PHYS 507
Lecture 10: Magnetic Fields in Matter

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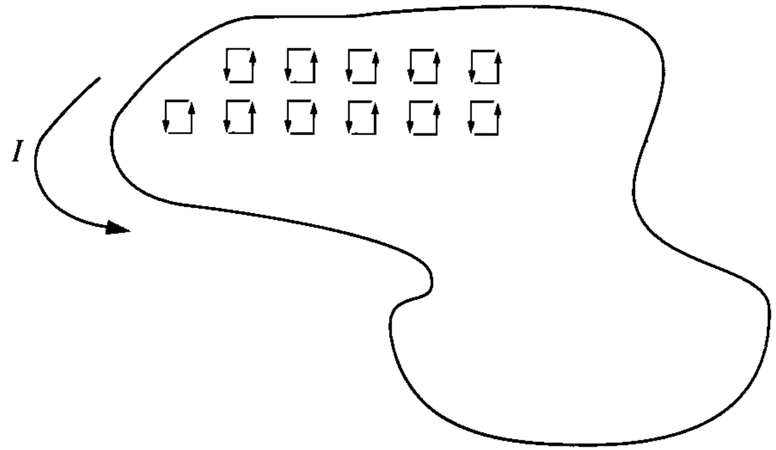
Diamagnetics – Paramagnets - Ferromagnets

- When a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or **magnetized**.
- Unlike electric polarization, which is almost always in the same direction **E**, some materials acquire a magnetization **parallel** to **B** (**paramagnets**) and some **opposite** to **B** (**diamagnets**).
- A few substances (called **ferromagnets**), retain their magnetization after the external field has been removed. For these the magnetization is not fetermined by the **presen**t field but by the whole magnetic “history” of the object.

Torques and Forces on Magnetic Dipoles

- A magnetic dipole inside a magnetic field experiences a torque just as an electric dipole inside an electric field.
- We start from a rectangular loop since any current loop can be built up from infinitesimal rectangles with all the “internal” sides cancelling as shown in figure.



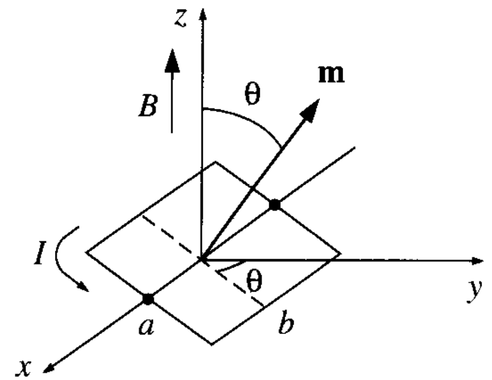
Torque on a rectangular loop-a

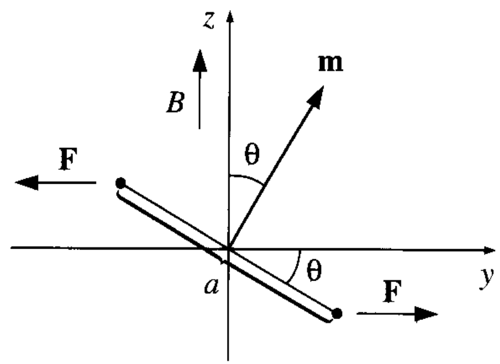
- It can be shown that the torque exerted on a square loop, with a current I in a magnetic field $\mathbf{B}$, is given by:

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}$$

- With $\mathbf{m}=I\mathbf{S}$ ($\mathbf{S}$: the vector for the surface).

$$\mathbf{m} = I\mathbf{S} = IS\hat{\mathbf{n}}$$



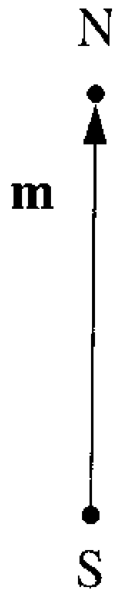


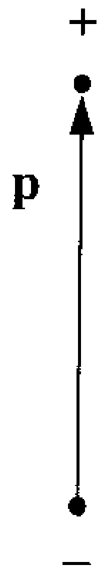
Torque on a rectangular loop-b

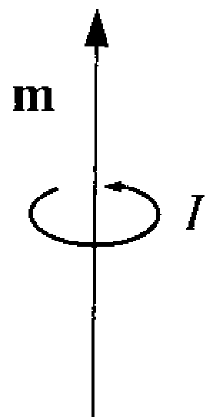
- Note that the torque from an electric field on an electric dipole was given by: $\mathbf{N} = \mathbf{p} \times \mathbf{E}$
- The torque is again in such a direction as to align the dipole moment of the loop with the magnetic field. It is this torque that accounts for **paramagnetism.**
- For an **infinitesimal** loop with dipole moment **m**, in a field **B**, the net force is:

$$\mathbf{F} = \nabla\left(\mathbf{m} \cdot \mathbf{B}\right)$$

Electricity and Magnetism







The "old"
Gilbert model
of magnetic dipole

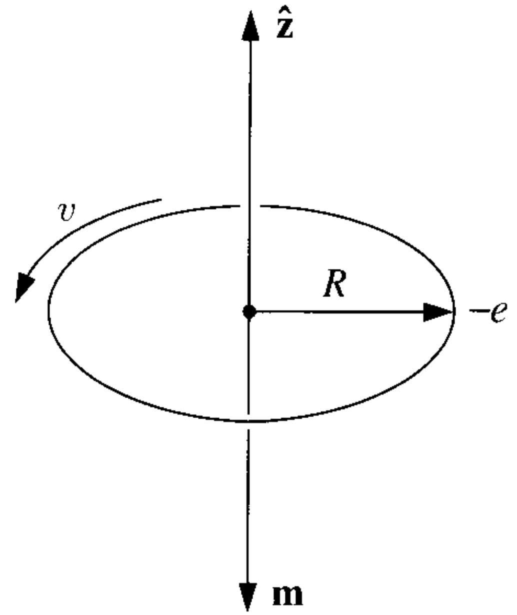
The electric dipole

The "modern"
Ampere model
of magnetic dipole

Effect of a Magnetic field on Atomic Orbits

- Electrons have not only **spin**; they also **revolve** around the nucleus. Their orbital dipole moment is:

$$\mathbf{m} = -\frac{1}{2} evR\hat{\mathbf{z}}$$



Magnetization-a

- **Paramagnetism:** The dipoles associated with the spins of unpaired electrons experience a torque tending to line them up parallel to the field.
- **Diamagnetism:** The orbital speed of electrons is altered in such a way as to change the orbital dipole moment in a direction opposite to the field.

Magnetization-b

- In general, when a material is placed in a region of nonuniform field, **the paramagnet is attracted into the field**, whereas the **diamagnetic is repelled away**.
- Whatever the cause, we describe the state of magnetic polarization by the vector quantity:
  - **M** = *magnetic dipole moment per unit volume*
- **M** is called the **magnetization** (it plays a role analogous to the polarization **P** in electrostatics).

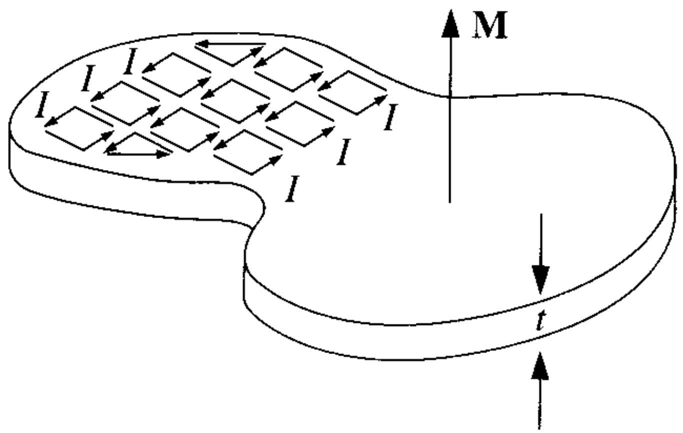
The Field of a Magnetized Object-a

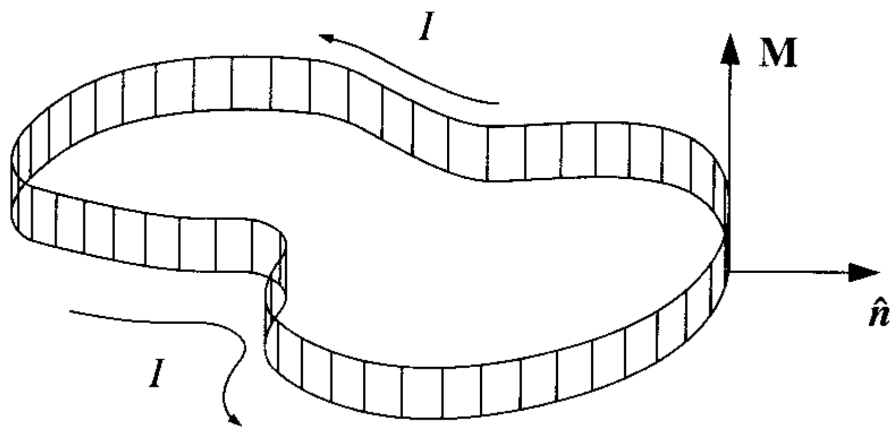
- Suppose we have a piece of magnetized material; the magnetic dipole moment per unit volume, **M**, is given. What field does this object produce?
- It can be proved that the field produced by the magnetized object is the same as would be produced by a volume current $\mathbf{J}_b = \nabla \times \mathbf{M}$ throughout the material plus a surface current $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$ on the boundary.

Recall the striking parallel with the electrical case: there the field of a polarized object was the same as that of the bound volume charge $\rho_b = -\nabla \cdot \mathbf{P}$ plus a bound surface charge $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$.

Physical Interpretation of Bound Currents

- All the “internal” currents cancel: every time there is one going to the right, a contiguous one is going to the left. However, at the edge there is **no adjacent loop to do the cancelling.**
- The whole thing is equivalent to a single ribbon of current *I*, flowing around the boundary.





The Magnetic Field Inside Matter – The Auxiliary Field H-a

- The total current inside matter is made up by a free and a bound charge: $\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$
- The Ampere's law takes the form:

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad \oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}}$$

- With the vector **H** given by:

$$\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

- **H** plays a role in magnetostatics analogous to **D** in electrostatics: Just as **D** allowed us to write Gauss's law in terms of the free charge alone, **H** permits us to express Ampere's law in terms of the free current alone.

The Magnetic Field Inside Matter – The Auxiliary Field H-b

- We must be careful for the correspondence with **E** and **D**. Remember that **D=**ε_0**E**, we cannot write **B=**μ_0**H** unless

$$\nabla \cdot \mathbf{M} = 0$$

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Boundary conditions

- The magnetostatic boundary conditions can be written in terms of **H** and the free current:

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -\left(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}\right)$$

$$\mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

- In the presence of the magnetic materials these are sometimes more useful than the corresponding boundary conditions on **B**

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0$$

$$\mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0\left(\mathbf{K}_f \times \hat{\mathbf{n}}\right)$$

Linear and Non-linear Media

- In most substances the magnetization is proportional to the field, provided the field is not too strong.

$$\mathbf{M} = \chi_m \mathbf{H}$$

- The constant of proportionality χ_m is called the **magnetic susceptibility**; Materials which obey the above relation are called **linear media.** For these materials:

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right) = \mu_0 \left(1 + \chi_m \right) \mathbf{H}$$

- Where μ is called the **permeability** of the material:

$$\mathbf{B} = \mu \mathbf{H}, \quad \mu = \mu_0 \left(1 + \chi_m \right)$$

- Incidentally, the volume bound current density in a homogeneous linear material is proportional to the **free** current density:

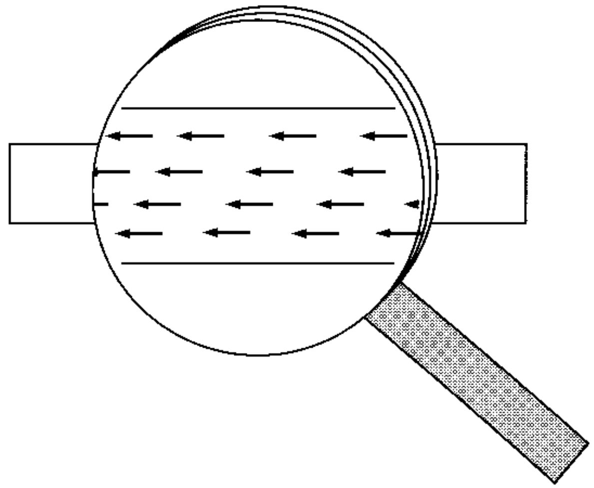
$$\mathbf{J}_b = \nabla \times \mathbf{M} = \nabla \times \left(\chi_m \mathbf{H} \right) = \chi_m \mathbf{J}_f$$

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.6×10^{-4}	Oxygen	1.9×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.1×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.8×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.8×10^{-4}
Carbon Dioxide	-1.2×10^{-8}	Liquid Oxygen ($-200°$ C)	3.9×10^{-3}
Hydrogen	-2.2×10^{-9}	Gadolinium	4.8×10^{-1}

Table 6.1 Magnetic Susceptibilities (unless otherwise specified, values are for 1 atm, 20° C). *Source: Handbook of Chemistry and Physics,* 67th ed. (Boca Raton: CRC Press, Inc., 1986).

Ferromagnetism-a

- Ferromagnets require no external fields to sustain the magnetization: the alignment is “frozen in”.
- The feature, which makes ferromagnetism so different from paramagnetism, is the interaction between nearby dipoles: each dipole “likes” to point in the same direction as its neighbors. The reason is of quantum mechanical explanation.

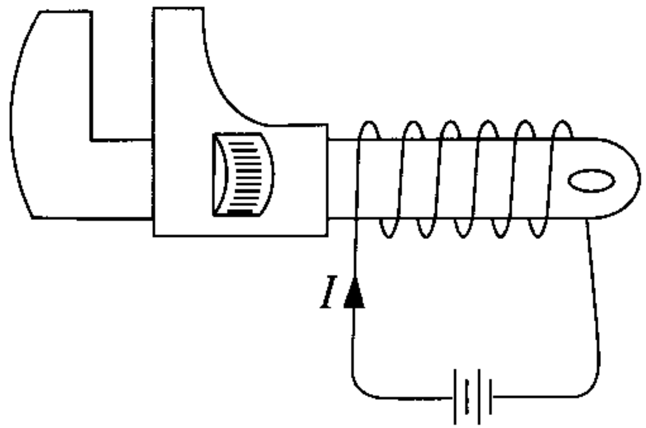


Ferromagnetism-b

- The alignment occurs in relatively small patches, called **domains**. Each domain contains billions of dipoles, all lined up. Each domain has its own orientation. That is why iron is not a permanent magnet.
- How the iron is magnetized?
- If we place iron in a strong magnetic field then the torque exerted on the domains by the field moves the domain boundaries: Domains parallel to the field grow, and the others shrink. If the field is strong enough, one domain takes over entirely, and the iron is said to be “saturated”.

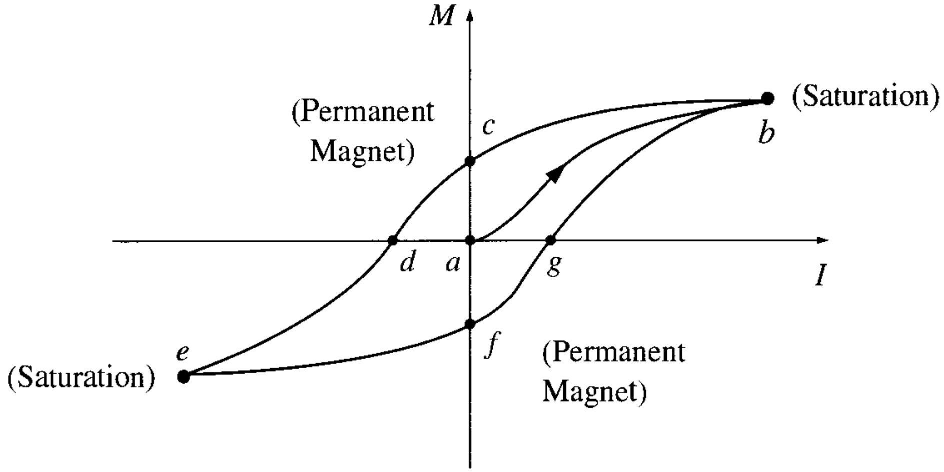
Ferromagnetism-c

- This process is not entirely reversible. When the field is switched off, there will be some return to randomly oriented domains. The object is now a permanent magnet.
- A simple way to accomplish this, in practice, is to wrap a coil of wire around the object to be magnetized. Run a current *I* through the coil.



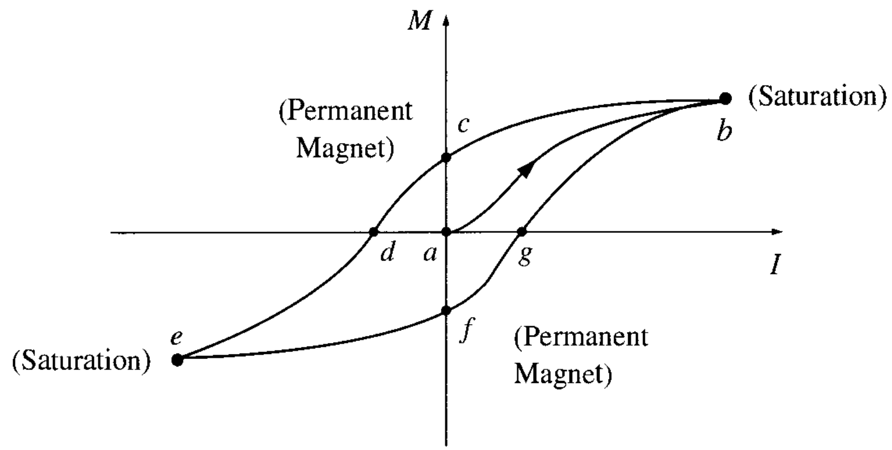
Ferromagnetism-d

- As you increase the current, the field increases, the domain boundaries move, and the magnetization grows. Eventually, you reach the saturation point, with all dipoles aligned. A further increase on current has no effect on **M**.



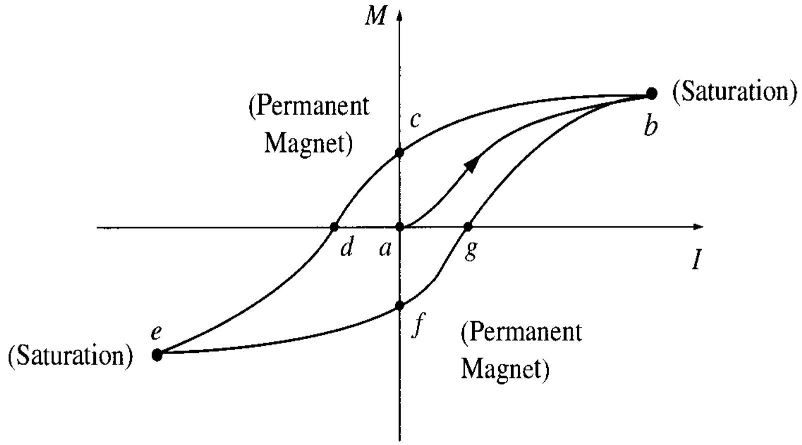
Ferromagnetism-e

- Now suppose you **reduce** the current. Instead of retracting the path back to $M = 0$, there is only a partial return to randomly oriented domains. M decreases, but even with the current off there is some residual magnetization. The wrench is now a permanent magnet.



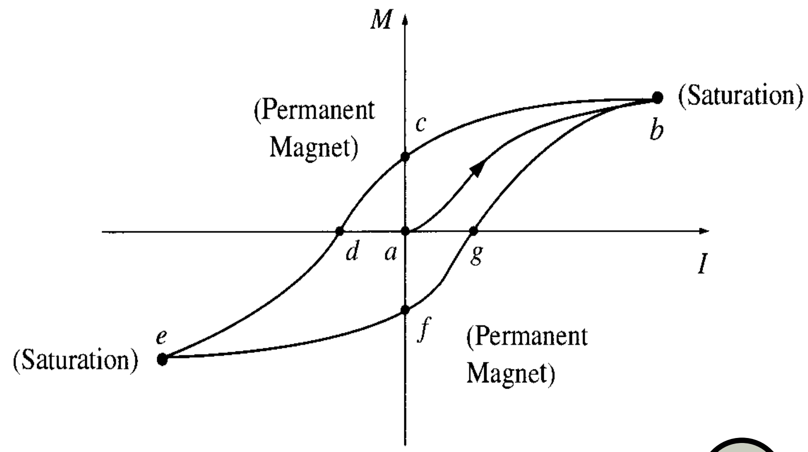
Ferromagnetism-f

- If you want to eliminate the remaining magnetization, you will have to run a current backwards through the coil (a negative *I*). Now the external field points to the right, and as you increase *I* (negatively), *M* drops down to zero (point *d*). By keep increasing you reach saturation in the other direction - all the dipoles now pointing to the right (e)



Ferromagnetism-g

- At this stage switching off the current will leave the wrench with a permanent magnetization to the right (point *f*). Turn now *I* on again in the positive sense: *M* returns to zero (point *g*) and eventually to the forward saturation point (*b*).
- The path we have traced out is called a **hysteresis loop**.



The magnetization depends not only on the applied field but also in the "history". At three different times in our experiment the current was zero (*a*, *c*, and *f*), yet the magnetization was different for each of them.

Ferromagnetism-h

- Ferromagnetism comes from the fact that the dipoles within a given domain line up parallel to one another.
- Random thermal motions compete with this ordering, but as long as the temperature does not get too high, they cannot destroy the alignment of dipoles.
- What is surprising is that alignment is destroyed at a precise temperature (770^0 C, for iron) which is called **Curie point**. Below this iron is ferromagnetic, above it is paramagnetic. This is an example of a phase transition.