# Phys 103

# Chapter 9

## Linear Momentum and Collisions

By

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# LECTURE OUTLINE

- 9.1 Linear Momentum and Its Conservation
- 9.2 Impulse and Momentum
- 9.3 Collisions in One Dimension
- 9.4 Two-Dimensional Collisions

# PROBLEMS

**Section 9.1 Linear Momentum and its Conservation**

**1.** A 3.00-kg particle has a velocity of (3.00**i** ˆ - 4.00**j** ˆ ) m/s.

(a) Find its *x* and *y* components of momentum.

(b) Find the magnitude and direction of its momentum.

**SOLUTIONS TO PROBLEM:**

$m = 3.00 \text{ kg}, \qquad \mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

# PROBLEMS

**Section 9.1 Linear Momentum and its Conservation**

**2.** A 0.100-kg ball is thrown straight up into the air with an initial speed of 15.0 m/s. Find the momentum of the ball

(a) at its maximum height and (b) halfway up to its maximum height.

**SOLUTIONS TO PROBLEM:**

(a) At maximum height $v = 0$, so $\mathbf{p} = \boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.100)\text{kg}(15.0\ \text{m/s})^2 = 11.2\ \text{J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62\ \text{J} = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$v = \sqrt{\frac{2 \times 5.62\ \text{J}}{0.100\ \text{kg}}} = 10.6\ \text{m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100\ \text{kg})(10.6\ \text{m/s})\hat{\mathbf{j}}$

$$\mathbf{p} = \boxed{1.06\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}.$$

# PROBLEMS

## Section 9.1 Linear Momentum and its Conservation

**4.** Two blocks of masses *M* and 3*M* are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after this, the block of mass 3*M* moves to the right with a speed of 2.00 m/s.

(a) What is the speed of the block of mass *M*? (b) Find the original elastic potential energy in the spring if *M* = 0.350 kg.

## SOLUTIONS TO PROBLEM:

(a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00\ \text{m/s})$

Solving gives $v_m = \boxed{-6.00\ \text{m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\ \text{J}}$

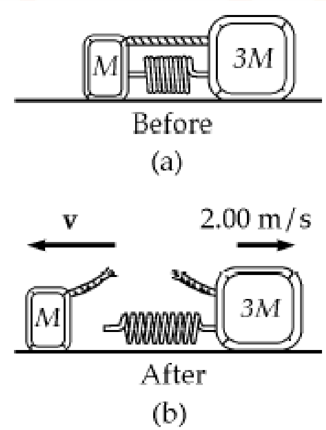

FIG. P9.4

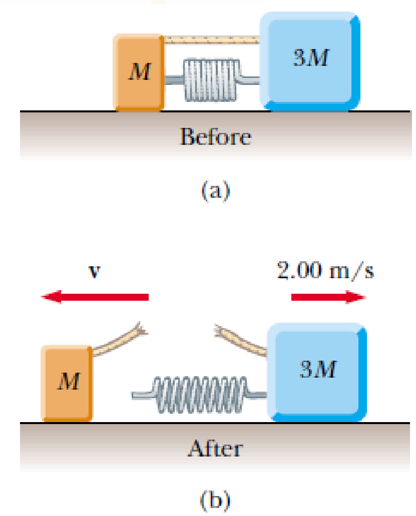

Figure P9.4

# PROBLEMS

## Section 9.1 Linear Momentum and its Conservation

**5.** (a) A particle of mass *m* moves with momentum *p*. Show that the kinetic energy of the particle is $K=p^2/2m$.

(b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

**SOLUTIONS TO PROBLEM:**

(a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

# PROBLEMS

## Section 9.2 Impulse and Momentum

**7.** An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.7. From this curve, determine

(a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

**SOLUTIONS TO PROBLEM:**

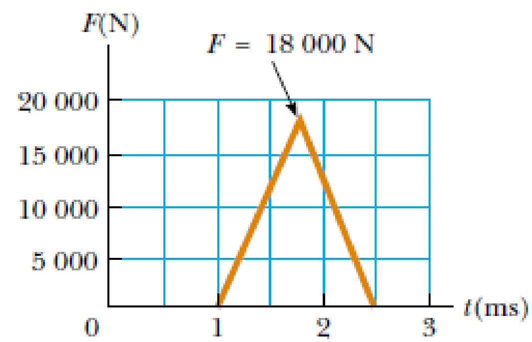

Figure P9.7

(a) $I = \int F dt = \text{area under curve}$

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\ \text{s}\right)\left(18\,000\ \text{N}\right) = \boxed{13.5\ \text{N}\cdot\text{s}}$$

(b) $$F = \frac{13.5\ \text{N}\cdot\text{s}}{1.50 \times 10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\ \text{kN}}$

# PROBLEMS

## Section 9.2 Impulse and Momentum

**8.** A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

## SOLUTIONS TO PROBLEM:

The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2 \text{ up} - mv_1 \text{ down} &= m(v_2 + v_1) \text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right) \text{ up} \\ &= 0.15 \text{ kg}\sqrt{2(9.8 \text{ m/s}^2)}\left(\sqrt{0.960 \text{ m}} + \sqrt{1.25 \text{ m}}\right) \text{ up} \\ &= \boxed{1.39 \text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

# PROBLEMS

## Section 9.2 Impulse and Momentum

**9.** A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle (Fig. P9.9). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball ?

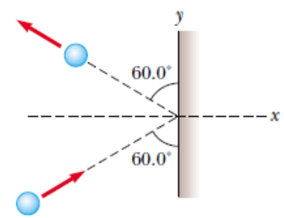


Figure P9.9

**SOLUTIONS TO PROBLEM:**

$$\Delta p = F\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866)$$

$$= -52.0\ \text{kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg}\cdot\text{m/s}}{0.200\ \text{s}} = \boxed{-260\ \text{N}}$$

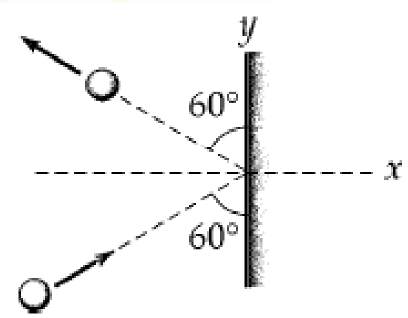


FIG. P9.9

# PROBLEMS

## Section 9.2 Impulse and Momentum

**10.** A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction.

(a) What is the impulse delivered to the ball by the racquet?

(b) What work does the racquet do on the ball?

**SOLUTIONS TO PROBLEM:**

Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}}\ \mathrm{N\cdot s}}$

(b) $\text{Work} = K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\ \mathrm{J}}$

# PROBLEMS

**Section 9.2 Impulse and Momentum**

**13.** A garden hose is held as shown in Figure P9.13. The hose is originally full of motionless water. What additional force is necessary to hold the nozzle stationary after the water flow is turned on, if the discharge rate is 0.600 kg/s with a speed of 25.0 m/s?

Figure P9.13

**SOLUTIONS TO PROBLEM:**

The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\ \text{kg})(25.0\ \text{m/s}) - 0}{1.00\ \text{s}} = \boxed{15.0\ \text{N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

# PROBLEMS

**Section 9.3 Collisions in One Dimension**

**15.** High-speed stroboscopic photographs show that the head of a golf club of mass 200 g is traveling at 55.0 m/s just before it strikes a 46.0-g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.

**SOLUTIONS TO PROBLEM:**

$$(200\text{ g})(55.0\text{ m/s}) = (46.0\text{ g})v + (200\text{ g})(40.0\text{ m/s})$$

$$v = \boxed{65.2\text{ m/s}}$$

# PROBLEMS

**Section 9.3 Collisions in One Dimension**

**16.** An archer shoots an arrow toward a target that is sliding toward her with a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the 300-g target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

**SOLUTIONS TO PROBLEM:**

$$(m_1 v_1 + m_2 v_2)_i = (m_1 v_1 + m_2 v_2)_f$$

$$22.5\ \text{g}(35\ \text{m/s}) + 300\ \text{g}(-2.5\ \text{m/s}) = 22.5\ \text{g} v_{1f} + 0$$

$$v_{1f} = \frac{37.5\ \text{g} \cdot \text{m/s}}{22.5\ \text{g}} = \boxed{1.67\ \text{m/s}}$$

# PROBLEMS

**Section 9.3 Collisions in One Dimension**

**17.** A 10.0-g bullet is fired into a stationary block of wood (*m* !5.00 kg). The relative motion of the bullet stops inside the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.600 m/s. What was the original speed of the bullet?

**SOLUTIONS TO PROBLEM:**

Momentum is conserved

$$\left(10.0\times10^{-3}\ \text{kg}\right)v=\left(5.01\ \text{kg}\right)\left(0.600\ \text{m/s}\right)$$

$$v=\boxed{301\ \text{m/s}}$$

# PROBLEMS

## Section 9.3 Collisions in One Dimension

**18.** A railroad car of mass 2.50 * 104 kg is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s. (a) What is the speed of the four cars after the collision?

## SOLUTIONS TO PROBLEM:

(a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4$ kg

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right] = \left(2.50 \times 10^4\right)(12.5 - 8.00 - 6.00) = \boxed{-3.75 \times 10^4 \text{ J}}$$

# PROBLEMS

## Section 9.3 Collisions in One Dimension

**21.** A 45.0-kg girl is standing on a plank that has a mass of 150 kg. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless supporting surface. The girl begins to walk along the plank at a constant speed of 1.50 m/s relative to the plank. (a) What is her speed relative to the ice surface? (b) What is the speed of the plank relative to the ice surface?

**SOLUTIONS TO PROBLEM:**

(a), (b) Let $v_g$ and $v_p$ be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0 v_g + 150 v_p = 0, \text{ or } v_g = -3.33 v_p$$

Putting this into the equation (1) above gives

$$-3.33 v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

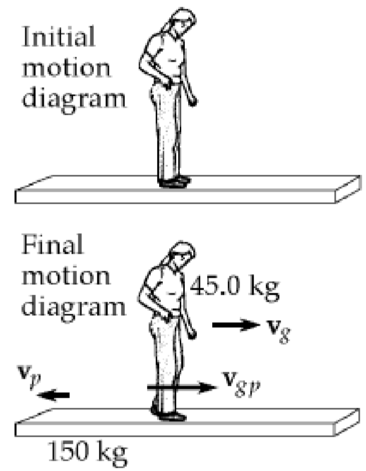


FIG. P9.21

# PROBLEMS

## Section 9.3 Collisions in One Dimension

**25.** A 12.0-g wad of sticky clay is hurled horizontally at a 100-g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

## SOLUTIONS TO PROBLEM:

At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112\text{ kg})v_2^2 = 0.650(0.112\text{ kg})(9.80\text{ m/s}^2)(7.50\text{ m})$$

$v_2^2 = 95.6\text{ m}^2/\text{s}^2$ $\qquad v_2 = 9.77\text{ m/s}$

$(12.0 \times 10^{-3}\text{ kg})v_1 = (0.112\text{ kg})(9.77\text{ m/s})$ $\qquad v_1 = \boxed{91.2\text{ m/s}}$

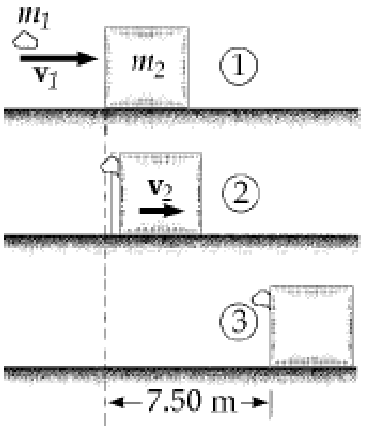


FIG. P9.25

# PROBLEMS

## Section 9.3 Collisions in One Dimension

**27.** (a) Three carts of masses 4.00 kg, 10.0 kg, and 3.00 kg move on a frictionless horizontal track with speeds of 5.00 m/s, 3.00 m/s, and 4.00 m/s, as shown in Figure P9.27. Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts. (b) **What If?** Does your answer require that all the carts collide and stick together at the same time? What if they collide in a different order?

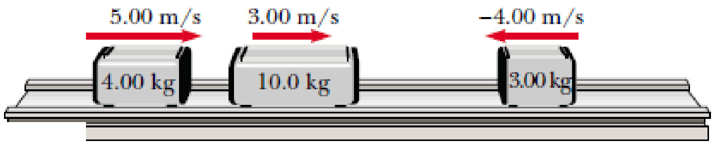


Figure P9.27

**SOLUTIONS TO PROBLEM:**

(a) Using conservation of momentum, $\left(\sum p\right)_{\text{after}} = \left(\sum p\right)_{\text{before}}$, gives

$$\left[(4.0+10+3.0)\text{ kg}\right]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24$ m/s, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

# PROBLEMS

## Section 9.4 Two-Dimensional Collisions

**32.** Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed $v_{2i}$. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within

truth?

## SOLUTIONS TO PI

We use conservation of momentum for the system of two vehicles for both northward and eastward components.

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

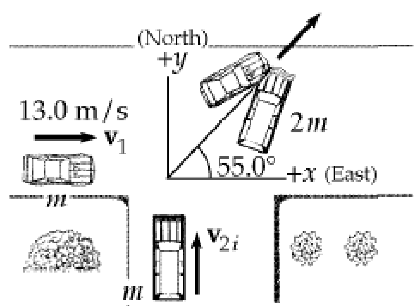


FIG. P9.32

# PROBLEMS

## Section 9.4 Two-Dimensional Collisions

**33.** A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves, at 4.33 m/s, at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

**SOLUTIONS TO PROBLEM:**

By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00\ \text{m/s} + 0 = (4.33\ \text{m/s})\cos 30.0° + v_{2fx}$$

$$v_{2fx} = 1.25\ \text{m/s}$$

$$0 = (4.33\ \text{m/s})\sin 30.0° + v_{2fy}$$

$$v_{2fy} = -2.16\ \text{m/s}$$

$$\mathbf{v}_{2f} = \boxed{2.50\ \text{m/s at } -60.0°}$$

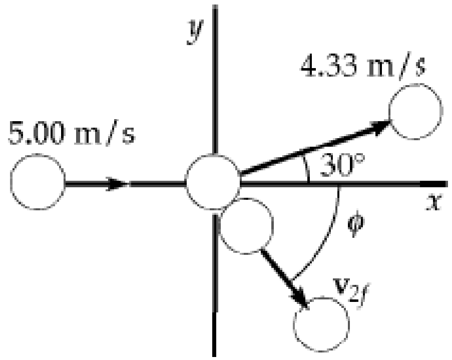


FIG. P9.33

Note that we did not need to use the fact that the collision is perfectly elastic.

# PROBLEMS

**Section 9.4 Two-Dimensional Collisions**

**35.** An object of mass 3.00 kg, moving with an initial velocity of 5.00ˆi m/s, collides with and sticks to an object of mass 2.00 kg with an initial velocity of "3.00ˆj m/s. Find the final velocity of the composite object.

**SOLUTIONS TO PROBLEM:**

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f: \qquad 3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$$

$$\mathbf{v} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\ \text{m/s}}$$

# Lecture Summary

**Perfectly Inelastic Collisions**

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

**Perfectly Elastic Collisions:**

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 \, {v_{1i}}^2 + \frac{1}{2} m_2 \, {v_{2i}}^2 = \frac{1}{2} m_1 \, {v_{1f}}^2 + \frac{1}{2} m_2 \, {v_{2f}}^2$$

When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. An inelastic collision is one for which the total kinetic energy of the system is not conserved. A perfectly inelastic collision is one in which the colliding bodies stick together after the collision. An elastic collision is one in which the kinetic energy of the system is conserved.

# Lecture Summary

**Two dimensional collisions**

For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

# Thank You


ACKNOWLEDGEMENTS