PHYS 507
Lecture 1: The Classical Theory of Electromagnetic Fields –An Introduction

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The classical theory of e/m fields

- Classical theory of the electromagnetic field or Classical Electrodynamics, formulated by Maxwell about 150 years ago, is now a well established theory with many applications in different areas of physics, chemistry and engineering. In this context 'classical' means 'non-quantum', but we would like to point out that the basic equations of electromagnetism, the Maxwell's equations, hold equally in quantum and classical field theory.

Elementary aspects of e/m fields-a

1) Electromagnetic interactions are ONE of FOUR fundamental types:

Type of interaction	Relative strength
Strong interaction (nuclear)	1
Electromagnetic	10^{-2}
Weak interaction (nuclear)	10^{-12}
Gravitation	10^{-40}

2)Electromagnetic (EM) forces or interactions are due to ELECTRIC CHARGE, which is NOT in turn explicable in terms of anything else.

Elementary aspects of e/m fields-b

3) Charges are of two kinds called positive and negative. In the static limit like charges repel and unlike attract.

4) Charges are quantized in units of $e = 1.6 \times 10^{-19}\ C$.

5) In the static limit the inverse square (Coulomb) law of force holds:

$$\mathbf{F}_{q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}, \qquad q_1 \xrightarrow{\hat{r}} q_2$$

that the charge q_1 acts on the charge q_2 with the force $\mathbf{F}_{q_2}$. The force acts along the distance r and the parameter ε_0 determines the property of the medium and is called the **electric permittivity**.

Elementary aspects of e/m fields-c

The Coulomb's law holds for microscopic charges, i.e. it holds only for charges whose the spatial dimensions are small com- pared with the distance separating them.

The Coulomb's law does not tell us how the first charge knows the other one is present. One usually assumes that the charge produces an electric field which then interacts with the other charges. To express this explicitly, the Coulomb force is often written as

Elementary aspects of e/m fields-d

$$\mathbf{F}_{q_2} = q_2 \mathbf{E}_{q_1}$$

where $\mathbf{E}_{q_1}$ is the electric field produced by charge q_1

6) Electric charge is conserved (algebraically)

$$\sum_{\substack{\text{whole} \\ \text{universe}}} q = \text{constant}$$

7) In the NON-STATIC case, i.e. the case of moving charges, the force is no longer given by Coulomb's Law. It is given by the Lorentz equation

Elementary aspects of e/m fields-e

$$\mathbf{F}_{q_2} = q_2\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

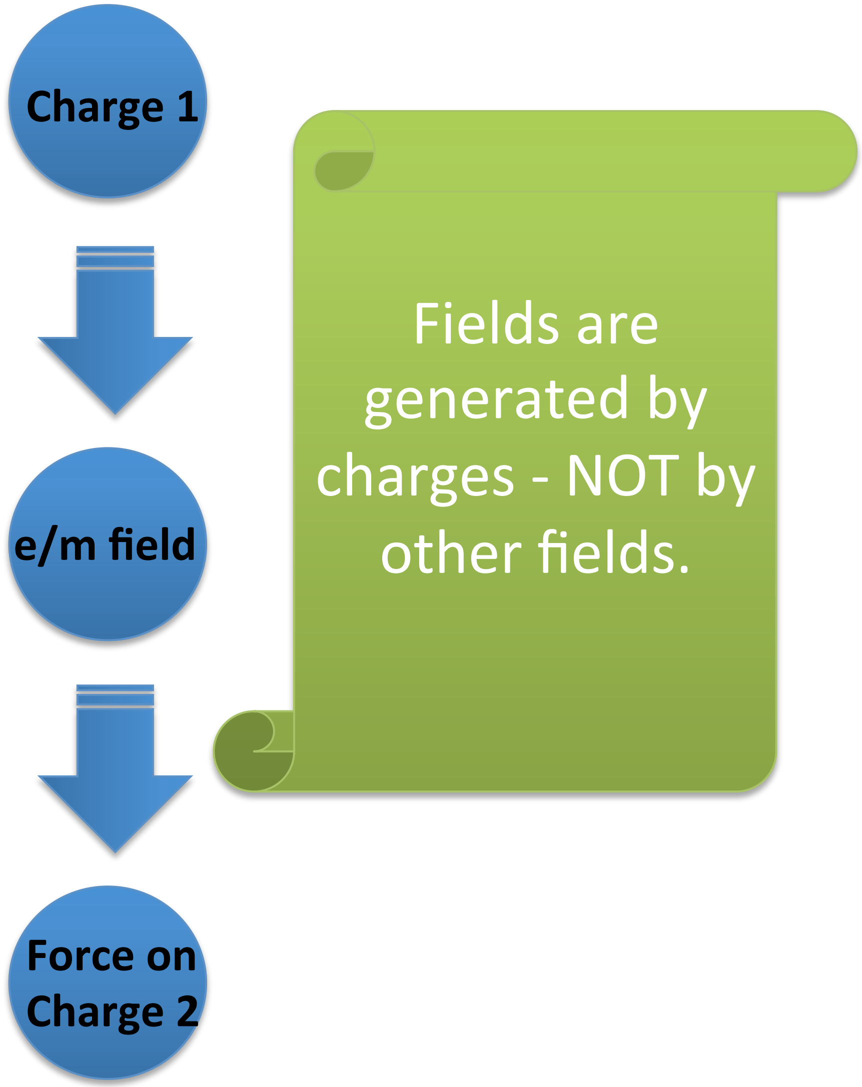
Where **E, B** are the electric field, **v** is the velocity and **B** the magnetic field.

8) In the electromagnetic theory, we assume that the fields **E** and **B** depend on the frame of reference of the observer, that **E**, **B** and the force **F** must follow the required relativistic transformation law.

Elementary aspects of e/m fields-f

The student has noticed that the Lorentz force involves both, the electric and magnetic fields. Why there must be a **B** and how **E** and **B** are computed for arbitrary motion of charges is the substance of electromagnetic theory.

We now have the following picture of the basis of the electromagnetic theory:



Macroscopic Charges/Currents a

- We know from the Millikan experiment that electric charge is quantized. The electron is a point charge on the smallest scale measurable. We may then speak, on a subatomic scale, of a microscopic theory of electromagnetism. On a subatomic scale there must be very strong and rapidly varying electric and magnetic fields on spatial scales ~ 10−8 m and temporal scales ~ 10−10 s.

Macroscopic Charges/Currents b

- When we measure the fields around a macroscopic circuit, clearly we are not looking at these fields. We are measuring fields on distance scales much larger than 10–8 m and time scales much longer than 10–10 s.
- In the macroscopic context, we do not distinguish individual charges. It is convenient and justifiable to regard the charge as a continuous ”fluid” dis- tributed over the volume or surface of a charged material.

Macroscopic Charges/Currents c

- Thus, we may introduce the concept of macroscopic charges and currents to indicate the macroscopic nature of electromagnetic phenomena.

Charge Density

- Charges may be distributed throughout the volume of a material, or on the surface of the material, or may be distributed along one dimensional wires. Thus, there are different spatial distributions of charges considered in the electromagnetic theory that result in three types of charge densities: volume charge density, surface charge density and linear charge density.

Volume Charge Density-a

When we encounter a large number of point charges in a finite volume, it is convenient to describe the source in terms of a volume charge density, defined as

$$\rho = \lim \frac{\sum q}{\Delta V}$$

where Σq is the algebraic sum of the charge in the volume ΔV .The limit is not to zero, but to a ΔV much larger than atomic scale size, which is still very small on the laboratory scale.

Volume Charge Density-b

- If the volume charge density is represented by a continuous function $\rho(\mathbf{r})$, the total charge Q in a volume V is given by

$$Q = \int_V \rho(\mathbf{r})\,dV$$

Note that the charge density is a function of the position which varies smoothly inside a charged material. An exception is a boundary between two charged materials where ρ may change discontinuously due to the presence of surface charges of a non-zero density. Thus, we may introduce the concept of **surface charge density**.

Surface Charge Density

- A surface charge density is defined, analogously to the volume charge density:

$$\sigma = \lim \frac{\sum q}{\Delta S}$$

where Σq is the algebraic sum of the charge on the surface ΔS. Then, the total charge continously distributed on a surface S is:

$$Q = \int_S \sigma\left(\mathbf{r}\right) dS$$

Linear Charge Density

- A linear charge density is defined analogously as:

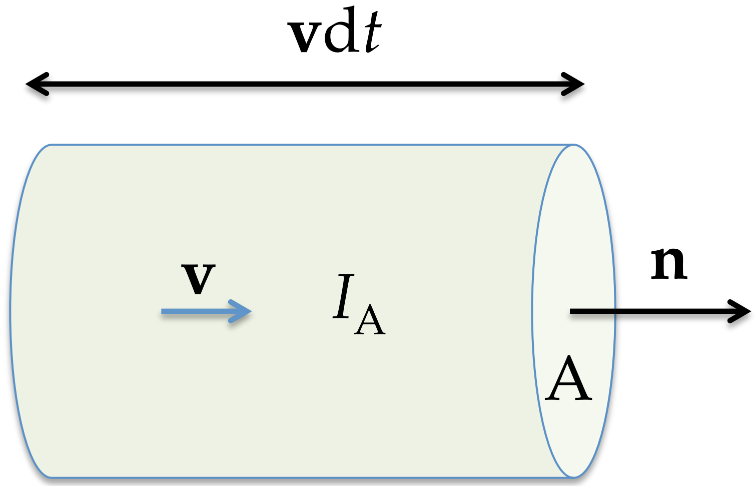
$$\lambda = \lim \frac{\sum q}{\Delta L}$$

where Σq is the total charge continuously distributed on the length L. Then, the total charge continously distributed on the length L is:

$$Q = \int_L \lambda(\mathbf{r})\, dL$$

Current density

- Current density measures the amount of current flowing through an area normal to the direction of the current.



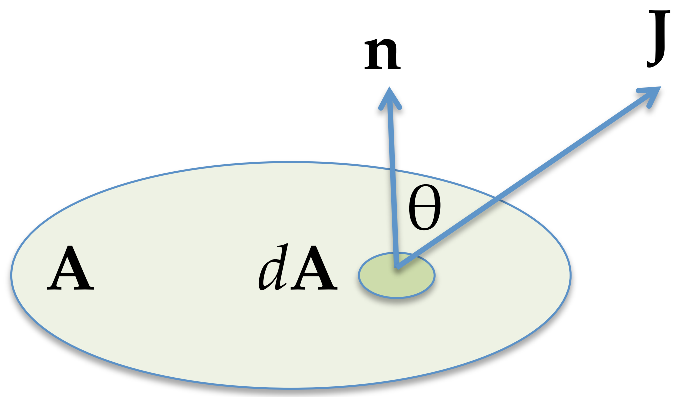
- We define the current density as follows:

$$\mathbf{J} = \lim \frac{I}{A}\mathbf{n} = \lim \frac{\delta q}{\delta t A}\mathbf{n} = \lim \frac{\rho A v \delta t}{\delta t A}\mathbf{n} \Rightarrow$$

$$\mathbf{J} = \rho \mathbf{v}$$

Total current through an arbitrary surface area

- For a surface of an arbitrary shape, the current may not be normal to the surface at all points on the surface.



$$I_A = \int \delta I = \int_A \mathbf{J} \cdot \mathbf{n} dA = \int_A \mathbf{J} d\mathbf{A}$$

In vector analysis it is common to represent a surface by a vector whose length corresponds to the magnitude of the surface area and whose the direction is specified by the unit vector **n** normal to the surface.

- The vector $d\mathbf{A} = dA\mathbf{n}$ is a vector representing the element dA of the surface A.