PHYS 507
Lecture 11: Electromotive Force

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Ohm’s Law

- For most of the materials, the current denasity **J** is proportional to the the **force per unit charge f**:

$$\mathbf{J}=\sigma\mathbf{f} \quad (11.1)$$

- The proportionality factor σ (must not be confused with surface charge density) is called **conductivity** and depends on the medium.
- Conductivity is related to **resistivity** ρ by the relation $\rho=1/\sigma$.
- For a metal σ is of the order of $10^{22.}$
- For a **perfect conductor** $\sigma=\infty$.

Ohm's Law

- For an electromagnetic force acting on a charge we have:

  $$\mathbf{J}=\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \quad (11.2)$$

- For most of the cases the velocity of charges is small thus:

  $$\mathbf{J}=\sigma\mathbf{E} \qquad (11.3)$$

- Eq. (11.3) is **Ohm's Law**.

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Salt water (saturated)	4.4×10^{-2}
Copper	1.68×10^{-8}	Germanium	4.6×10^{-1}
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2.5×10^{3}
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.58×10^{-7}	Water (pure)	2.5×10^{5}
Nichrome	1.00×10^{-6}	Wood	$10^{8} - 10^{11}$
Manganese	1.44×10^{-6}	Glass	$10^{10} - 10^{14}$
Graphite	1.4×10^{-5}	Quartz (fused)	$\sim 10^{16}$

Table 7.1 Resistivities, in ohm-meters (all values are for 1 atm, 20° C).
Source: Handbook of Chemistry and Physics, 78th ed.
(Boca Raton: CRC Press, Inc., 1997).

Ohm's Law- a confusion

- In lecture 2 we have said that **E**=0 inside a conductor! What happens then?
- In that case we were talking about stationary charges (**J**=0). But for **perfect conductors** **E**=**J**/σ=**J**/∞= 0 even when current is flowing.
- In practice, metals are such good conductors that the electric field required to drive current in them is negligible. Thus we routinely treat the connecting wires in electric circuits as equipotentials.
- **Resistors**, by contrast, are made from **poorly** conducting materials.

Ohm’s Law

- A direct consequence of Eq.(11.3), in a resistor, is the familiar relation, *V*=*IR*.
- The constant of proportionality *R* is called the **resistance** and is measured in **ohms** (Ω).
- For *steady* currents and *uniform* conductivity we get

$$\nabla \cdot \mathbf{E} = \frac{1}{\sigma} \nabla \cdot \mathbf{J} = 0 \qquad (11.4)$$

- As a result the charge density is zero; any unbalanced charge resides on the *surface*. This means that Laplace’s equation holds within a homogeneous ohmic material carrying a steady current. All the techniques we have learned for computing the potential in the first lectures are valid here.

Ohm's Law - discussion

- Ohm's law ever holds. But how can this conform with the fact that a field **E** which exerts a force $q\mathbf{E}$ on a charge q will accelerate it? And if the charges are accelerating why the current does not increase?
- Our analysis has ignored first the collisions electrons make as they pass down the wire and then the thermal motion of the electrons.

Joule heating law

- As a result of all the collisions, the work done by the electrical force is converted into heat in the resistor. Since the work done per unit charge is V and the charge flowing per unit time is I, the power delivered is

- $$P = VI = I^2 R \qquad (11.5)$$

Electromotive Force

- The are actually **two** forces involved in driving current around a circuit: the **source force** $\mathbf{f}_S$ which is ordinarily confined to one portion of the loop (a battery, say), and the **electrostatic** force which serves to smooth out the flow and communicate the influence of the source to distant parts of the circuit:

$$\mathbf{f} = \mathbf{f}_S + \mathbf{E} \qquad (11.6)$$

- The physical agency responsible for $\mathbf{f}_S$ can be any one of many different things: in a battery it is a chemical force, in a piezoelectric crystal mechanical pressure is converted into an electrical impulse, in a thermocouple it is a temperature gradient, in a photelectric cell it is light.

Electromotive Force

- Whatever is the mechanism, its net effect is determined by the line integral of **f** around the circuit:

$$E \equiv \oint \mathbf{f} \cdot d\mathbf{l} = \oint \mathbf{f}_S \cdot d\mathbf{l} \qquad (11.7)$$

- Question: Why in 11.6 it does not matter whether you use **f** or $\mathbf{f}_S$.
- The quantity E is called the **electromotive force** or **emf**. Be careful it is not a force at all –it's the integral of a force per unit charge.

Electromotive Force

- Within an ideal source of emf (e.g. a resistanceless battery), the **net** force on the charges is zero (Eq.11.1 with $\sigma=\infty$), so **E=-f**$_{S.}$ The potential difference between the terminals *a* and *b* is therefore

$$V = -\int_a^b \mathbf{E}\cdot d\mathbf{l} = \int_a^b \mathbf{f}_S \cdot d\mathbf{l} = \oint \mathbf{f}_S \cdot d\mathbf{l} = E \quad (11.8)$$

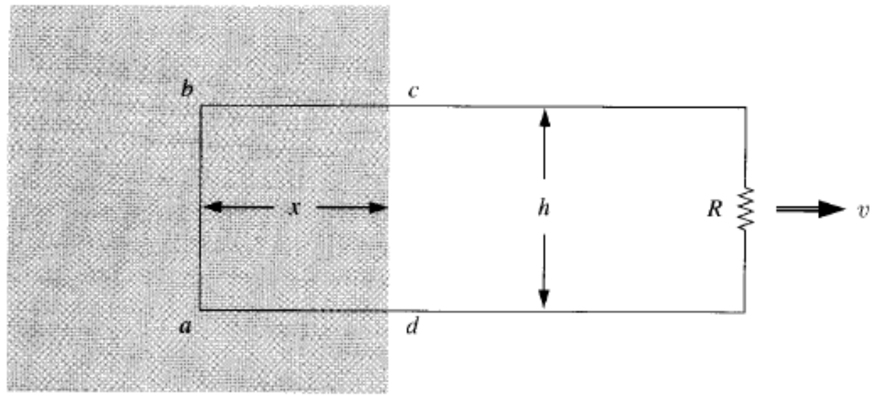
Why the integral can be extended to the whole loop?

- The emf can be interpreted as the work done per unit charge by the source.

Real batteries have a certain **internal** resistance, *r*, and the potential difference between their terminals is *E-Ir*, when a current *I* is flowing. For a nice discussion on how batteries work see D. Roberts, *Am. J. Phys*. **51**, 829 (1983).

Motional emf

- **Motional emf** arises when you move a wire through a magnetic field.
- Generators exploit motional emf.
- The figure shows a primitive model of a generator. The circuit moves in a uniform magnetic field (shaded region)



Motional emf

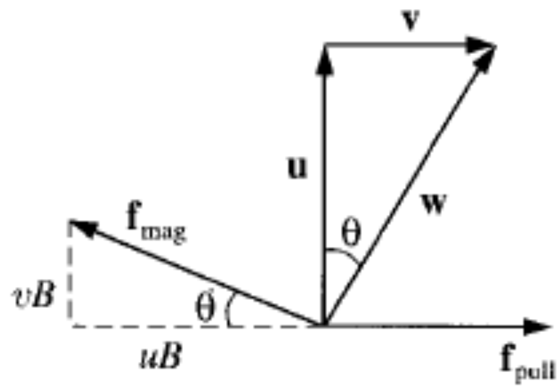
- If the entire loop is pulled to the right with a speed v, the charges in segment ab experience a magnetic force whose vertical component qvB drives the current around the loop, in the clockwise direction. The generated emf is:

$$E = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh \qquad (11.9)$$

where h is the width of the loop.

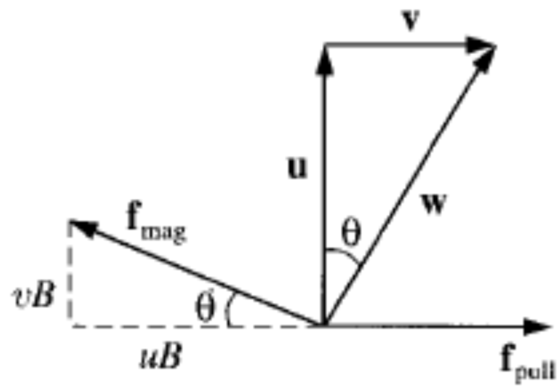
Motional emf - discussion

- The magnetic force establishes emf but – do not forget this- does not produce work. Who is supplying the energy that heats the resistor?
- Answer: The person who is pulling the loop! With the current flowing, the charges in segment *ab* have a vertical velocity (call it **u**) in addition to the horizontal velocity **v** they inherit from the motion of the loop.



Motional emf − discussion

- To counteract this, the person pulling on the wire must exert a force per unit charge $f_{\text{pull}} = uB$ to the right. This force is transmitted to the charge by the structure of the wire.
- The particle has a resultant velocity **w** and a total displacement $(h/\cos\theta)$. The work per unit charge is therefore



$$\int \mathbf{f}_{pull} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = E$$

Motional emf − discussion

- As it turns out, then, **the work done per unit charge is exactly equal to the emf**, though the integrals are taken along entirely different paths and completely different forces are involved.
- To calculate the emf you integrate around the loop at one instant, but to calculate the work done you follow a charge in its motion around the loop.
- $\mathbf{f}_{pull}$ conntributes nothing to the emf, because it is perpendicular to the wire; whereas $\mathbf{f}_{mag}$ contributes nothing to work because it is perpendicular to the motion of the charge.

Motional emf and flux of B

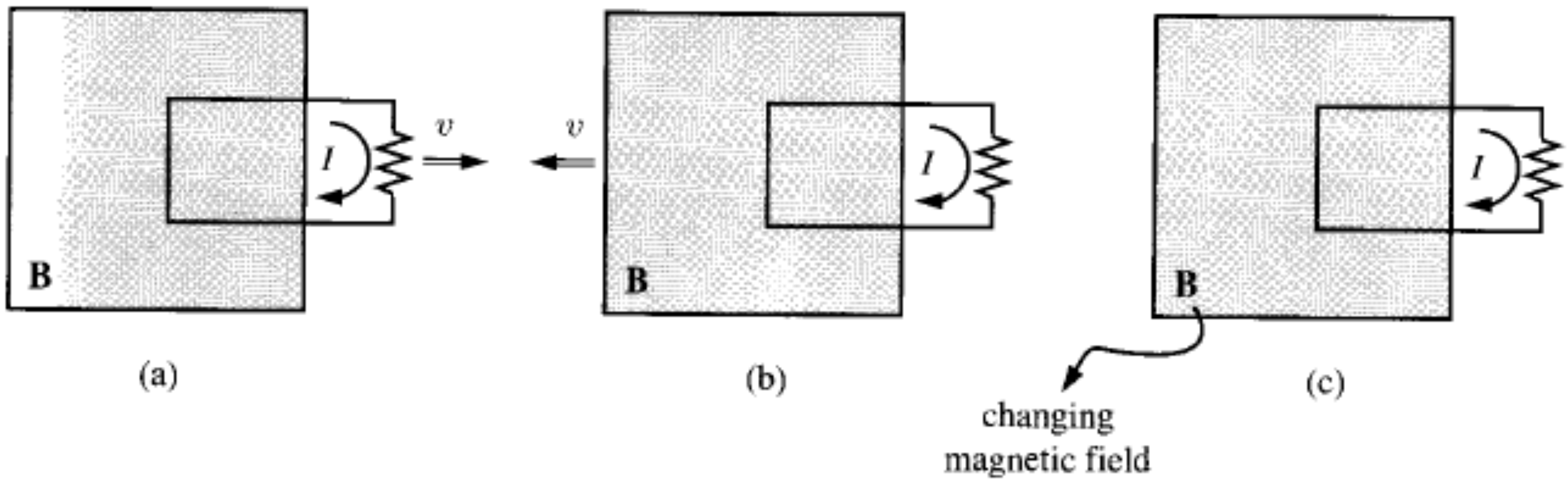
- It can be shown that the motional emf and the flux of the magnetic field are related through the so called **flux rule** for motional emf.

$$E = -\frac{d\Phi}{dt} \qquad (11.10)$$

- Apart from its simplicity, it applies to any shaped loop moving in arbitrary directions through nonuniform magnetic fields. The loop, in fact, need not even maintain a fixed shape.

Electromagnetic Induction

- In 1831 Michael Faraday performed the following three experiments:



Electromagnetic Induction

- After these experiments Faraday arrived at the following genious conclusion:

A changing magnetic field induces an electric field

- This conclusion can be expressed mathematically by the following integral and differential forms:

$$E = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

Whenever and for whatever reason the magnetic flux through a loop changes this emf will appear in the loop.

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \Phi}{\partial t} \cdot d\mathbf{a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

The induced electric field

- Faraday's discovery tells us that there are two distinct kinds of electric fields: those attributable directly to electric charges, and those associated with changing magnetic fields. The former are calculated by Coulomb's law and the second from the relation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

in exactly the same way as magnetostatic fields are generated by $\mu_0\mathbf{J}$.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

The induced electric field

- In particular, if symmetry permits, we can use all the tricks associated with Ampere’s law in integral form:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$$

but only this time it is Faraday’s law in integral form:

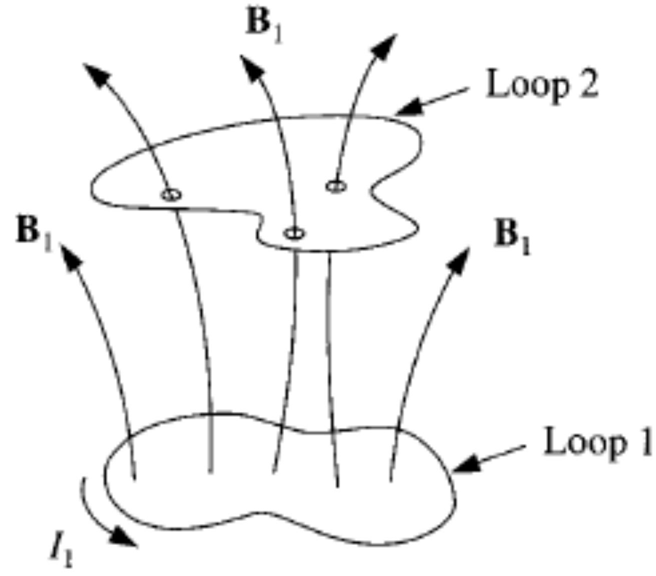
$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

- The rate of change of magnetic flux through the Amperian loop plays the role formerly assigned to $\mu_0 I_{\mathrm{enc}}$

Inductance

- When we have two circuits nearby any change of the current in one of them induces an emf to the other circuit.
- Indeed the flux through the second loop is related to the current in the first circuit with the relation:

$$\Phi_2 = M_{21} I_1$$



Inductance

- The quantity M_{21} is a constant of proportionality known as the **mutual inductance** of the two loops.
- The mutual inductance is given by Neumann formula:

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r}$$

1. M_{21} is a purely geometrical quantity, having to do with the sizes, shapes, and relative positions of the two loops.

2. The integral is unchanged if we switch the roles of loops 1 and 2, i.e. $M_{21} = M_{12}$.

- Whatever the shapes and positions of the loops the flux through 2 when we run a current *I* around 1 is identical to the flux through 1 when we send the same current *I* around 2. We may as well drop the subscripts and call them *M.*

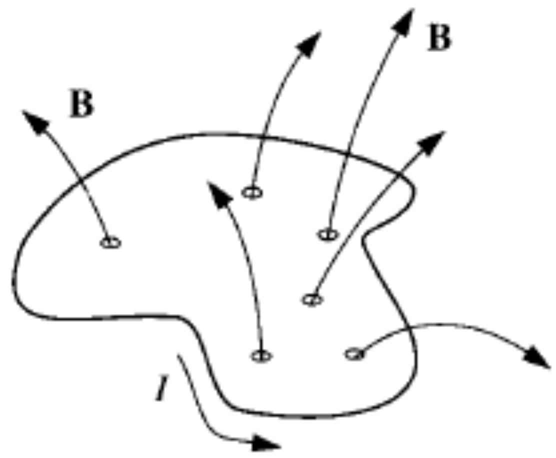
Self-Inductance

- A changing current not only induces an emf in any nearby loops, it also induces an emf in the source loop **itself**.
- Once again, the field is proportional to the current

$$\Phi = LI$$

- The constant of proportionality *L* is called the **self-inductance** (or simply the **inductance**)
- The emf induced in the loop is

$$E = -L\frac{dI}{dt}$$



Self-Inductance

- Inductance is a positive quantity.
- Lenz's law, which is enforced by the minus sign, dictates that the emf is in such a direction at to **oppose** any change in current. For this reason, it is called a **back emf**. Whenever you try to alter the current in a wire, you must fight against this back emf. Thus inductance plays somewhat the same role in electric circuits that **mass** plays in mechanical systems. The greater L is, the harder it it to change the current.

Energy in Magnetic Fields

- It takes an amount of energy to start a current flowing in a circuit because you must do work **against the back emf** to get current going.
- This is a **fixed** amount and it is **recoverable**: you get it back ehen the current is turned off.
- This is given by the following formula:

$$W = \frac{1}{2} L I^2$$

Energy in Magnetic Fields

- It can be shown that this energy is stored in the magnetic field and can take the following simple form:

$$W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau$$