Phys 103

Chapter 9

Linear Momentum and Collisions

By

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LECTURE OUTLINE

- 9.1 Linear Momentum and Its
- Conservation
- 9.2 Impulse and Momentum
- 9.3 Collisions in One Dimension
- 9.4 Two-Dimensional Collisions

Introduction

Consider what happens when a bowling ball strikes a pin, as in the opening photograph. The pin is given a large velocity as a result of the collision; consequently, it flies away and hits other pins or is projected toward the backstop. Because the average force exerted on the pin during the collision is large, the pin achieves the large velocity very rapidly and experiences the force for a very short time interval. According to Newton's third law, the pin exerts a reaction force on the ball that is equal in magnitude and opposite in direction to the force exerted by the ball on the pin.

This reaction force causes the ball to accelerate, but because the ball is so much more massive than the pin, the ball's acceleration is much less than the pin's acceleration.

Introduction

- Momentum Analysis Models Force and acceleration are related by Newton's second law. When force and acceleration vary by time, the situation can be very complicated. The techniques developed in this chapter will enable you to understand and analyze these situations in a simple way. Will develop momentum versions of analysis models for isolated and non-isolated systems These models are especially useful for treating problems that involve collisions and for analyzing rocket propulsion.

9.1 Linear Momentum and Its Conservation

Consider two particles m1and m2with v1and v2collide as in figuer:

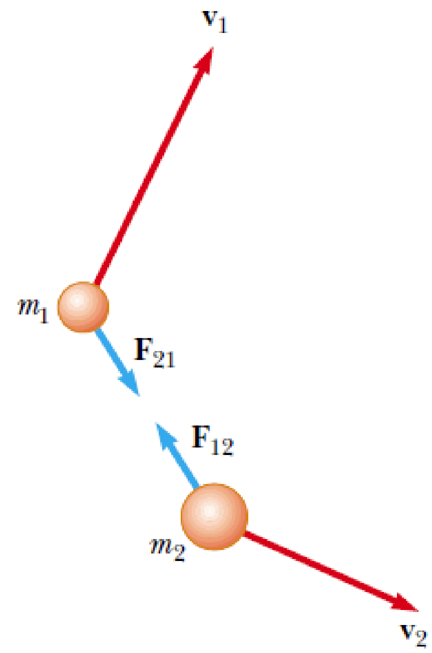
If a force from particle 1 acts on particle 2, then there must be a second force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1. That is, they form a Newton's third law action–reaction pair, so that F_{12}=-F_{21}. We can express this condition as:

$$F_{12} + F_{21} = 0$$

Using Newton's 2nd law:

$$m_1 a_1 + m_2 a_2 = 0$$

$$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$



9.1 Linear Momentum and Its Conservation

If the masses m1and m2are constant, we can bring them into the derivatives, which gives:

$$\frac{d(m_1 v_1)}{dt} + \frac{d(m_2 v_2)}{dt} = 0 \text{ so } \frac{d(m_1 v_1 + m_2 v_2)}{dt} = 0$$

- To finalize this discussion, note that the derivative of the sum $(m_1 v_1 + m_2 v_2)$ with respect to time is zero. Consequently, this sum must be constant. We learn from this discussion that the quantity mv for a particle is important, in that the sum of these quantities for an isolated system is conserved. We call this quantity linear momentum linear momentum of a particle or an object is defind as:

p= m**v**

Linear momentum is a vector quantity.

Its direction is the same as the direction of the velocity.

The dimensions of momentum are ML/T. The SI units of momentum are kg · m / s.

9.1 Linear Momentum and Its Conservation

If a particle is moving in 3-D then: $\mathbf{p}_x = m\mathbf{v}_x$

- Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle:

$$\sum F_x = ma = m\frac{dv}{dt}$$

- In Newton's second law, the mass m is assumed to be constant. Thus, we can bring m inside the derivative notation to give us:

$$\sum F_x = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$$

- *This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.*
    - **This is the form in which Newton presented the Second Law.**
    - **It is a more general form than the one we used previously.**
    - **This form also allows for mass changes.**

9.1 Linear Momentum and Its Conservation

- Using the definition of momentum, $\frac{d(m_1 v_1 + m_2 v_2)}{dt} = 0$ can be written:

$$\frac{d(p_1 + p_2)}{dt} = 0$$

$$\frac{d(p_1 + p_2)}{dt} = \frac{dp_{tot}}{dt} = 0$$

$$p_{tot} = p_1 + p_2 = constant$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

Conservation of Linear Momentum

This is the mathematical statement of a new analysis model, the isolated system (momentum).

- *Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.*
- This law tells us that the total momentum of an isolated system at all times equals its initial momentum.

9.1 Linear Momentum and Its Conservation

Momentum and Kinetic Energy

Momentum and kinetic energy both involve mass and velocity. There are major differences between them:

- Kinetic energy is a scalar and momentum is a vector.
- Kinetic energy can be transformed to other types of energy.

There is only one type of linear momentum, so there are no similar transformations.

Analysis models based on momentum are separate from those based on energy. This difference allows an independent tool to use in solving problems.

9.2 Impulse and Momentum

- To build a better, let us assume that a single force **F**acts on a particle and that this force may vary with time. According to Newton's second law:

$$F = \frac{dp}{dt}, \qquad dp = F dt$$

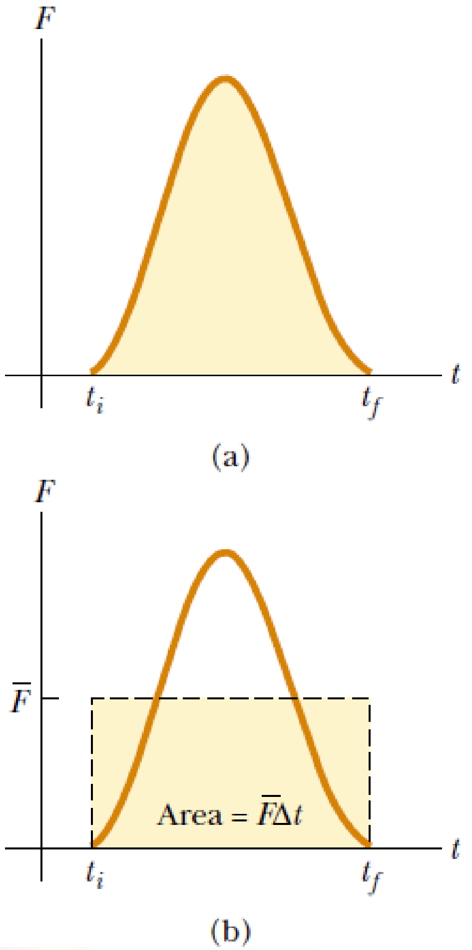
Inegrating for time t_i to t_f:

$$\Delta p = p_f - p_i = \int_{t_i}^{t_f} F dt$$

Or

$$I = \int_{t_i}^{t_f} F dt$$

The integral is called the impulse, I, of the force acting on an object over Δ t.



9.2 Impulse and Momentum

Forces and Conservation of Momentum

- In conservation of momentum, there is no statement concerning the types of forces acting on the particles of the system. The forces are not specified as conservative or non-conservative. There is no indication if the forces are constant or not. The only requirement is that the forces must be internal to the system.
- This gives a hint about the power of this new model.

9.2 Impulse and Momentum

- The quantity in ($I = \int_{t_i}^{t_f} F dt$) is called: Impulse. ($\Delta p = p_f - p_i = \int_{t_i}^{t_f} F dt$) is called: **Impulse-Momentum Theorm**.
- The impulse of the force **F** acting on a particle equals the change in the momentum of the particle.
- Because the force imparting an impulse can generally vary in time, it is convenient to define a time-averaged force:

$$\overline{F} = \frac{1}{\Delta t}\int_{t_i}^{t_f} F dt \quad or\ I = \overline{F}\Delta t$$

In principle, if **F** is known as a function of time, the impulse can be calculated from Equation ($I = \int_{t_i}^{t_f} F dt$). The calculation becomes especially simple if the force acting on the particle is constant. In this case: **I= FΔt**

9.2 Impulse and Momentum

Impulse-Momentum Theorem

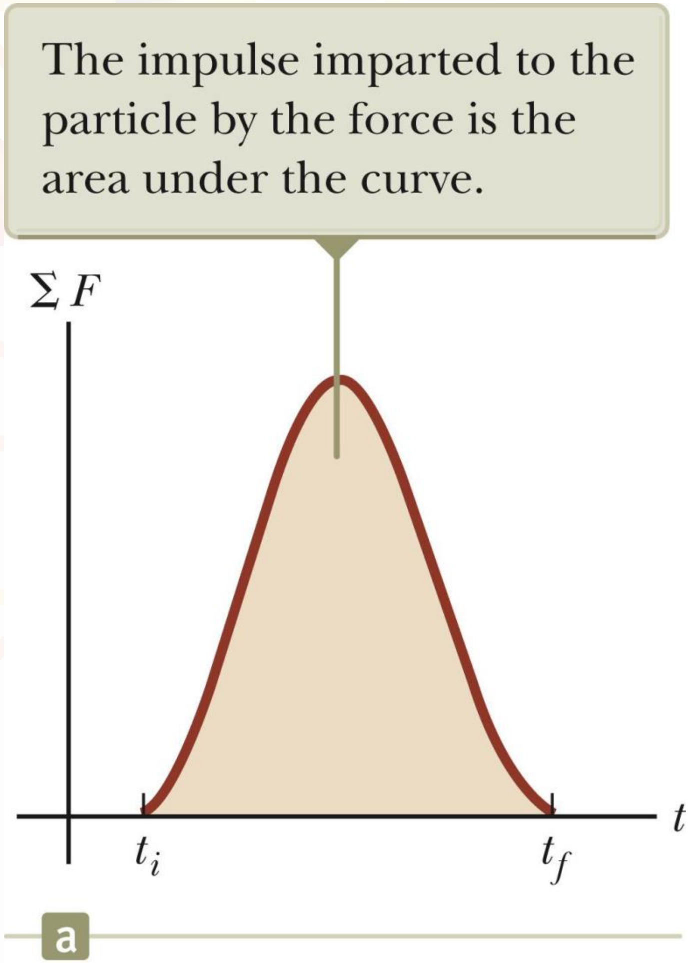
- This equation expresses the **impulse-momentum theorem**: The change in the momentum of a particle is equal to the impulse of the new force acting on the particle.
- This is equivalent to Newton's Second Law.
- This is identical in form to the conservation of energy equation.
- This is the most general statement of the principle of conservation of momentum and is called the conservation of momentum equation.
- This form applies to non-isolated systems.
- This is the mathematical statement of the **non-isolated system (momentum) model.**

Impulse is a vector quantity.

9.2 Impulse and Momentum

More About Impulse

- Impulse is a vector quantity. The magnitude of the impulse is equal to the area under the force-time curve.
- The force may vary with time. Dimensions of impulse are M L / T Impulse is not a property of the particle, but a measure of the change in momentum of the particle.



Example 9.4 How Good Are the Bumpers?

In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are vi= -15 m/s and vf= 2.6 m/s , respectively. If the collision lasts for 0.150 s, *find the impulse caused by the collision and the average force exerted on the car.*

- ***Solution**:*

<u>Conceptualize</u>

- The collision time is short.
- We can image the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

<u>Categorize</u>

- Assume net force exerted on the car by wall and friction with the ground is large compared with other forces.
- Gravitational and normal forces are perpendicular and so do not effect the horizontal momentum.

Example 9.4 How Good Are the Bumpers?

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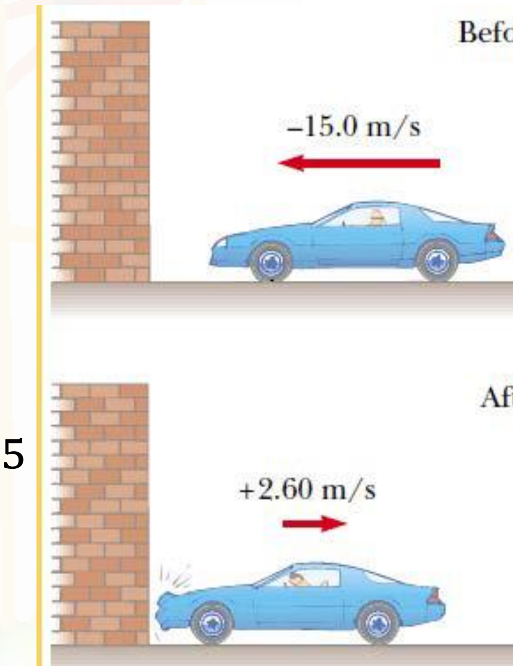
- ***Solution:***

$$\because I = \Delta p = p_f - p_i$$
$$= mv_f - mv_i$$
$$= (1500)(2.6\hat{\imath}) - (1500)(-15\hat{\imath})$$
$$= 2.64 \times 10^4\ \hat{\imath}\ kg.m/s$$
$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4}{0.15} = 1.76 \times 10^5$$



Lecture Summary

- The linear momentum **p**of a particle , m moving with a velocity **v**is:
- **p**= m**v** (9.2)
- The law of conservation of linear momentum indicates that the total momentum of an isolated system is conserved. If two particles form an isolated system, the momentum of the system is conserved regardless of the nature of the force between them. Therefore, the total momentum of the system at all times equals its initial total momentum, or

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

- The impulse imparted to a particle by a force **F is equal to the change in the momentum** of the particle:

$$I = \int_{t_i}^{t_f} F dt = \Delta p$$

- This is known as the impulse–momentum theorem.

Thank You


ACKNOWLEDGEMENTS