Phys 103

Chapter 4

Motion in Two Dimensions

By

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LECTURE OUTLINE

<u>4.1 The Position, Velocity, and Acceleration Vectors</u>

<u>4.2 Two-Dimensional Motion with Constant Acceleration</u>

4.3 Projectile Motion

4.4 Uniform Circular Motion

4.5 Tangential and Radial Acceleration

4.1 The Position, Velocity, and Acceleration Vectors

Kinematics in Two Dimensions

Will study the vector nature of position, velocity and acceleration in greater detail.

Will treat projectile motion and uniform circular motion as special cases

Discuss relative motion

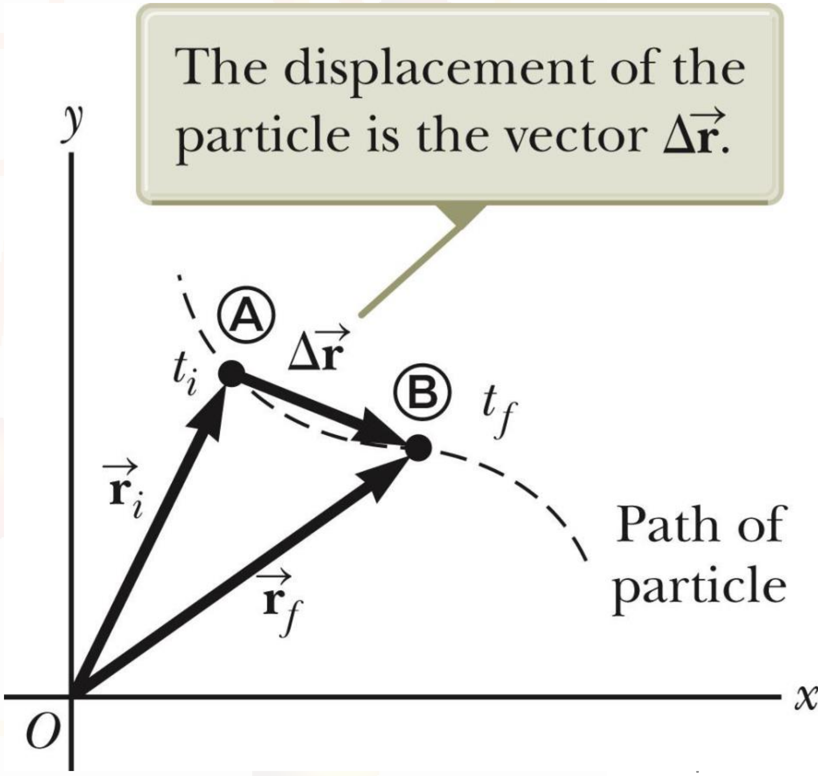
4.1 The Position, Velocity, and Acceleration Vectors

Position and Displacement

The position of an object is described by its position vector, $\vec{\boldsymbol{r}}$.

The **displacement** of the object is defined as the **change in its position.**

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i$$



4.1 The Position, Velocity, and Acceleration Vectors

General Motion Ideas

In two- or three-dimensional kinematics, everything is the same as in one-dimensional motion except that we must now use full vector notation.

Positive and negative signs are no longer sufficient to determine the direction.

4.1 The Position, Velocity, and Acceleration Vectors

Average Velocity

The average velocity is the ratio of the displacement to the time interval for the displacement.

$$\vec{\boldsymbol{v}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$$

The direction of the average velocity is the direction of the displacement vector.

The average velocity between points is independent of the path taken.

This is because it is dependent on the displacement, which is also independent of the path.

4.1 The Position, Velocity, and Acceleration Vectors

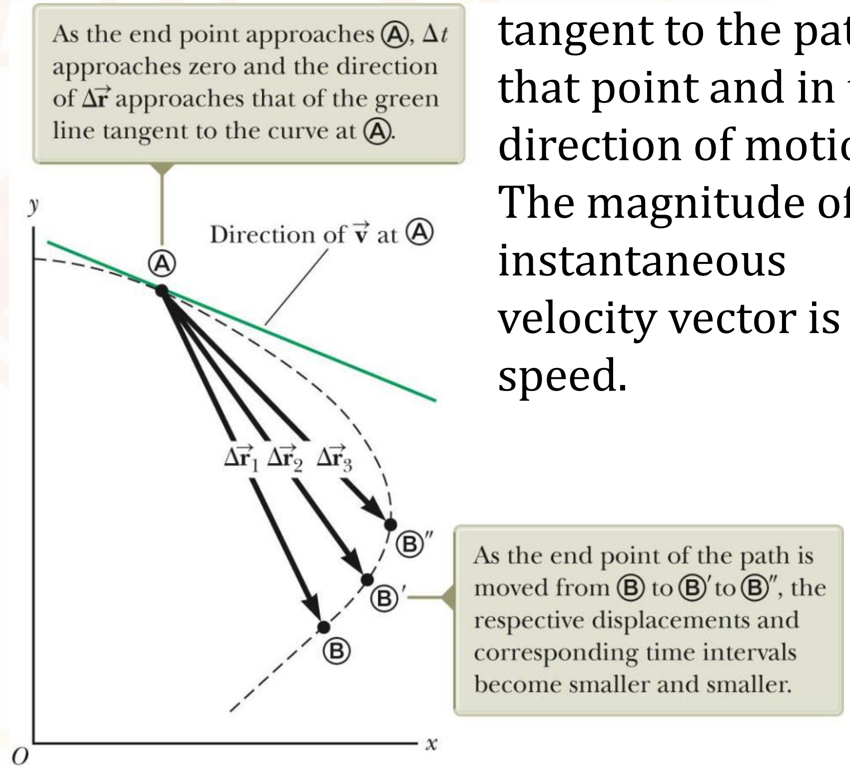
Instantaneous Velocity

The instantaneous velocity is the limit of the average velocity as Δt approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve.

- The speed is a scalar quantity.



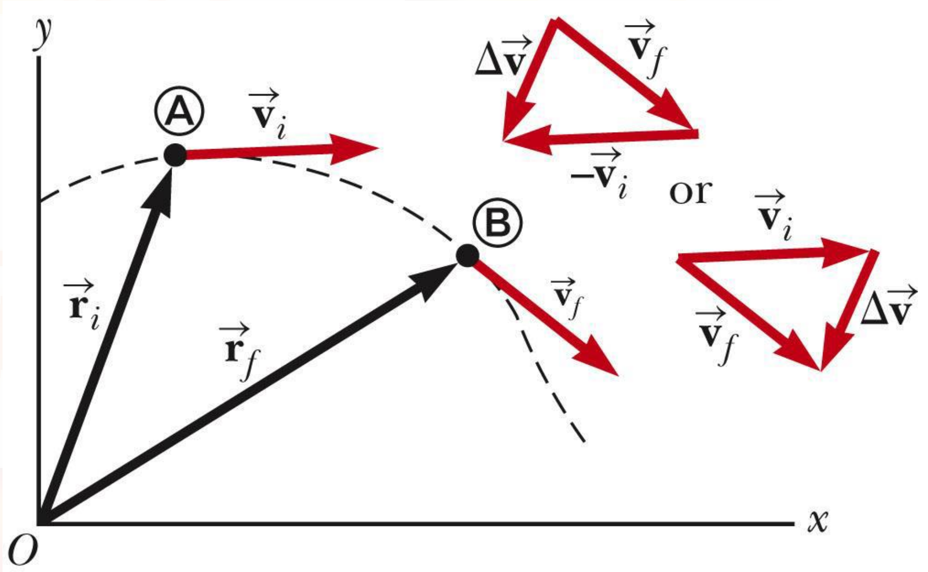
The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion.

The magnitude of the instantaneous velocity vector is the speed.

4.1 The Position, Velocity, and Acceleration Vectors

Average Acceleration

The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.



$$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

As a particle moves, the direction of the change in velocity is found by vector subtraction.

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

The average acceleration is a vector quantity directed along $\Delta\vec{v}$.

4.1 The Position, Velocity, and Acceleration Vectors

Instantaneous Acceleration

The instantaneous acceleration (acceleration as a function of time) is the limiting value of the ratio as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{v}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{d}t}$$

The instantaneous equals the derivative of the velocity vector with respect to time.

Note: the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion.

- the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (2-d) motion.

4.1 The Position, Velocity, and Acceleration Vectors

Producing An Acceleration

- Various changes in a particle's motion may produce an acceleration.
- The magnitude of the velocity vector may change.
- The direction of the velocity vector may change.
  - Even if the magnitude remains constant
- Both may change simultaneously

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

The Let us consider 2-D motion during which the acceleration remains constant in both magnitude and direction.

The position vector for a particle moving in the yy plane can be written:

$$\vec{\boldsymbol{r}} = x\hat{\imath} + \mathrm{y}\hat{\jmath}$$

- The velocity vector can be found from the position vector.

$$\vec{v} = \frac{\boldsymbol{d}\vec{\boldsymbol{r}}}{\boldsymbol{d}t} = v_x\hat{\imath} + v_y\hat{\jmath}$$

Since acceleration is constant, we can also find an expression for the velocity as a function of time:

$$\vec{\boldsymbol{v}}_{\boldsymbol{f}} = \overrightarrow{\boldsymbol{v}_{\boldsymbol{i}}} + \vec{a}t$$

Because **a** is assumed constant, its components a_x and a_y also are also constants.

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

For constant acceleration

$a_x = Cons$ **and** $\Delta \boldsymbol{t} = t$

$$\overline{v_x} = \frac{v_{xi} + v_{xf}}{2}$$

For 2-D motion we will have 2 sets of Equations; one for each direction.

- For x-direction; we have:

$$v_{xf} = v_{xi} + a_x t$$

$$\boldsymbol{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

$$v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x(x_f - x_i)$$

For y-direction; we have:

$$v_{yf} = v_{yi} + a_y t$$

$$\boldsymbol{y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2}$$

$$v_{yf}^{\ 2} = v_{yi}^{\ 2} + 2a_y(y_f - y_i)$$

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

vectors of velocity v and position r are.

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$
$$v_{xf} = v_{xi} + a_x t$$
$$v_{yf} = v_{yi} + a_y t$$
$$\overrightarrow{v_f} = v_{x\mathrm{f}}\hat{\imath} + v_{y\mathrm{f}}\hat{\jmath}$$
$$\overrightarrow{v_f} = (v_{xi} + a_x t)\,\hat{i} + (v_{yi} + a_y t)\hat{\jmath}$$
$$\overrightarrow{v_f} = \left(v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}\right) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$$

Because **a** is assumed constant, its components a_x and a_y also are also constants.

$$a_{x=}a_x = a$$

$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

vectors of velocity v and position r are.

$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

$$\boldsymbol{r_f} = \left(\boldsymbol{v_{xi}t} + \frac{1}{2}\boldsymbol{a_x}\mathbf{t}^2\right)\hat{i} + \left(\boldsymbol{v_{yi}t} + \frac{1}{2}\boldsymbol{a_y}\mathbf{t}^2\right)\hat{j}$$

$$= (\boldsymbol{v_{xi}}\hat{i} + \boldsymbol{v_{yi}}\hat{j})\boldsymbol{t} + \frac{1}{2}(\boldsymbol{a_x}\hat{i} + \boldsymbol{a_y}\hat{j})\mathbf{t}^2$$

$$= v_i\boldsymbol{t} + \frac{1}{2}\boldsymbol{a}\,\mathbf{t}^2$$

Lecture Summary

Displacement of a particle in 2-D is:

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i$$

The average velocity is defined as:

$$\vec{\boldsymbol{v}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$$

Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{r}}}{\boldsymbol{d}t}$$

The average acceleration is defined as:

$$\vec{\boldsymbol{a}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{v}}}{\Delta \boldsymbol{t}} = \frac{\vec{\boldsymbol{v}}_{\boldsymbol{f}} - \vec{\boldsymbol{v}}_{\boldsymbol{i}}}{t_f - t_i}$$

The instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{d}t}$$

Lecture Summary

Constant Acceleration motion of a particle in 2-D:

$v_{xf} = v_{xi} + a_x t$ $\boldsymbol{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$ $v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$	$v_{yf} = v_{yi} + a_y t$ $\boldsymbol{y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2}$ $v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$

Velocity and position in Vector form in 2-D motion:

$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ $\overrightarrow{v_f} = (v_{xi} + a_x t)\hat{\imath} + (v_{yi} + a_y t)\hat{\jmath}$ $\overrightarrow{v_f} = \left(v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}\right) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$ $\overrightarrow{v_f} = \overrightarrow{v_i} + at$	$\boldsymbol{r_f = \left(v_{xi}t + \frac{1}{2}a_x t^2\right)\hat{\imath} + \left(v_{yi}t + \frac{1}{2}a_y t^2\right)\hat{\jmath}}$ $\boldsymbol{= (v_{xi}\hat{\imath} + v_{yi}\hat{\jmath})t + \frac{1}{2}(a_x\hat{\imath} + a_y\hat{\jmath})t^2}$ $\boldsymbol{= v_i t + \frac{1}{2}a\, t^2}$

Thank You


ACKNOWLEDGEMENTS