PHYS 111

1ST semester 1446

Prof. OMAR H. M. ABD-ELKADER

Lecture 13

NUCLEUS STRUCTURE

NUCLEUS STRUCTURE

The nucleus consists of PROTONS and NEUTRONS called NUCLEONS, the sum of which is called the total number A.

- **The number of protons in the nucleus is the atomic number Z.**
- **The number of neutrons in the nucleus is N, where A=Z+N.**

There are 92 elements in nature, starting with hydrogen (Z=1) to uranium (Z=92).

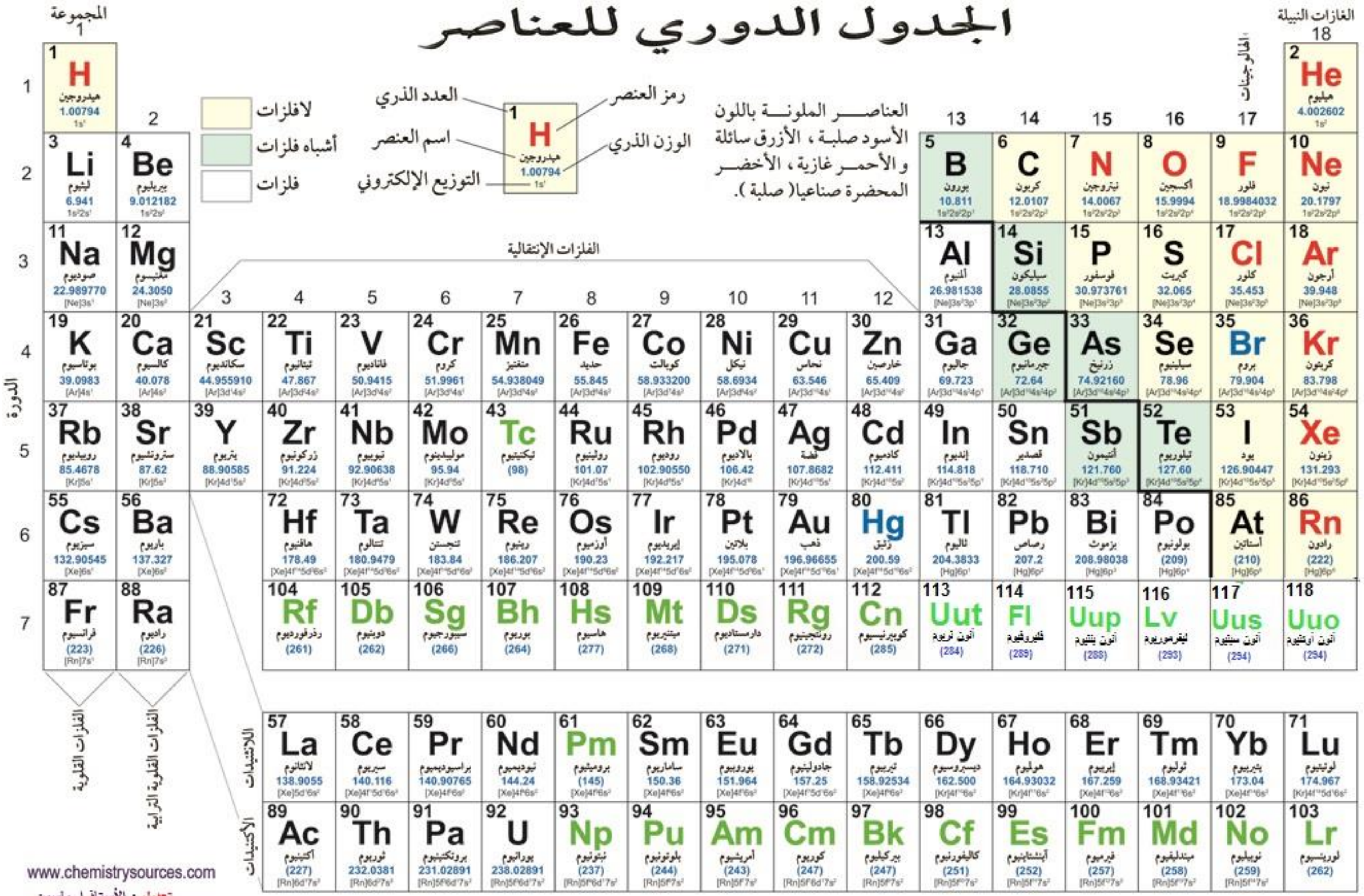
- **Using nuclear accelerators, it was possible to produce artificial elements whose atomic number reached Z=118.**
- $_{Z}^{A}X$

الجدول الدوري للعناصر

لافلزات
أشباه فلزات
فلزات

العدد الذري — رمز العنصر — اسم العنصر — الوزن الذري — التوزيع الإلكتروني

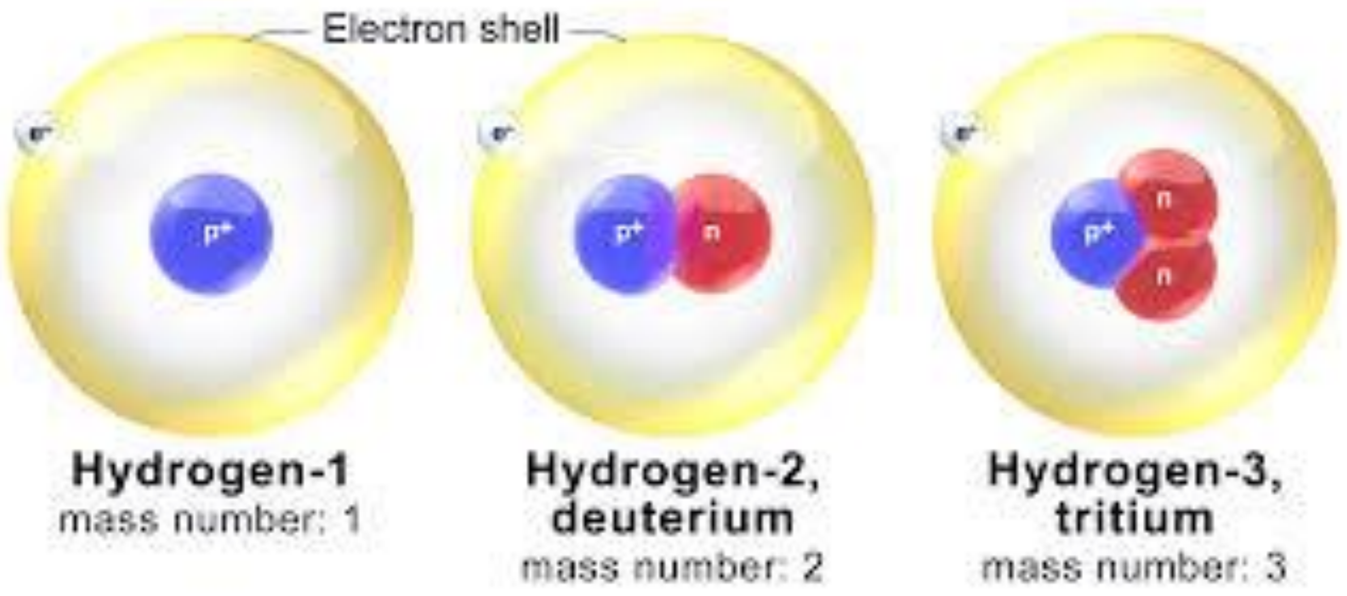
العناصر الملونة باللون الأسود صلبة ، الأزرق سائلة والأحمر غازية ، الأخضر المحضرة صناعيا(صلبة).



www.chemistrysources.com
تعديل : الأستاذ ا. ياسين

ISOTOPES.

- **Atoms with the same number of protons but different numbers of neutrons are called isotopes.**
- **They share almost the same chemical properties, but differ in mass and therefore in physical properties.**
- **There are stable isotopes, which do not emit radiation, and there are unstable isotopes, which do emit radiation.**


Electron shell
p+
n
Hydrogen-1
mass number: 1
Hydrogen-2,
deuterium
mass number: 2
Hydrogen-3,
tritium
mass number: 3

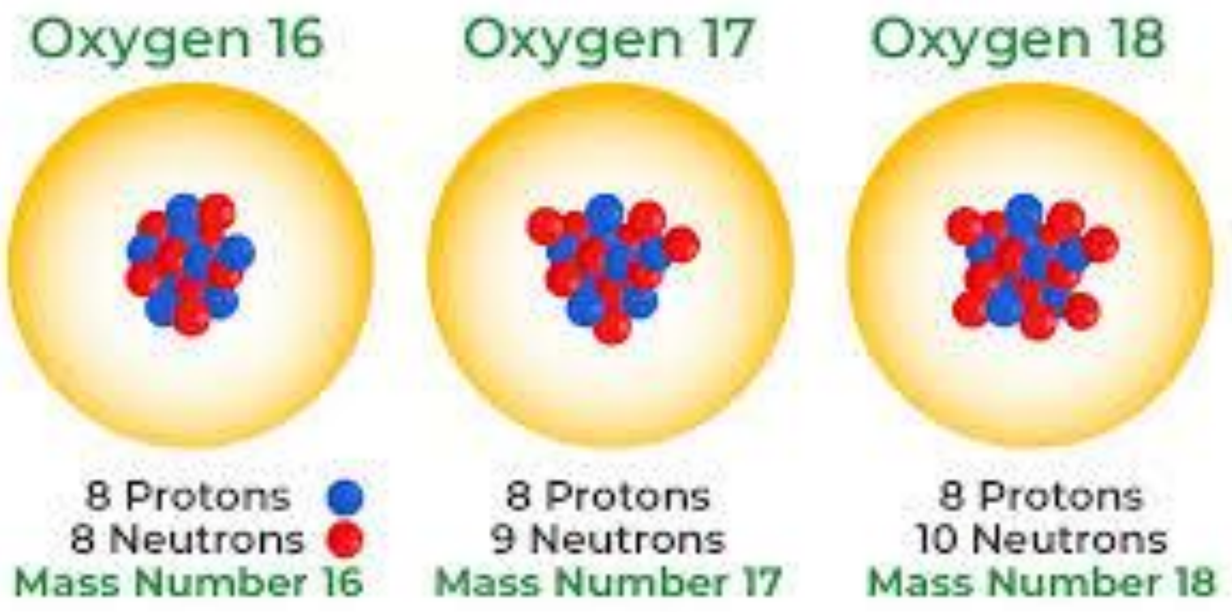

Oxygen 16
Oxygen 17
Oxygen 18
8 Protons
8 Neutrons
Mass Number 16
8 Protons
9 Neutrons
Mass Number 17
8 Protons
10 Neutrons
Mass Number 18

Table 44.1 Masses of Selected Particles in Various Units

Particle	Mass kg	u	MeV/c^2
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron (β particle)	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
$^1_1$H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
$^4_2$He nucleus (α particle)	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
$^4_2$He atom	$6.646\,48 \times 10^{-27}$	4.002 603	3 728.40
$^{12}_6$C atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9

It is often convenient to express the atomic mass unit in terms of its *rest-energy equivalent*. For one atomic mass unit,

$$E_R = mc^2 = (1.660\,539 \times 10^{-27}\ \text{kg})(2.997\,92 \times 10^8\ \text{m/s})^2 = 931.494\ \text{MeV}$$

where we have used the conversion

$$1\ \text{eV} = 1.602\,176 \times 10^{-19}\ \text{J}.$$

1 u= 1.660566 × 10^{-27} Kg

$$m_p = 1.672648 \times 10^{-27} kg = 1.007276u$$

$$m_n = 1.674955 \times 10^{-27} kg = 1.008665u$$

$$m_e = 9.109 \times 10^{-31} kg = 0.000549u$$

$$E = mc^2 = (mass of\, 1u) \times c^2$$

$$E = 1.660566 \times 10^{-27} kg \times (3 \times 10^8 m/s)^2$$

$$= 0.149 \times 10^{-9} J$$

$$E = \frac{0.149 \times 10^{-9}}{1.6 \times 10^{-19}} = 9.315 \times 10^8 eV = 931.5 MeV$$

1u= 931.5 MeV

$$m_p = 1.007276u = 938.28MeV$$

$$m_n = 1.008665u = 939.57MeV$$

$$m_e = 0.000549u = 0.511MeV$$

- **The size of the nucleus depends on the number of nucleons (the sum of protons and neutrons).**

- **It was found that the radius of the nucleus is proportional to the mass number as follows:**

$$r^3 \propto A$$

$$r \propto A^{\frac{1}{3}} \Rightarrow r = r_0 A^{\frac{1}{3}},$$

$$r_0 = 1.2 \times 10^{-15} m$$

Considering that the nucleus is a sphere, its volume is given by:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(r_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi\left(r_0^3 A\right)$$

Example: Calculate the volume of the carbon atom's nucleus and its density, where A = 12

$$V = \frac{4}{3}\pi\left(r_0^3 A\right) = \frac{4}{3}\pi\left(1.2\times10^{-15}\right)^3 \times 12 = 8.6859\times10^{-44}\,m^3$$

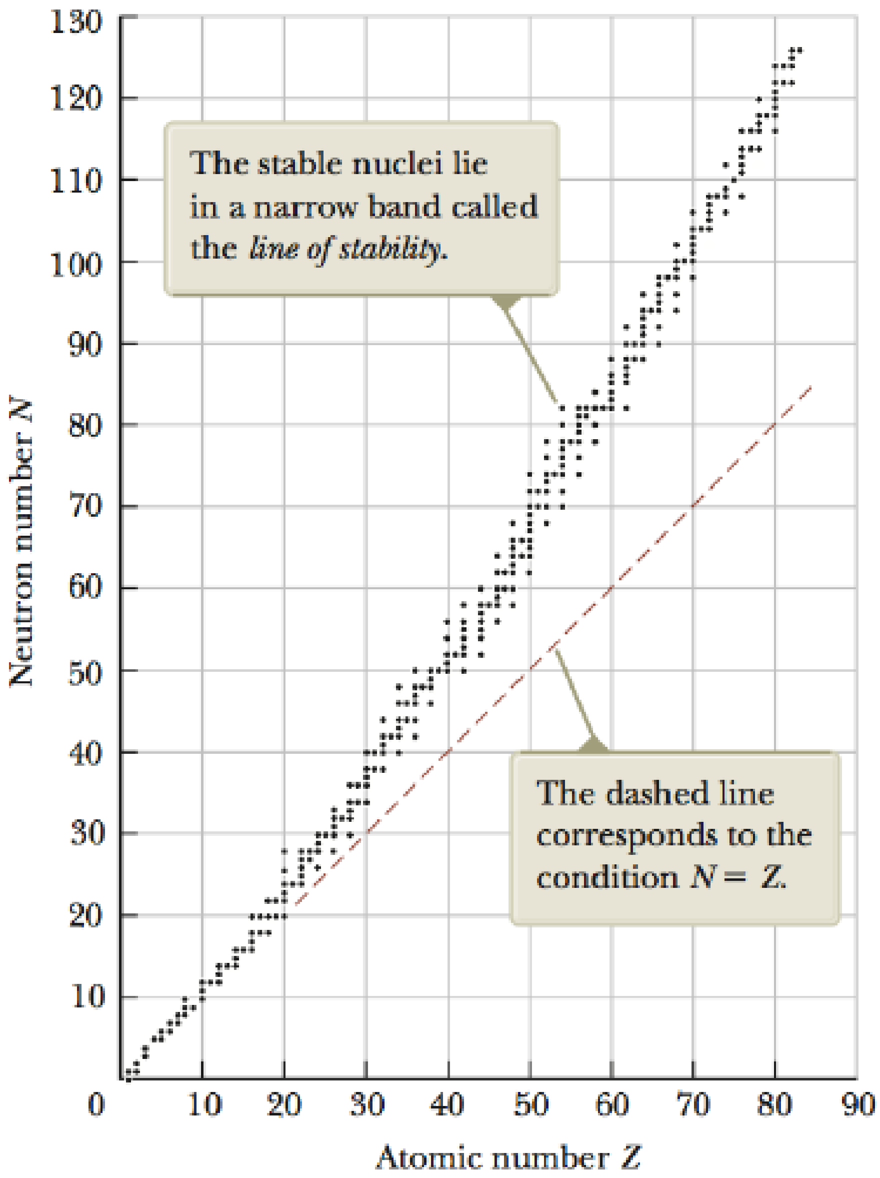
$$\rho = \frac{M}{V} \approx \frac{12\times1.660566\times10^{-27}}{8.6859\times10^{-44}} = 2.3\times10^{17}\,kg/m^3$$

Nuclear Stability

- You might expect that the very large repulsive Coulomb forces between the protons in a nucleus should cause the nucleus to fly apart. Because that does not happen, there must be a attractive force.
- The **nuclear force** is a very short range (about 2 fm) attractive force that acts between all nuclear particles.
- The protons attract each other by means of the **nuclear force**, and, at the same time, they repel each other through the **Coulomb force**.
- The nuclear force also acts between pairs of neutrons and between neutrons and protons.
- The nuclear force is independent of charge. In other words, the forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are the same.

Neutron number *N* versus atomic number *Z* for stable nuclei

- The **stable nuclei** are represented by the black dots, which lie in a narrow range called the *line of stability.*
- The **light stable nuclei** contain an equal number of protons and neutrons; that is, $N = Z$.
- In **heavy stable nuclei**, the number of neutrons exceeds the number of protons: above $Z = 20$, the line of stability deviates upward from the line representing $N = Z$.
- This deviation can be understood by recognizing that as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart.
- As a result, more neutrons are needed to keep the nucleus stable because neutrons experience only the attractive nuclear force.



44.2 Nuclear Binding Energy

- The total mass of a nucleus is less than the sum of the masses of its individual nucleons.
- Therefore, the rest energy of the bound system (the nucleus) is less than the combined rest energy of the separated nucleons.
- This difference in energy is called the **binding energy** of the nucleus and can be interpreted as the energy that must be added to a nucleus to break it apart into its components.
- Therefore, to separate a nucleus into protons and neutrons, energy must be delivered to the system.

- The mass of the nucleus is equal to the mass of the nucleons minus the mass of the electrons, meaning its mass is always less than the mass of the nucleons.
- For example, the mass of the carbon-12 nucleus can be calculated as follows:

$$M\left({}^{12}_{6}C\right) = 12u - 6 \times 0.000549u = 11.996706u$$

$$6m_p + 6m_n = 6 \times 1.007276 + 6 \times 1.008665 = 12.095646u$$

$$\Delta m = 12.095646u - 11.996706u = 0.09894u$$

- **The equivalent energy of this difference in mass can be calculated according to Einstein as follows:**

$$\Delta mc^2 = E = 0.09894u \times 931.5MeV/u = 92.2MeV$$

- **The difference in mass Δm is equivalent to the energy used to bind the nucleons together and is called binding energy.**

The binding energy per nucleon in the carbon-12 nucleus is:

$$\frac{B.E.}{A} = \frac{92.2MeV}{12} = 7.68MeV$$

- **The binding energy of any nucleus can be written as follows:**

$$B.E = \left(Zm_p + Zm_e + Nm_n - M\left({}^A_Z X\right)\right) \times 931.5MeV$$
$$= \left(Zm_H + Nm_n - M\left({}^A_Z X\right)\right) \times 931.5MeV,$$
$$m_H = m_p + m_e = 1.007276u + 0.000549u = 1.007825u$$

Example: Calculate the binding energy per nucleon of the nucleus of the iron isotope 56 $\left({}^{56}_{26}Fe\right)$ $M\left({}^{56}_{26}Fe\right)=55.934937u$

$$B.E=\left(Zm_H+Nm_n-M\left({}^{A}_{Z}X\right)\right)\times 931.5MeV,$$

$$Z=26, N=A-Z=56-26=30$$

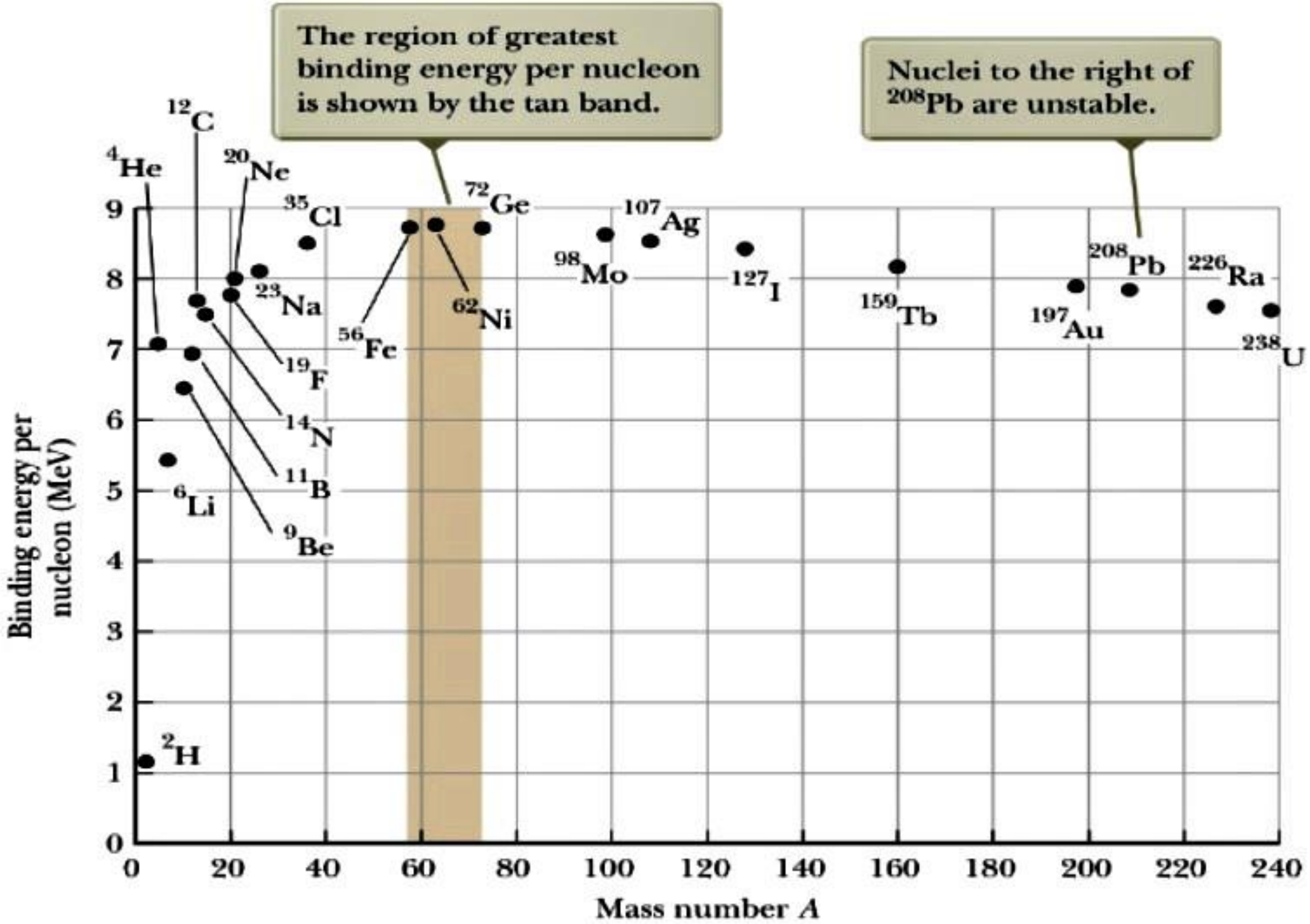
$$\therefore B.E=\left(26\times 1.007825+30\times 1.008665-55.934937\right)\times 931.5MeV$$

$$=492.2663MeV$$

$$\therefore \frac{B.E}{A}=\frac{492.2663}{56}=8.79MeV/Nucleon$$

44.2 Nuclear Binding Energy

Binding energy per nucleon E_b/A as a function of mass number A for various stable nuclei.



RADIOACTIVITY

- In 1896, Becquerel accidentally discovered that uranyl potassium sulfate crystals emit an invisible radiation that can darken a photographic plate even though the plate is covered to exclude light.

- This process of spontaneous emission of radiation by uranium was soon to be called radioactivity.

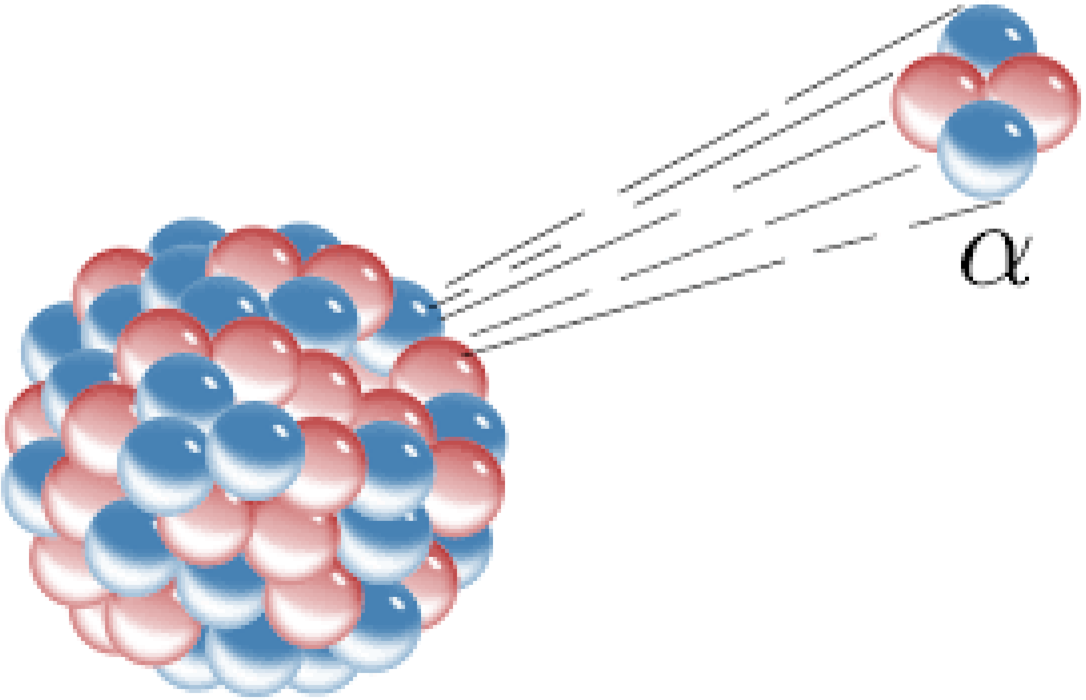
- Additional experiments, including Rutherford's famous work on alpha-particle scattering, suggested that radioactivity is the result of the decay, or disintegration, of unstable nuclei.

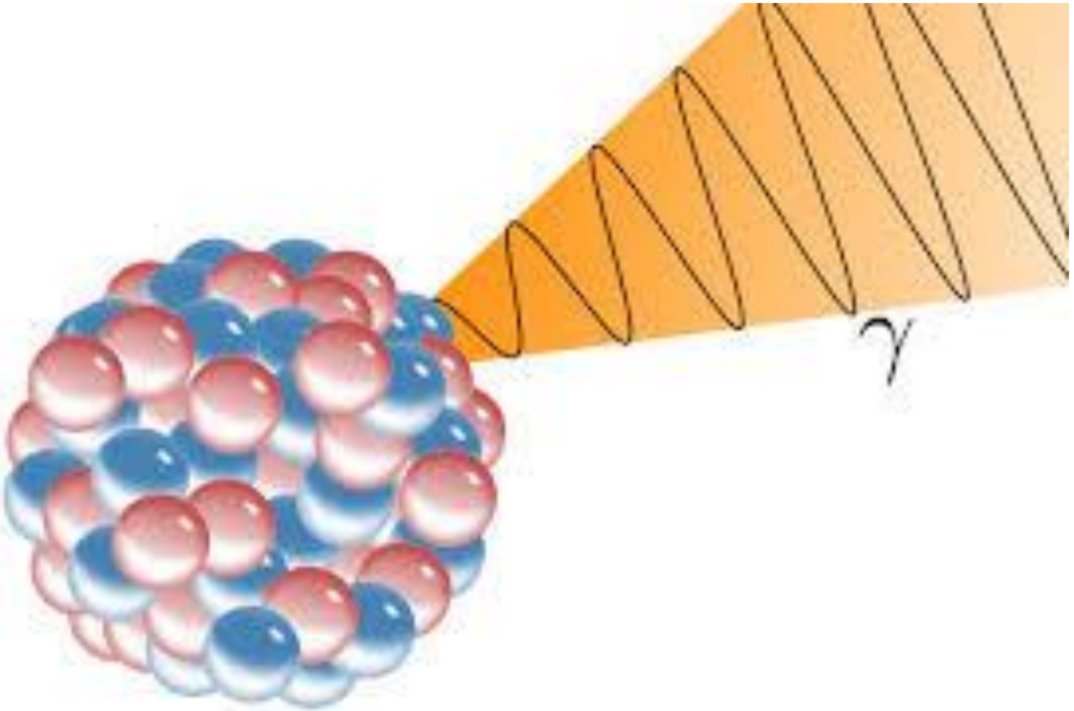
NATURAL RADIOACTIVITY

The radiation emitted by radioactive (or radioactive) materials, according to their electrical charges and their ability to penetrate the material, has been classified into three types:

1. Alpha particles (α): They are the nucleus of a helium atom, their charge is positive and equal to +2e, and their range is short in the air (3cm).

2. Gamma radiation (ϒ): These are electromagnetic radiations that have no charge or mass, their wavelength is very short, and their range in the air is very long.

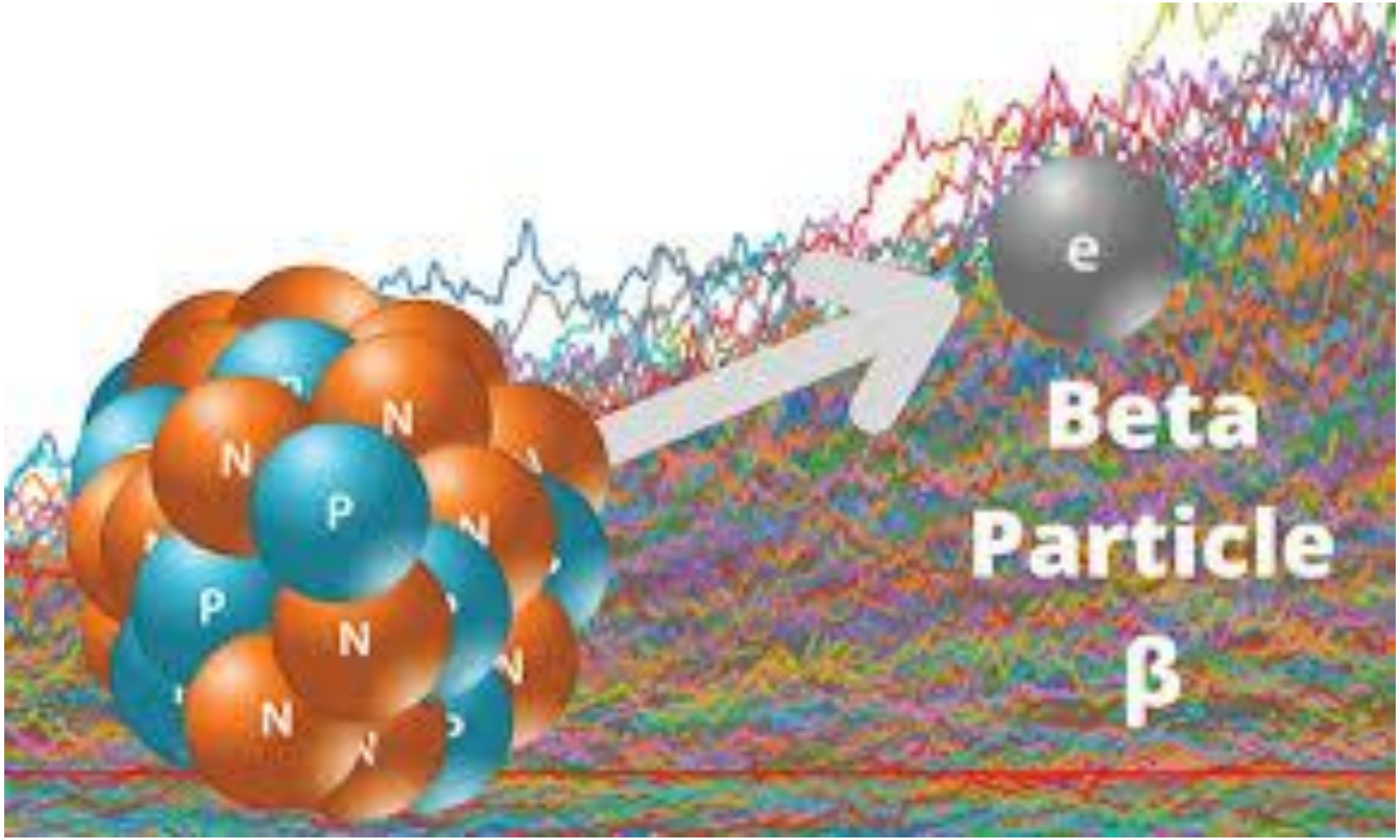

α


γ

Beta particles (β): They are charged particles that penetrate a long distance in the air (3 m). β particles are classified into two types:

1. **Positive beta particles (β^+): They are positively charged particles that are numerically equal to the charge of the electron and whose mass is equal to the mass of the electron.**

2. **Negative beta particles (β^-): They are negatively charged particles that are numerically equal to the charge of the electron and their mass is equal to the mass of the electron. However, their source is the nucleus, while the electrons come from the orbitals of atoms.**


e
Beta
Particle
β
N
P

❑Alpha decay

$$ {}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He \quad Example: {}_{92}^{238}U \rightarrow {}_{90}^{234}Th + {}_{2}^{4}He $$

The energy released as a result of alpha decay is called decay energy and is usually symbolized by Q, meaning:

$$ Q = [M(x) - \left(M(y) + M_{\alpha}\right)] \times 931.5 MeV $$

In order for the nucleus to emit alpha particles, it must be Q > 0

Example: Does polonium 210 emit alpha particles if it is known that:

$$M(^{210}Po) = 209.98285u$$

$$M(^{206}Pb) = 205.97440u$$

$$M_{\alpha} = 4.002603u$$

Solution: The decay equation is:

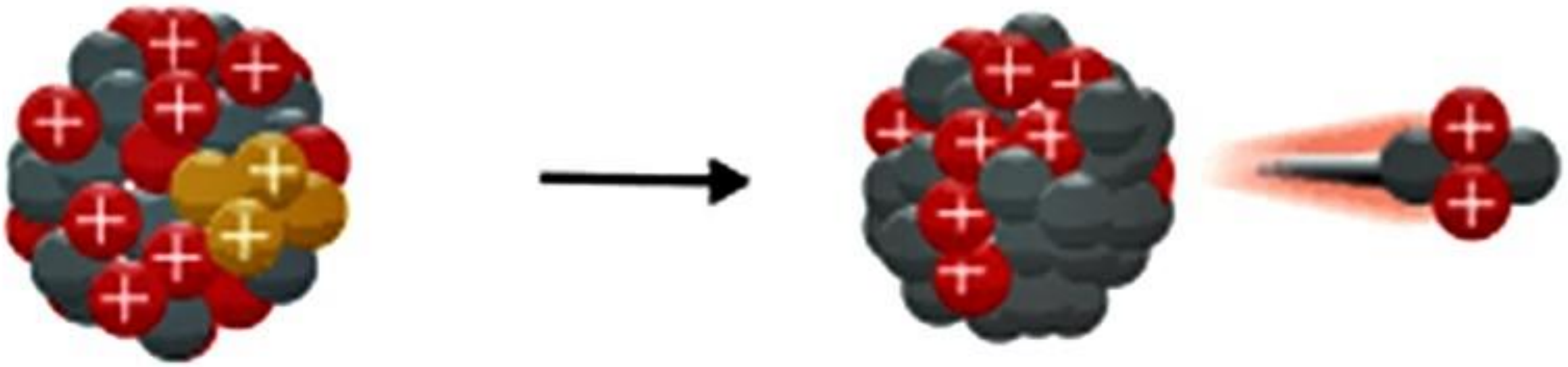
$$^{210}_{84}Po \rightarrow ^{206}_{82}Pb + ^{4}_{2}He$$

$$\therefore Q = \left[M(^{210}Po) - \left(M(^{206}Pb) + M_{\alpha}\right)\right] \times 931.5MeV$$

$$= [209.98285 - (205.97440 + 4.002603] \times 931.5$$

$$= 5.45MeV \succ 0$$

So polonium 210 can decay by emitting alpha particles



Parent

Daughter

Alpha Particle

$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

β- decay

Beta- minus Decay

$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + \beta^{-} + \overline{\nu} $$

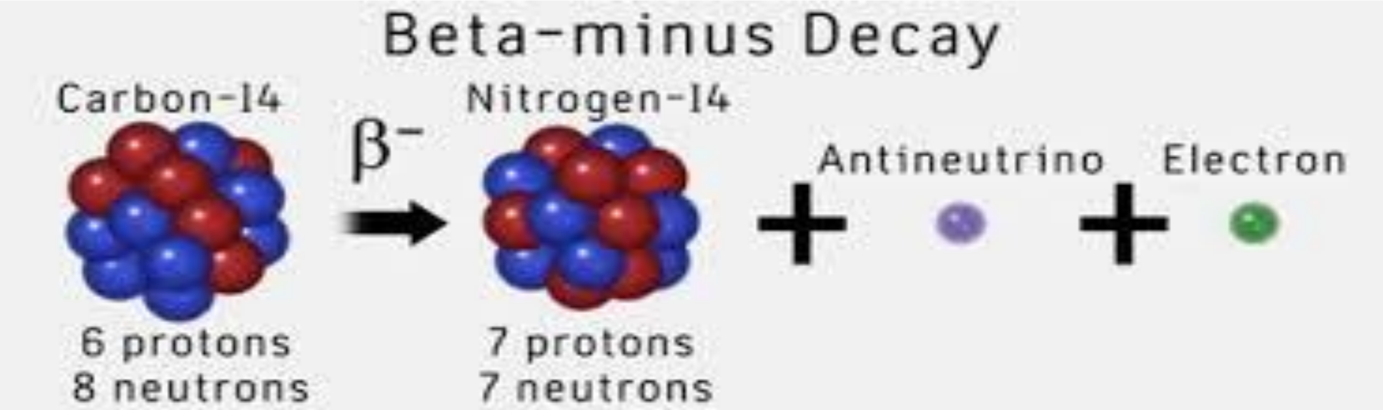
$$ {}_{0}^{1}n \rightarrow {}_{+1}^{1}p + {}_{-1}^{0}e + \overline{\nu} $$

This transformation occurs when the ratio of neutrons to protons in the nucleus is greater than its value in the stability region. In order for stability to occur, the neutron turns into a proton and the negative beta particle is released from the nucleus:

$$ Q = [M(x) - (M(y) + m_e)] \times 931.5 MeV $$

We obtain the highest value for the decay energy of negative beta particles by taking the atomic masses instead of the nuclear ones

$$ Q = [M(x) - M(y)] \times 931.5 MeV $$

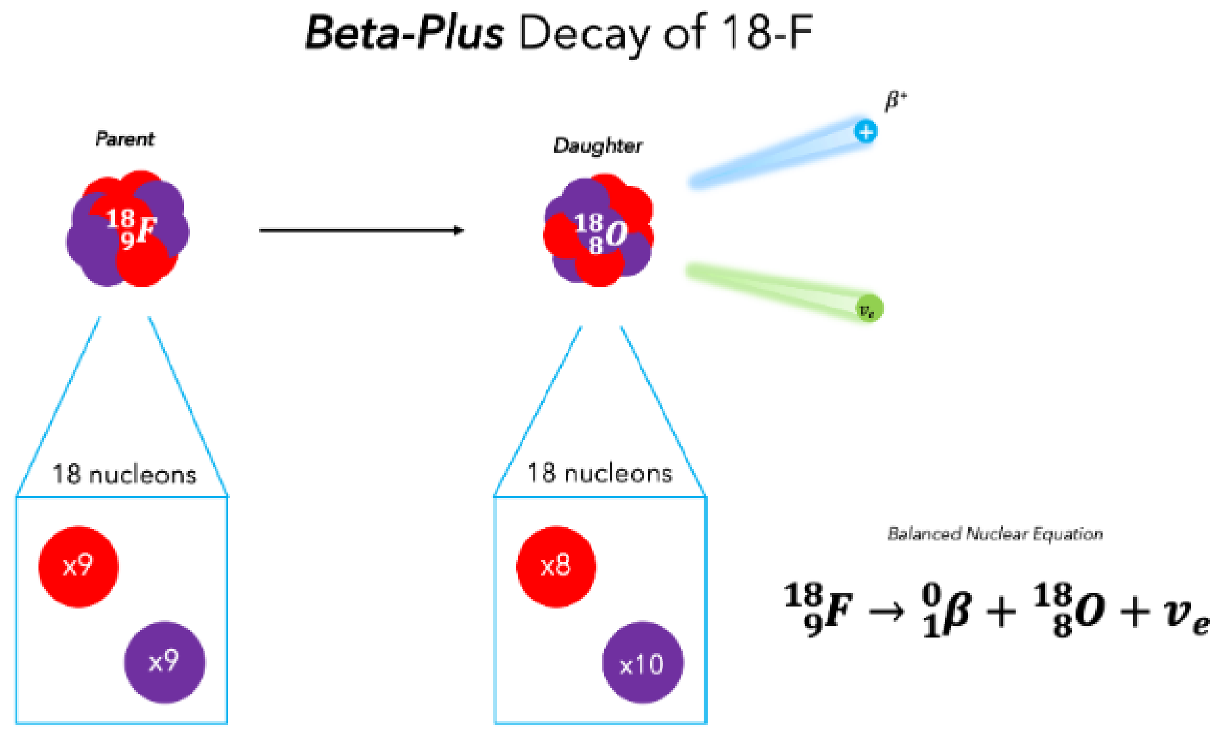


Beta- Plus Decay

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + \beta^{+} + \nu $$

In order for stability to occur, the proton turns into a neutron, and the positive beta particle (positron) is released from the nucleus:

$$ {}_{+1}^{1}p \rightarrow {}_{0}^{1}n + {}_{+1}^{0}e + \nu $$



$$Q = [M(x) - (M(y) + m_e)] \times 931.5 MeV$$

We obtain the highest value for the decay energy of negative beta particles by taking the atomic masses instead of the nuclear ones

$$Q = [M(x) - (M(y) + 2m_e)] \times 931.5 MeV$$

$$^{13}_{7}N \rightarrow {}^{13}_{6}C + \beta^{+} + \nu$$

There are two particles that accompany negative and positive beta decay: the neutrino ν and the antineutrino. $\bar{\nu}$ They are without charge and without mass, but their presence is necessary so that the energy, linear momentum, and angle before and after the decay remain preserved according to the principle of conservation of energy and momentum.

Example:

Calculate the disintegration energy Q for the beta decay $^{32}P \rightarrow {}^{32}S + e^- + \nu$.
Atomic masses: $m(^{32}P) = 31.97391$ u, $m(^{32}S) = 31.97207$ u

- **Solution:**
  $Q = m_ic^2 - m_fc^2$.
  $m_i = m_{nuc}(^{32}P) = m(^{32}P) - 15*m(e^-)$,
  $m_f = m_{nuc}(^{32}S) + m(e^-) = m(^{32}S) - 16*m(e^-) + m(e^-) = m(^{32}S) - 15*m(e^-)$
  The mass of the neutrino is negligibly small.
  $Q = m_ic^2 - m_fc^2 = (m(^{32}P) - m(^{32}S))c^2 = 1.71$ MeV.
  In β^- decay subtracting the atomic masses automatically takes into account the mass of the emitted electron. This is not true for β^+ decay.

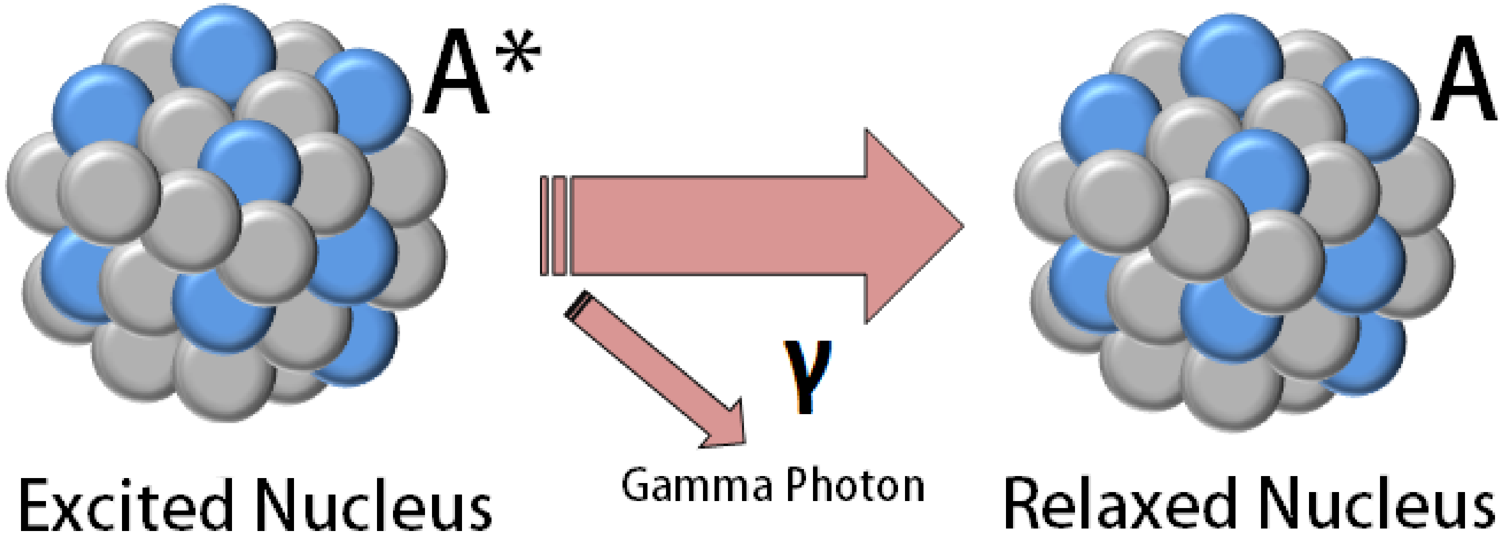
Calculate the disintegration energy Q for the beta decay $^{64}Cu \rightarrow {}^{64}Ni + e^+ + \nu$.
Atomic masses: $m(^{64}Cu) = 63.929766$ u, $m(^{64}Ni) = 63.927968$ u

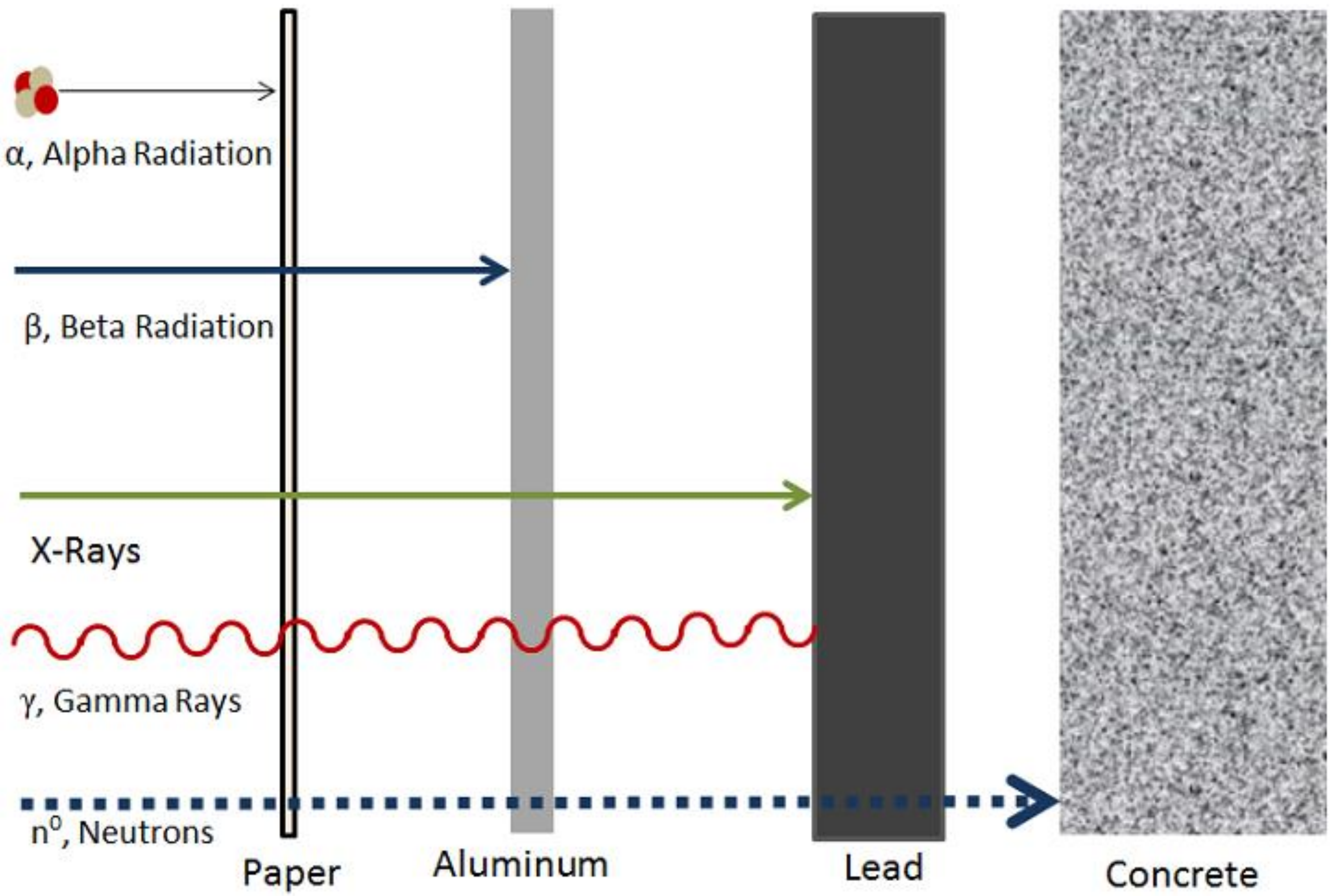
- **Solution:**
  $Q = m_ic^2 - m_fc^2$.
  $m_i = m_{nuc}(^{64}Cu) = m(^{64}Cu) - 29*m(e^-)$,
  $m_f = m_{nuc}(^{64}Ni) + m(e^+) = m(^{64}Ni) - 28*m(e^-) + m(e^+) = m(^{64}Ni) - 27m(e^-)$
  The mass of the neutrino is negligibly small and $m(e^+) = m(e^-)$.
  $Q = m_ic^2 - m_fc^2 = (m(^{64}Cu) - m(^{64}Ni))c^2 - 2*m(e^-)c^2 = 1.6748$ MeV $- 2*0.511$ MeV $= 0.653$ MeV.
  In β^+ decay subtracting the atomic masses does not automatically take into account the mass of the emitted positron.

❑ ϒ- decay

$$ {}_{Z}^{A}X^{*} \rightarrow {}_{Z}^{A}X + {}_{0}^{0}\gamma $$

Gamma radiation comes from an excited nucleus, while X-rays come from an excited atom.





α, Alpha Radiation
β, Beta Radiation
X-Rays
γ, Gamma Rays
n0, Neutrons
Paper
Aluminum
Lead
Concrete

Radioactive decay law

- **The intensity of radiation emitted by a radioactive material does not depend on temperature, pressure, or any external effect, but depends only on the number of unstable nuclei in the sample (N).**

- **The number of nuclei likely to spontaneously decay per unit time, i.e. the decay rate (ΔN/Δt) increases with the increase in the number of unstable nuclei in the sample, according to the law.**

$$\frac{N - N_0}{t - t_0} = \frac{\Delta N}{\Delta t} = -\lambda N$$

- **Where λ is the decay constant and its value is positive.**

The law of radioactive decay at any instant of time t can be written as follows

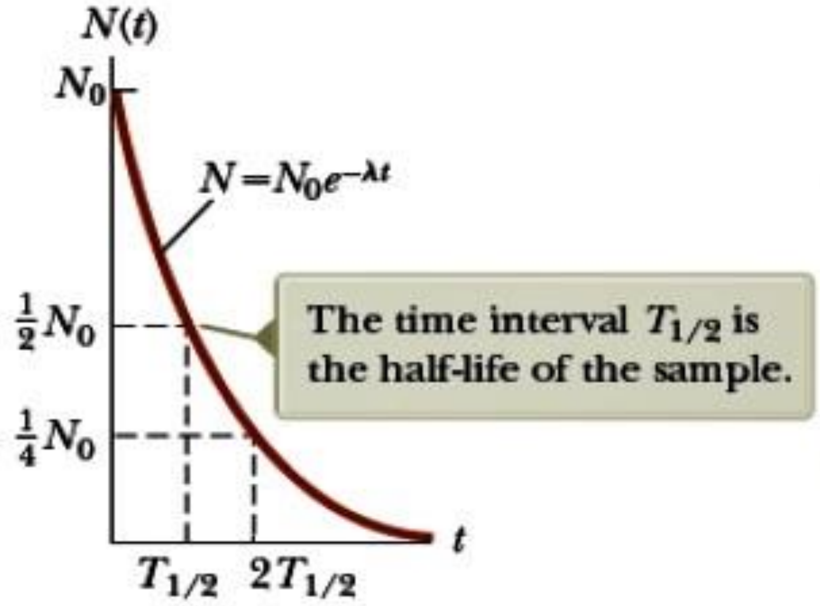
$$N = N_0 e^{-\lambda t}$$

If N is the number of undecayed radioactive nuclei present at some instant, the rate of change of N with time is

$$\frac{N - N_0}{t - t_0} = \frac{\Delta N}{\Delta t} = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$

where the constant N_0 represents the number of undecayed radioactive nuclei at $t = 0$.



$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\therefore 2 = e^{\lambda T_{1/2}} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- **Each radioactive isotope has its own λ and thus a specific half-life.**

The half-lives of radioactive isotopes range from 10-20 sec to 10^{16} years.

The number of nuclei in the sample cannot be counted directly, but the decay rate R (the number of decays per unit time), which is called the radioactive intensity, can be measured:

$$\frac{dN}{dt} = -\lambda N$$

$$\therefore R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$\therefore R = R_0 e^{-\lambda t}, R_0 = \lambda N_0$$

Ro is the sample decomposition rate at t=0, and R is the decomposition rate at time t.

Radiation intensity is measured in curies (Ci).

The curio is defined as the radiation intensity of a mass of one gram of radium 226, which is equal to: 1 Ci = 3.7×10^{10} decay/sec, but in SI System the unit is Bq

1 Bq= 1 decay/sec,

1 Ci= 3.7x1010 Bq

Example: A sample of cobalt 60 (Co^{60}), which has a half-life of 5.26 years, has $3x10^{16}$ radioactive nuclei. What is its radiation intensity in curie units after 15.78 years?

N_o = 3 x 10 16 nucleus, $T_{1/2}$= 5.26 years, t = 15.78 y

$T_{1/2}$ = 5.26 x 3.16 x 10 7 sec/ y = 1.66216 x 108 sec

$R = R_o\, e^{-\lambda t}$

$R = N_o\, \lambda\, e^{-\lambda t}$

$T_{1/2} = 0.693/\lambda$

λ = 0.693/1.66216 x 10^8 = 4.17016 x 10^{-9} decay/ sec

$R_o = N_o\, \lambda$ = 3 x 10^{16} x 4.17 x 10^{-9} = 1.25 x 10^8 decay/sec

t = 15.78 y x 3.16 x 10^7 = 4.986 x 10^8

$R = R_o\, e^{-\lambda t}$

R = 1.25 x 10^8 x $e^{-(4.17x\ 10\text{-}9\ x\ 4.98\ x\ 108)}$

R = 1.562 x 10^7 decay/sec = 1.562 x 10^7 Bq .

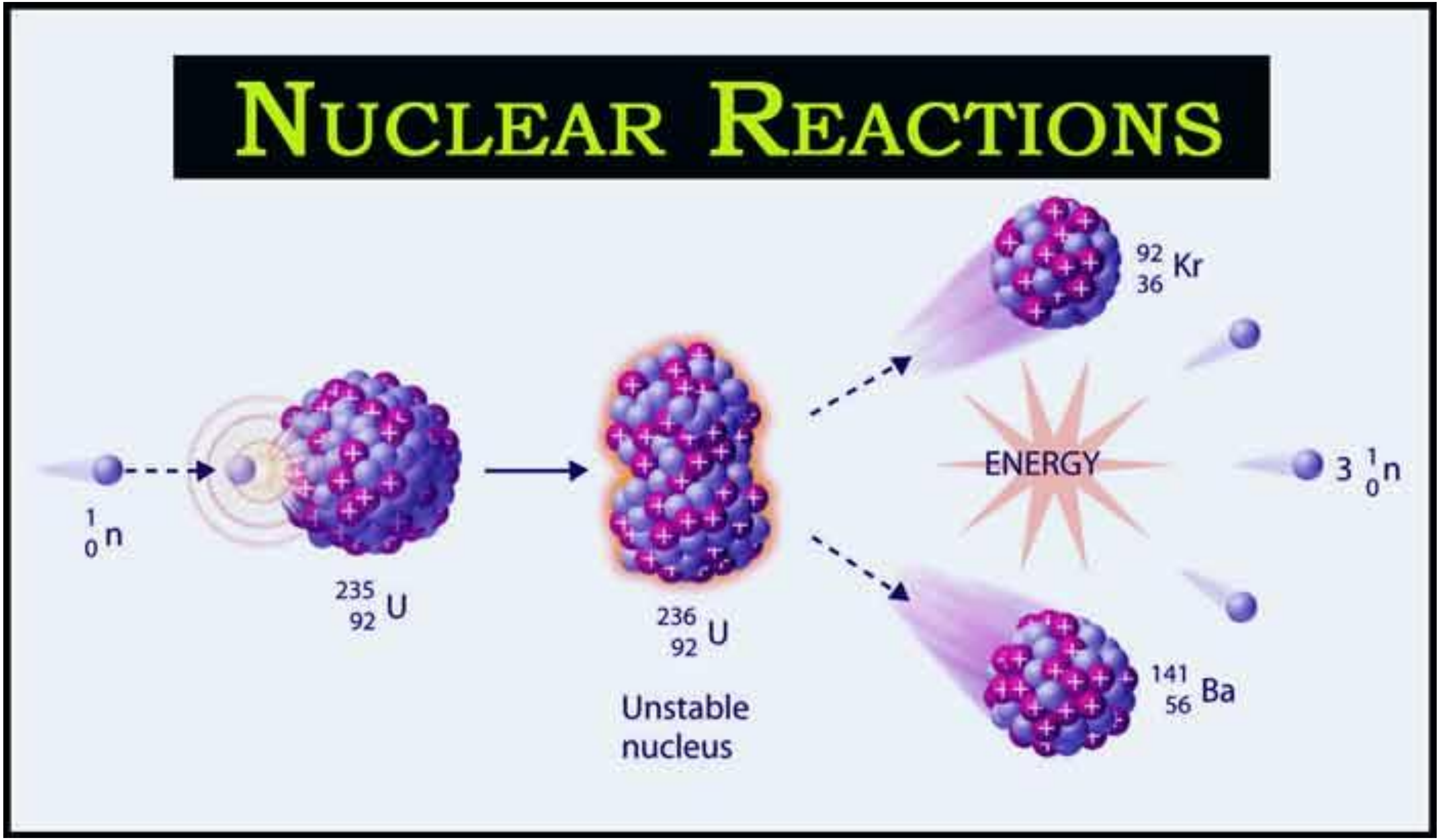
1 Ci = 3.7 x 10 ^{10} Bq and 1 Bq = 1/ 3.7 x 10^{10} Ci

R= 1.5625 x 10^7 x 1/ 3.7 x 10 ^{10} =4.223 x 10^{-4} Ci = 422.3 μCi

Nuclear reactions

Nuclear reactions are processes in which one or more nuclides are produced from a collision between two nuclei or one nucleus and a subatomic particle.

A nuclear reaction that occurs naturally is due to the interaction between cosmic rays and matter. A nuclear reaction is that is employed artificially is to obtain nuclear energy, at an adjustable rate and on-demand. Thou the most notable nuclear reactions are the nuclear chain reactions. It is in fissionable materials that produce induced nuclear fission. The various nuclear fusion reactions of light elements power the energy production of the Sun and stars.


NUCLEAR REACTIONS
1
0 n
235
92 U
236
92 U
Unstable
nucleus
92
36 Kr
ENERGY
3 1
0 n
141
56 Ba

$$ {}_{2}^{4}He + {}_{7}^{14}N \rightarrow \left[{}_{9}^{18}F \right] \rightarrow {}_{8}^{17}O + {}_{1}^{1}H \longrightarrow $$ عملية تحول العناصر

مستقر مشع مستقر

Nuclear reactions can be expressed by the following equation

$$ \boldsymbol{x + X \rightarrow Y + y} $$

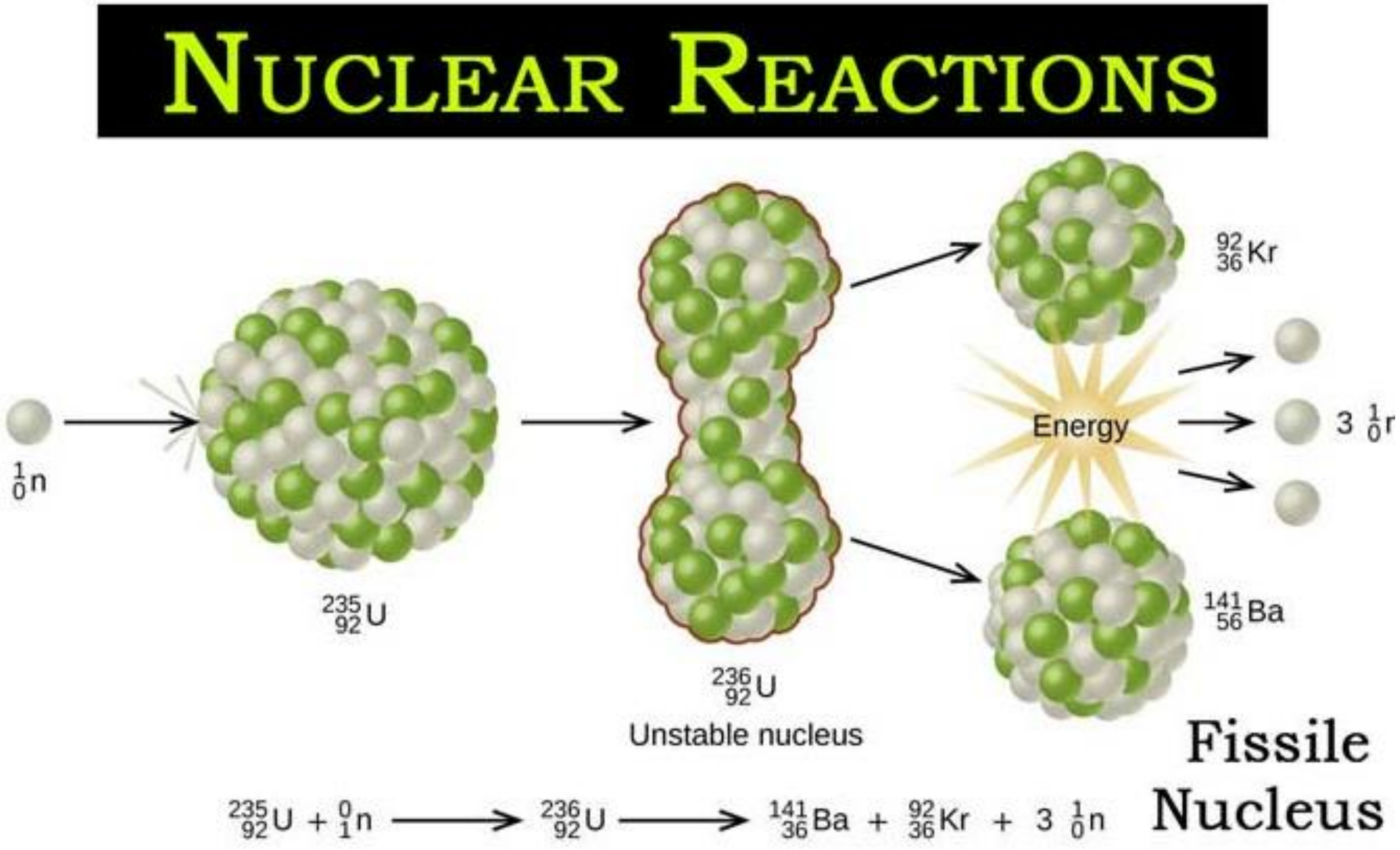
The energy released is equal to:

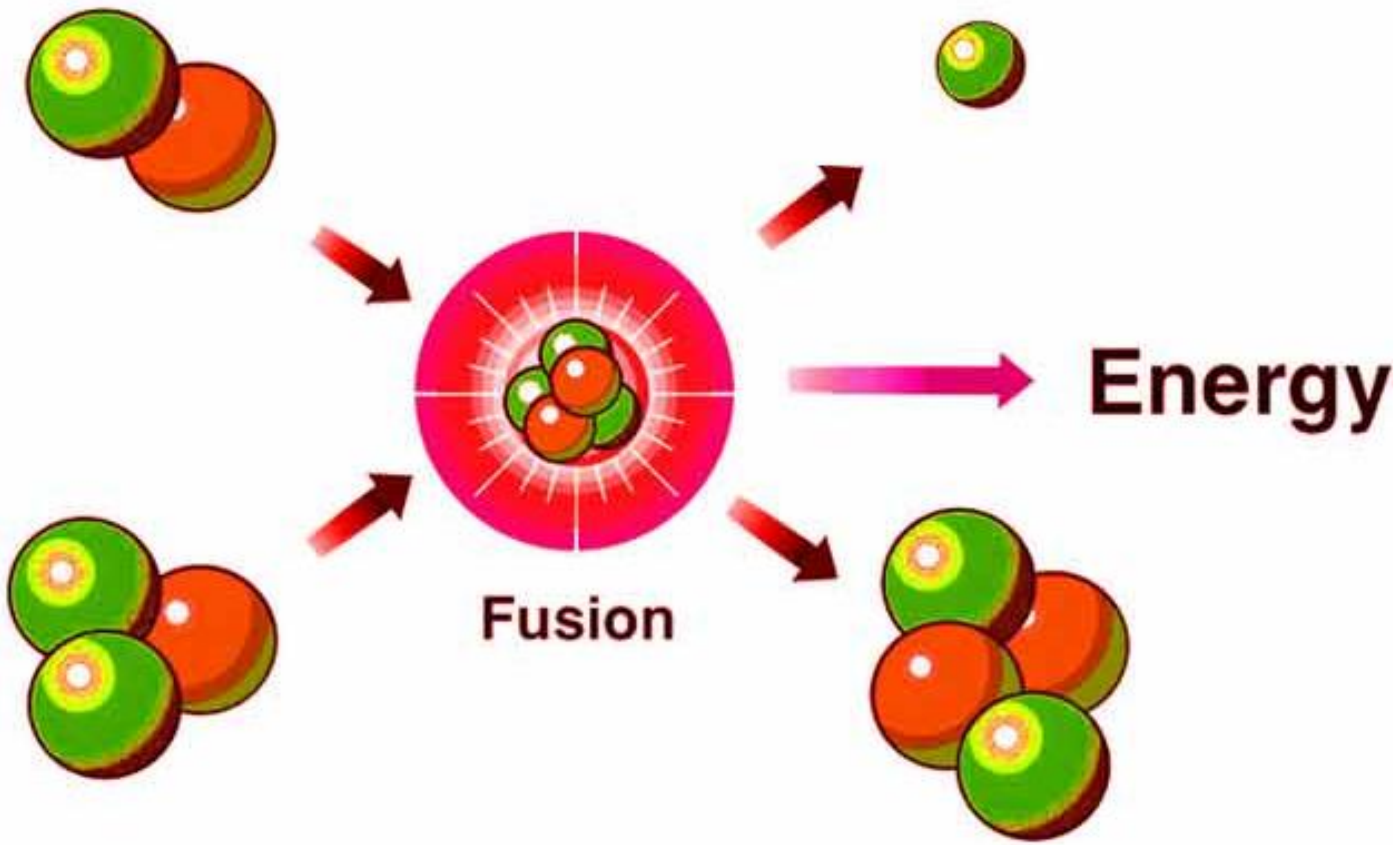
$$ Q = [M(x) + M(X) - \left(M(Y) + M(y)\right)] \times c^2 \, joule $$
$$ Q = [M(x) + M(X) - \left(M(Y) + M(y)\right)] \times 931.5 MeV $$

$\alpha + N \rightarrow F \rightarrow O + P \Longrightarrow Q = -1.19 \text{ MeV}$

Since the calculated value of the released energy is negative, for the reaction to occur, there must be a C Alpha is kinetic energy converted into mass so that the left side is equal to or greater than the sum of the masses on side A To the right for the reaction to take place

Example: When electrons are showered on Uranium-235 at a slow speed, the heavy nucleus of Uranium-235 splits into two nuclei barium-141 and krypton-92 and three neutrons are emitted. A large amount of energy is produced in this reaction.





$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$