Phys 103

Chapter 2

Motion in One Dimension

By

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LECTURE OUTLINE

- 2.1 Position, Velocity, and Speed
- 2.2 Instantaneous Velocity and Speed
- **2.3 Acceleration**
- **2.5 One-Dimensional Motion with Constant Acceleration**
- **2.6 Freely Falling Objects**

Introduction

When the velocity of a particle changes with time, the particle is said to be accelerating.

2.3 Acceleration

- **Average Acceleration**

- The average acceleration $\overline{\boldsymbol{a}_x}$ of the particle is defined as the *change* in velocity $\Delta \boldsymbol{v}_x$divided by the time interval $\Delta \boldsymbol{t}$ during which that change occurs:

$$\overline{\boldsymbol{a}_x} = \frac{\Delta \boldsymbol{v}_x}{\Delta \boldsymbol{t}} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

- Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T2. The SI unit of acceleration is meters per second squared (m/s2).

In one dimension, positive and negative can be used to indicate direction.

2.3 Acceleration

- **Instantaneous acceleration**

Instantaneous acceleration define as the limit of the average acceleration as Δt approaches zero.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

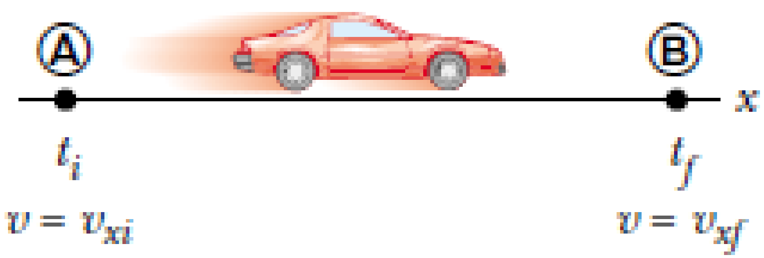
That is, the instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity–time graph.

The term acceleration will mean instantaneous acceleration. If average acceleration is wanted, the word average will be included.

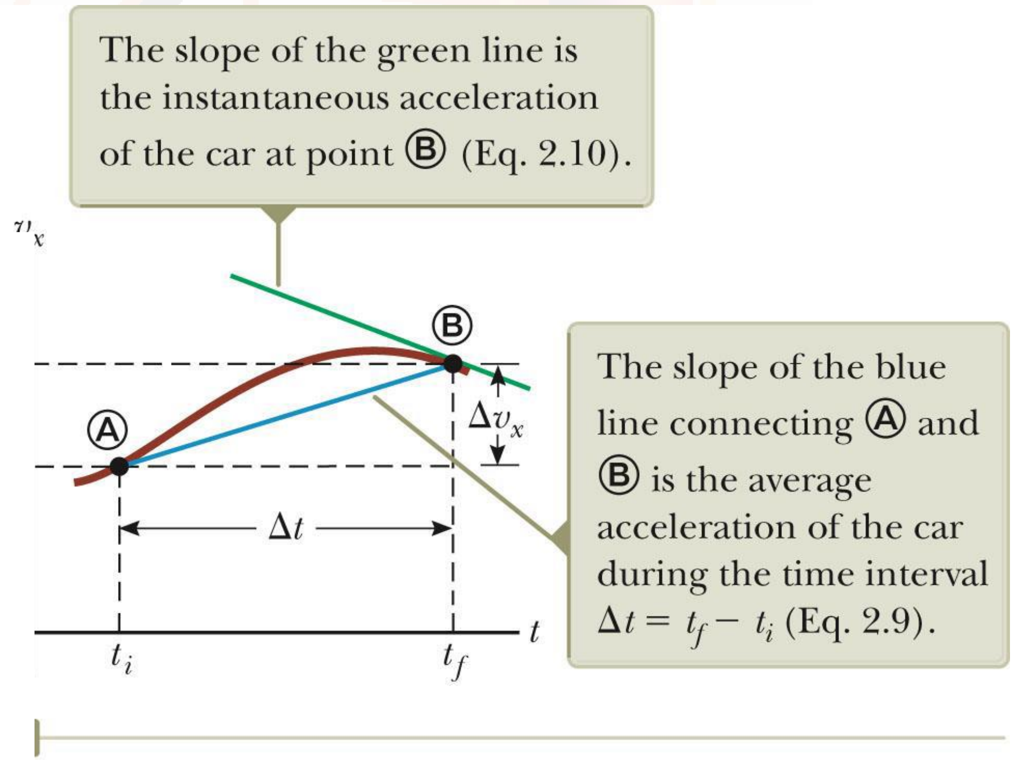
2.3 Acceleration

- **Instantaneous acceleration**

The slope of the velocity-time graph is the acceleration. The green line represents the instantaneous acceleration. The blue line is the average acceleration.



(a)



2.3 Acceleration

- **Acceleration and Velocity, Directions**

When an object’s velocity and acceleration are in the same direction, the object is speeding up. When an object’s velocity and acceleration are in the opposite direction, the object is slowing down.

2.3 Acceleration

Example 2.5 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

$$v_{xB} = (40 - 5t_B^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}}$$

$$= -10\text{ m/s}^2$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

2.3 Acceleration

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\ \text{m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t\ \text{m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0)\ \text{m/s}^2 = \boxed{-20\ \text{m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

Note that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.8 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Note also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.5.

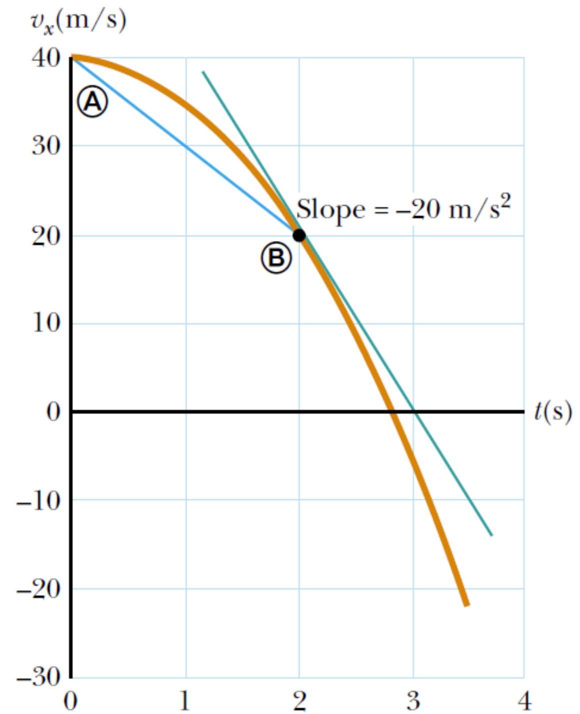


Figure 2.8 (Example 2.5) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

2.3 Acceleration

ple, suppose x is proportional to some power of t, such as in the expression

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is

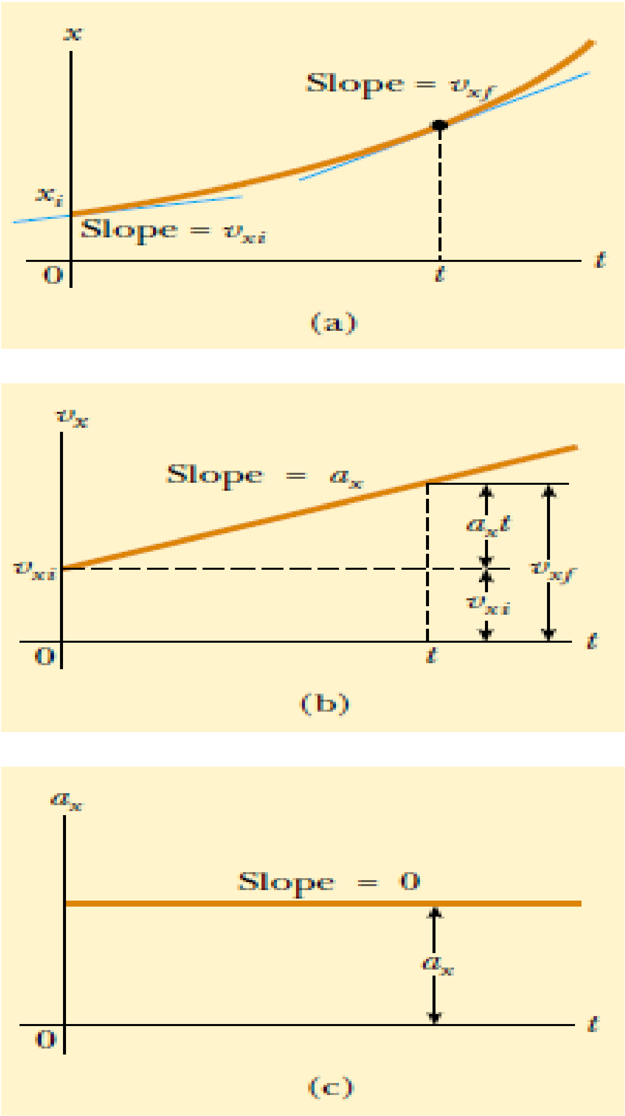
$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.5, in which $v_x = 40 - 5t^2$, we find that the acceleration is $a_x = dv_x/dt = -10t$.

2.5 One-Dimensional Motion with Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze.

- **Simple type of 1-D motion is that in which the acceleration is constant. When this is the case, the average acceleration over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.**



2.5 One-Dimensional Motion with Constant Acceleration

average acceleration

$$\overline{\boldsymbol{a}_x} = \frac{\Delta \boldsymbol{v}_x}{\Delta \boldsymbol{t}} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

average velocity

$$\overline{\boldsymbol{v}_x} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \frac{x_f - x_i}{\Delta \boldsymbol{t}}$$

or

$$x_f - x_i = \overline{\boldsymbol{v}_x}\, \Delta \boldsymbol{t}$$

$$\Delta x = x_f - x_i$$

$$\Delta \boldsymbol{t} = t_f - t_i$$

2.5 1-D Motion with Constant Acceleration

for situations $a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = Cons$ and $\Delta t = t_f - t_i = t - 0 = t$

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - 0} \quad or \quad v_{xf} = v_{xi} + a_x t$$

$$\overline{v_x} = \frac{v_{xi} + v_{xf}}{2}$$

$$While\ x_f - x_i = \overline{v_x}\,\Delta t = 1/2\left(v_{xf} + v_{xi}\right)t$$

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xi} + a_x t\ )t = x_i + \frac{1}{2}(2v_{xi} + a_x t\ )t$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \left(\frac{{v_{xf}}^2 - {v_{xi}}^2}{2a_x}\right)$$

$${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$$

2.5 One-Dimensional Motion with Constant Acceleration

For constant acceleration

$a_x = Cons$ **and** $\Delta \boldsymbol{t} = t$

$$\overline{v_x} = \frac{v_{xi} + v_{xf}}{2}$$

This applies only in situations where the acceleration is constant.

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

This gives you the position of the particle in terms of time and velocities. Doesn't give you the acceleration

$$\boldsymbol{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

Gives final position in terms of velocity and acceleration Doesn't tell you about final velocity

$$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$$

Gives final velocity in terms of acceleration and displacement Does not give any information about the time

2.5 One-Dimensional Motion with Constant Acceleration

For motion at zero acceleration

$a_x = 0$ **and** $v_x = C$

$$v_{x\boldsymbol{f}} = v_{x\boldsymbol{i}} = v_x$$

$$x_f = x_i + \frac{1}{2}(v_x + v_x)t$$

$$x_f = x_i + v_x t$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time.

The constant acceleration model reduces to the constant velocity model.

2.6 Freely Falling Objects

A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

- At the Earth's surface, the value of g is approximately 9.80 m/s2.
- we always choose a_y=-g=-9.80 m/s2

Lecture Summary

The average acceleration of a particle is defined as the ratio of the change in its velocity v_x divided by the time interval Δt during which that change occurs:

$$\overline{a_x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

The instantaneous acceleration is equal to the limit of the ratio $\frac{\Delta \mathrm{v_x}}{\Delta \mathrm{t}}$ as Δt approaches 0. By definition, this limit equals the derivative of $\mathrm{v_x}$ with respect to t, or the time rate of change of the velocity:

$$\mathrm{a_x} = \lim_{\Delta \mathrm{t} \to 0} \frac{\Delta \mathrm{v_x}}{\Delta \mathrm{t}} = \frac{\mathrm{dv_x}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right) = \frac{\mathrm{d^2x}}{\mathrm{dt^2}}$$

When the object’s velocity and acceleration are in the same direction, the objectis speed in gup. On the other hand, when the object’s velocity and acceleration are in opposite directions, the objectis slowing down.

Lecture Summary

The equations of kinematics for a particle moving along the x axis with uniform acceleration a_x are:

Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Thank You


ACKNOWLEDGEMENTS