PHYS 507
Lecture 13: Electromagnetic Waves

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The Wave Equation in one dimension

- A wave propagating along the z-direction with a speed v is described by the following equation:

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

- This is the so called **wave equation**. It is a linear differential equation and it admits as solutions all the functions of the form $f(z, t) = g(z \pm vt)$. The solution with – represents a wave propagating along the positive z-direction while the solution with + a wave along the negative z-direction.
- In general $f(z, t) = g(z-vt)+h(z+vt)$.

Sinusoidal Waves

- The most common solution to the wave equation are the sinusoidal waves:

$$f(z,t) = A\cos\left[k(z - vt) + \delta\right] \longrightarrow \text{phase}$$

- A is the amplitude of the wave (positive)
- δ is a phase constant
- k is the wavenumber $k = 2\pi/\lambda$, where λ is the wavelength.
- The wave is a periodical phenomenon with a frequency f given by $f=v/\lambda$ and an angular frequency $\omega = 2\pi f = 2\pi v/\lambda = \mathrm{k}v$

The complex notation

- Making use of the Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, the wave equation can be written as:

$$f(z,t) = \mathrm{Re}\left[Ae^{i(kz-\omega t+\delta)}\right]$$

- Or if we introduce the following function

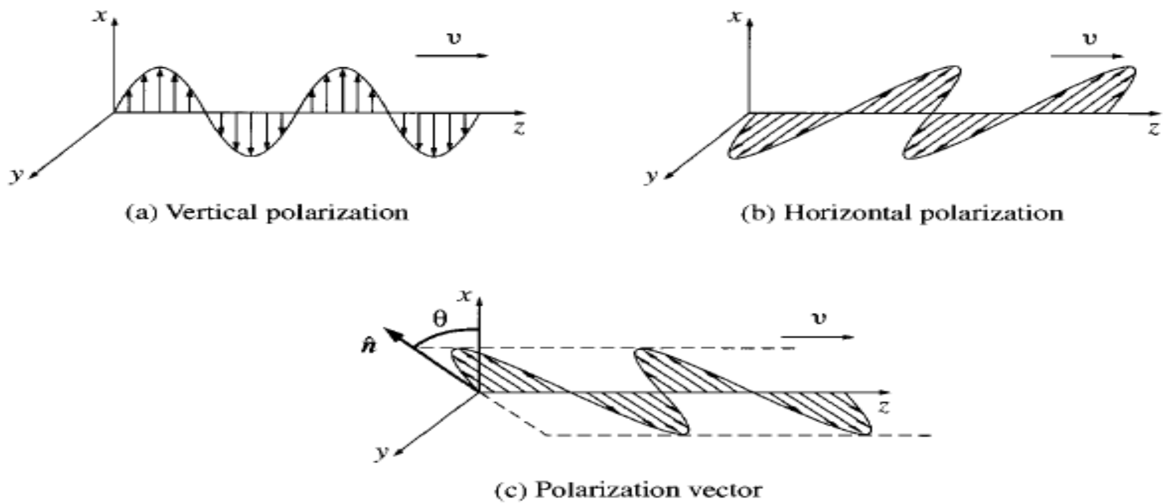
$$\tilde{f}(z,t) \equiv \tilde{A}e^{i(kz-\omega t)}$$

Where the complex amplitude $\tilde{A} \equiv Ae^{i\delta}$ absorbs the phase constant then the actual wave is the

$$f(z,t) = \mathrm{Re}\left[\tilde{f}(z,t)\right]$$

Polarization

- Waves can be classified in **transverse** and **longitudinal** ones. Electromagnetic waves far from their sources are **transverse**.
- Electromagnetic waves are characterized from the direction along which the electric field oscillates. In other words of their **polarization**.



(a) Vertical polarization (b) Horizontal polarization

(c) Polarization vector

E-M Waves

- The electric and magnetic fields of an E-M wave do satisfy the following relations:

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \qquad \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathrm{B}}{\partial t^2}$$

- And its of their components satisfy the relations:

$$\nabla^2 E_i = \mu_0 \varepsilon_0 \frac{\partial^2 E_i}{\partial t^2}, \qquad \nabla^2 B_i = \mu_0 \varepsilon_0 \frac{\partial^2 B_i}{\partial t^2}, \quad i = x, y, z$$

Monochromatic Plane Waves

- E-M fields with a single frequency propagating along a direction and do not depend on the other two directions they are called monochromatic plane waves. They are described by the following relations:

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}, \qquad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)}$$

- These waves are transverse and for them we can show the following relations:

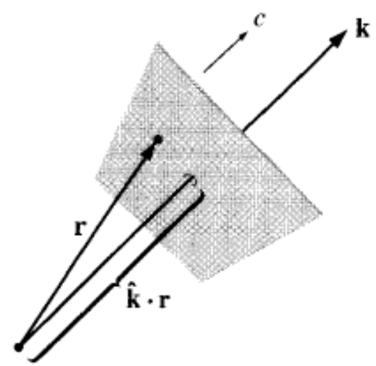
$$\tilde{\mathbf{B}}_0 = \frac{k}{\omega}\left(\hat{\mathbf{z}} \times \tilde{\mathbf{E}}_0\right) \qquad\qquad B_0 = \frac{k}{\omega} E_0$$

Propagation in a random direction

- A plane E-M wave can travel in any direction. The description is facilitated by the introduction of the **propagation wave vector, k.**

$$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{n}}, \qquad \tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\left(\hat{\mathbf{k}}\times\hat{\mathbf{n}}\right) = \frac{1}{c}\left(\hat{\mathbf{k}}\times\tilde{\mathbf{E}}\right)$$

- Where $\hat{\mathbf{n}}$ is the polarization vector. Because **E** is transverse $\hat{\mathbf{n}}\cdot\hat{\mathbf{k}} = 0$.



Energy and Momentum

- As the wave propagates it carries energy with it. The **energy flux**, i.e energy per unit area, per unit time, is given by the Poynting vector:

$$\mathbf{S} = \frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right)$$

- It also carries a momentum. The momentum density in the fields is:

$$\wp = \frac{1}{c^2}\mathbf{S}$$

Intensity-Radiation Pressure

- The average power per unit area transported by an electromagnetic wave is called the **intensity**:

$$I \equiv \langle S \rangle = \frac{1}{2} c \varepsilon_0 E_0^2$$

- When light falls on a perfect absorber it delivers its momentum to the surface. In a time Δt the momentum transfer is $\Delta \mathbf{p} = \langle \wp \rangle A c \Delta t$, so the **radiation pressure** (average force per unit area) is:

$$P = \frac{1}{A} \frac{\Delta p}{\Delta t} = \frac{1}{2} \varepsilon_0 E_0^2 = \frac{I}{c}$$