased vibrational motion of the atoms the make up the lattice further inhibiting the motion of the charge carriers

The relationship between the resistivity and temperature is given approximately by

$$\rho(T) = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

Temperature Variation of Resistance

- Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance.

$$R = R_o[1 + \alpha(T - T_o)]$$

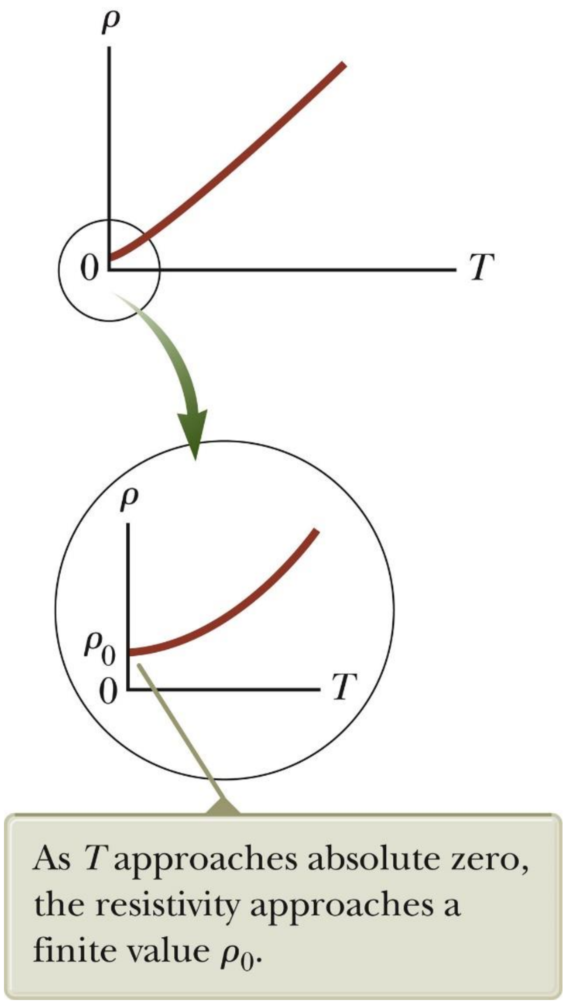
- Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

This relationship really only holds if the the length and the cross sectional area of the material being used does not appreciably change with temperature

Section 27.4

Resistivity and Temperature, Graphical View

- For some metals, the resistivity is nearly proportional to the temperature.
- A nonlinear region always exists at very low temperatures.
- The resistivity usually reaches some finite value as the temperature approaches absolute zero.



Residual Resistivity

- The residual resistivity near absolute zero is caused primarily by the collisions of electrons with impurities and imperfections in the metal.
- High temperature resistivity is predominantly characterized by collisions between the electrons and the metal atoms.
  - This is the linear range on the graph.

Section 27.4

Semiconductors

- Semiconductors are materials that exhibit a decrease in resistivity with an increase in temperature.
- α is negative
- There is an increase in the density of charge carriers at higher temperatures.

Energy

As a charge "moves" through a circuit, work is done that is equal to

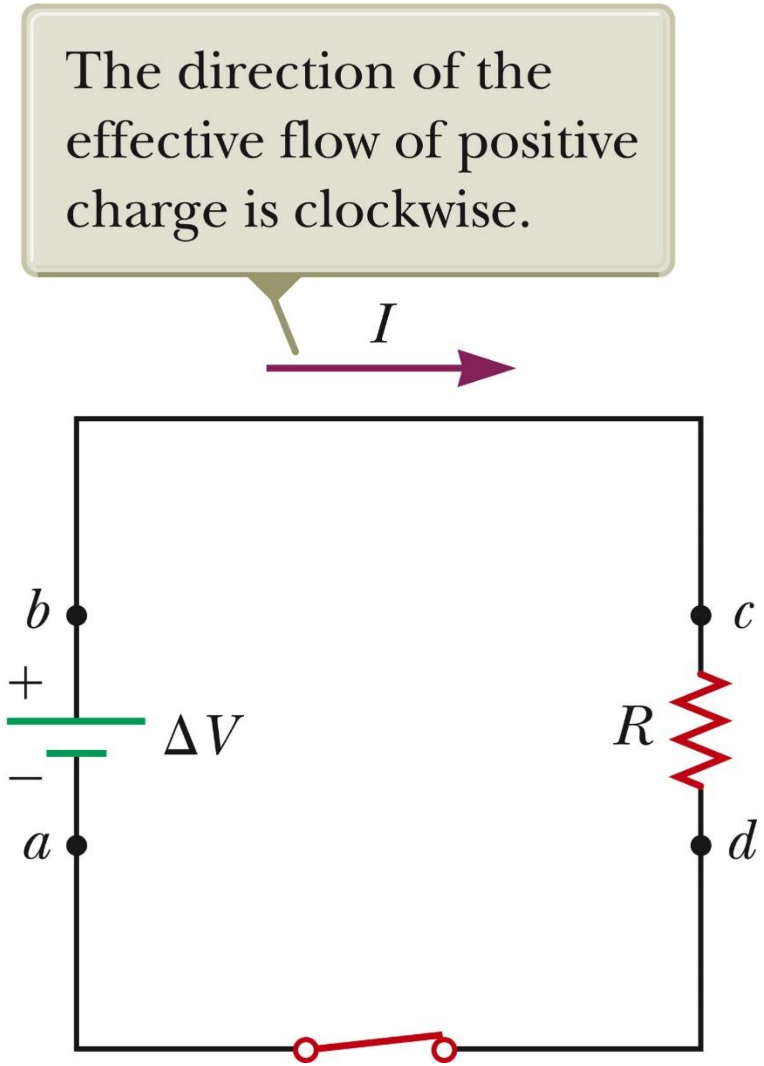
$$qV_{ab}$$

This work does not result in an increase in the kinetic energy of the charge, because of the collisions that occur

Instead, this energy is transferred to the circuit or circuit element within the complete circuit

Electrical Power

- Assume a circuit as shown
- The entire circuit is the system.
- As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$.
  - The chemical energy in the battery must decrease by this same amount.
- This electric potential energy is transformed into internal energy in the resistor.
  - Corresponds to increased vibrational motion of the atoms in the resistor



Section 27.6

Electric Power, 2

- The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.
- The resistor also emits thermal radiation.
- After some time interval, the resistor reaches a constant temperature.
    - The input of energy from the battery is balanced by the output of energy by heat and radiation.
- The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.
- The **power** is the rate at which the energy is delivered to the resistor.

Section 27.6

Electric Power, final

- The power is given by the equation P = I ΔV.
- Applying Ohm's Law, alternative expressions can be found:

$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

- Units: I is in A, R is in Ω, ΔV is in V, and P is in W

Some Final Notes About Current

- A single electron is moving at the drift velocity in the circuit.
  - It may take hours for an electron to move completely around a circuit.
- The current is the same everywhere in the circuit.
  - Current is not "used up" anywhere in the circuit
- The charges flow in the same rotational sense at all points in the circuit.

Electric Power Transmission

•Real power lines have resistance.

•Power companies transmit electricity at high voltages and low currents to minimize power losses.

Summary الخلاصة

Electron current

i_e = rate of electron flow

$N_e = i_e \Delta t$

Conventional current

I = rate of charge flow $= e i_e$

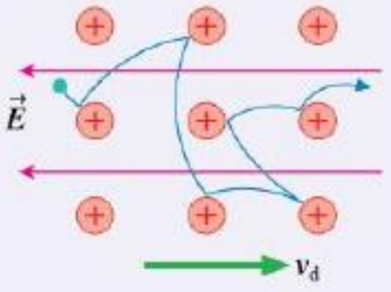
$Q = I \Delta t$

Current density

$J = I/A$

Sea of electrons

Conduction electrons move freely around the positive ions that form the atomic lattice.



Conduction

An electric field causes a slow drift at speed v_d to be superimposed on the rapid but random thermal motions of the electrons.

Collisions of electrons with the ions transfer energy to the atoms. This makes the wire warm and lightbulbs glow. More collisions mean a higher resistivity ρ and a lower conductivity σ.

Summary الخلاصة

The **drift speed** is $v_d = \frac{e\tau}{m}E$, where τ is the mean time between collisions.

The electron current is related to the drift speed by

$$i_e = n_e A v_d$$

where n_e is the electron density.

An electric field E in a conductor causes a current density $J = n_e e v_d = \sigma E$, where the **conductivity** is

$$\sigma = \frac{n_e e^2 \tau}{m}$$

The **resistivity** is $\rho = 1/\sigma$.

Summary الخلاصة

Resistors

A potential difference ΔV_{wire} between the ends of a wire creates an electric field inside the wire:

$$E_{wire} = \frac{\Delta V_{wire}}{L}$$

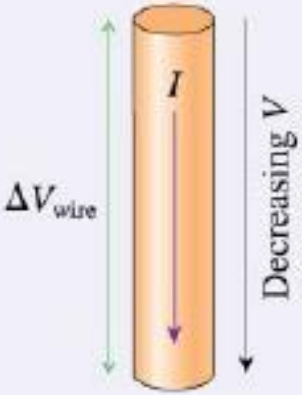
The electric field causes a current in the direction of decreasing potential.

The size of the current is

$$I = \frac{\Delta V_{wire}}{R}$$

where $R = \frac{\rho L}{A}$ is the wire's **resistance.**

This is **Ohm's law.**



Thank You

ACKNOWLEDGEMENTS