PHYS 111

1ST semester 1446

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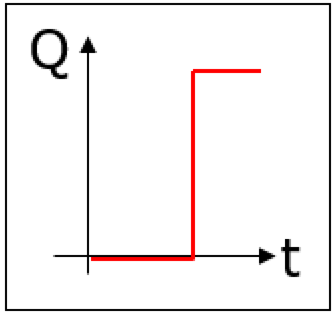
Lecture 7

RC Circuits

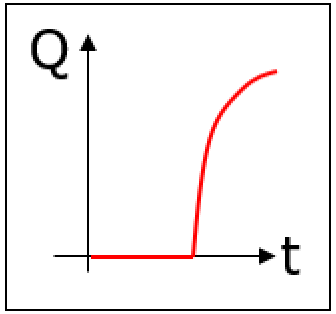
RC circuits contain both a resistor R and a capacitor C (duh).

Until now we have assumed that charge is instantly placed on a capacitor by an emf.



The approximation resulting from this assumption is reasonable, provided the resistance between the emf and the capacitor being charged/discharged is small.

If the resistance between the emf and the capacitor is finite, then the capacitor does not change instantaneously.



Charging a Capacitor

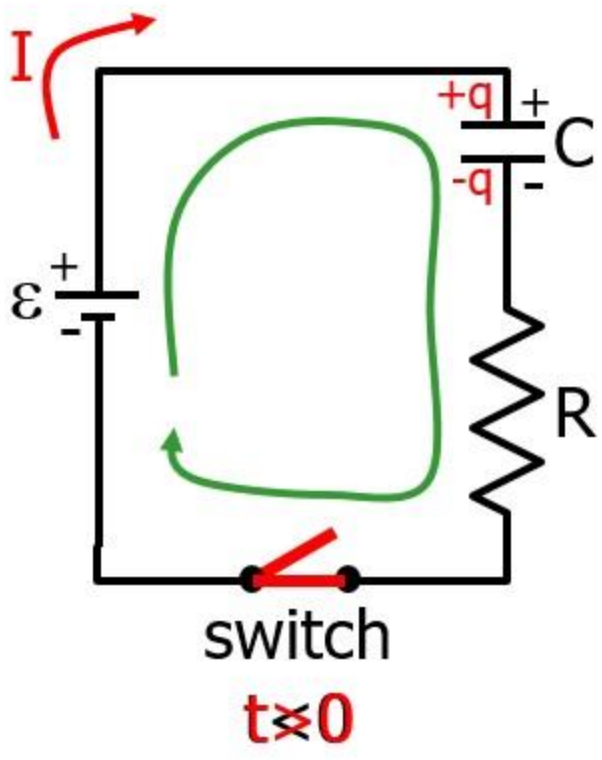
Switch open, no current flows.

Close switch, current flows.

Apply Kirchoff's loop rule* (green loop) at the instant charge on C is q.

$$\varepsilon - \frac{q}{C} - IR = 0$$

This equation is deceptively complex because I depends on q and both depend on time.



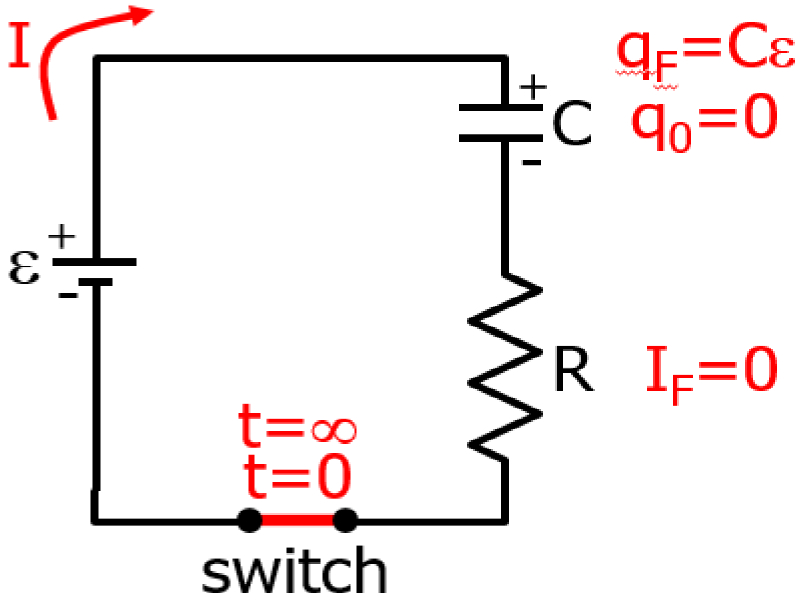
*Convention for capacitors is "like" batteries: negative if going across from + to -.

Limiting Cases

$$\varepsilon - \frac{q}{C} - IR = 0$$

When t=0, *q=0 and $I_0 = \varepsilon/R$.

When t is "large," the capacitor is fully charged, the current "shuts off," and $Q = C\varepsilon$.



*q=0 at t=0 only if the capacitor is initially uncharged (read the problems carefully!)

$\varepsilon = IR$ is true only at time t=0 or when q=0! $V_R = IR$ is always true, but V_R is the potential difference across the resistor, which you may not know. Using V = IR to find the voltage across the capacitor is likely to lead to mistakes unless you are very careful.

Math:

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$I = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = \frac{C\varepsilon - q}{RC}$$

$$\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

More math:

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\int_0^t \frac{dt}{RC}$$

$$\ln\left(q - C\varepsilon\right)\Big|_0^q = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-\frac{t}{RC}}$$

$$q - C\varepsilon = -C\varepsilon\, e^{-\frac{t}{RC}}$$

Still more math:

$$q = C\varepsilon - C\varepsilon\, e^{-\frac{t}{RC}}$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\boxed{q(t) = Q\left(1 - e^{-\frac{t}{RC}}\right)}$$

Why not just solve this for q and I?

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$I(t) = \frac{dq}{dt} = C\varepsilon\left(\frac{1}{RC}e^{-\frac{t}{RC}}\right) = \frac{C\varepsilon}{RC}e^{-\frac{t}{RC}} = \boxed{\frac{\varepsilon}{R}e^{-\frac{t}{RC}}} = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}}$$

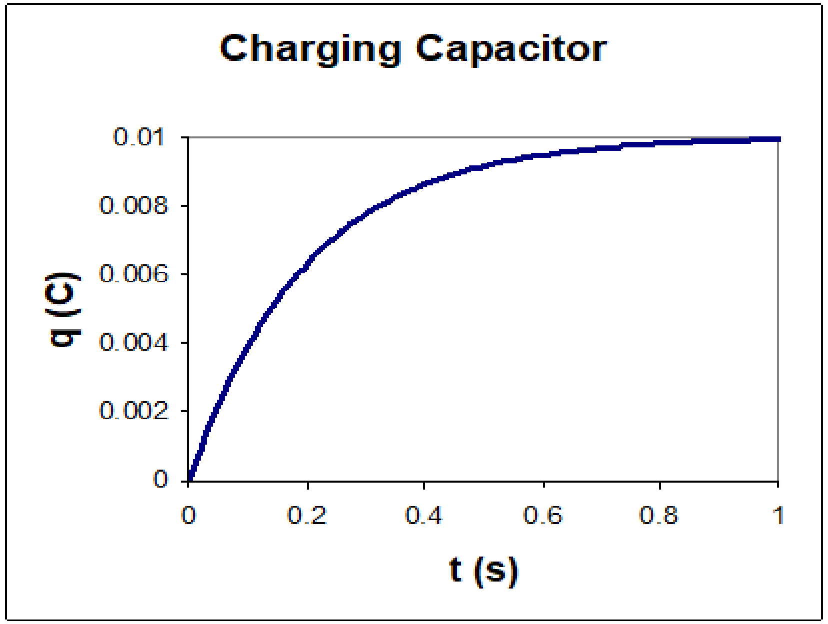
τ = RC is the "time constant" of the circuit; it tells us "how fast" the capacitor charges and discharges.

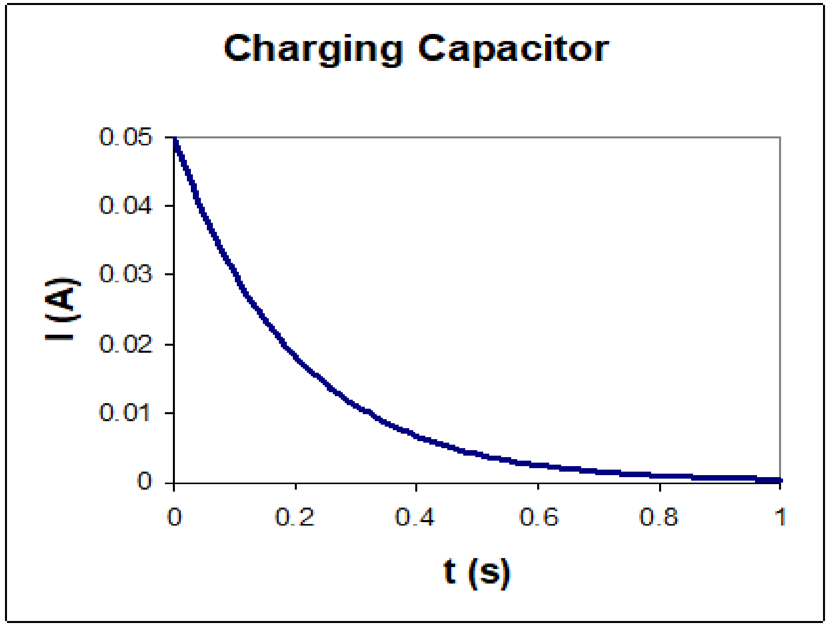
Charging a capacitor; summary:

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

recall that this is I_0, also called I_{max}

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

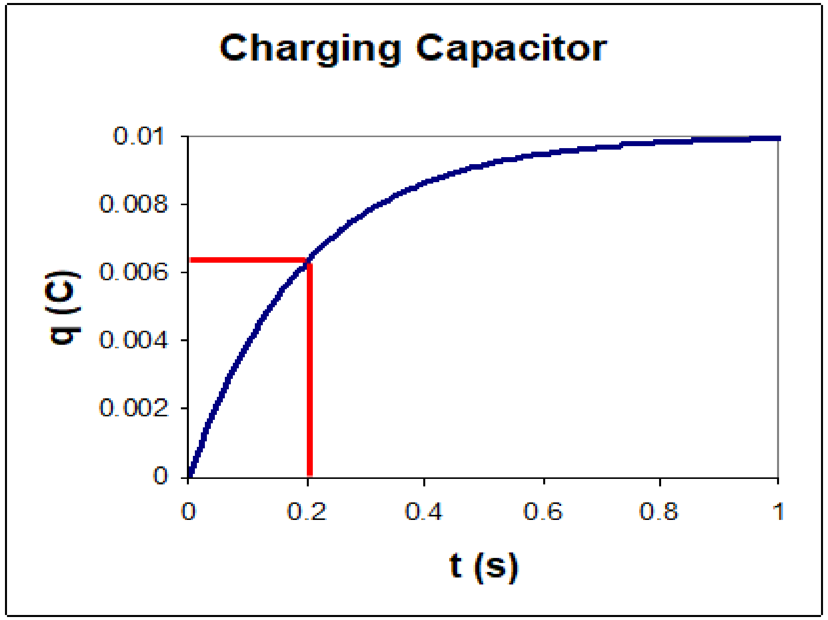


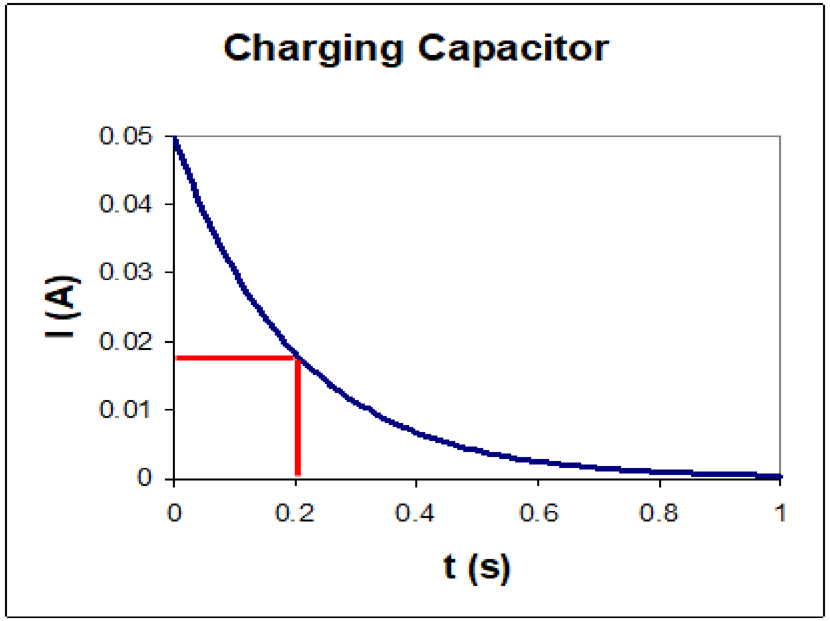


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor charges to $Q(1-e^{-1})$ or 63% of its capacity…

…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

$\tau=RC$ is called the **time constant** of the RC circuit

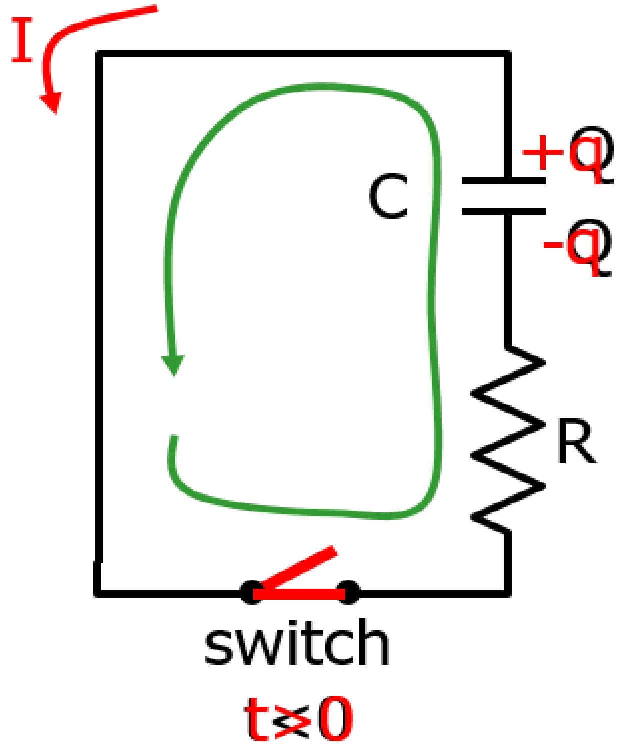
Discharging a Capacitor

Capacitor charged, switch open, no current flows.

Close switch, current flows.

Apply Kirchoff's loop rule* (green loop) at the instant charge on C is q.

$$\frac{q}{C} - IR = 0$$



*Convention for capacitors is "like" batteries: positive if going across from - to +.

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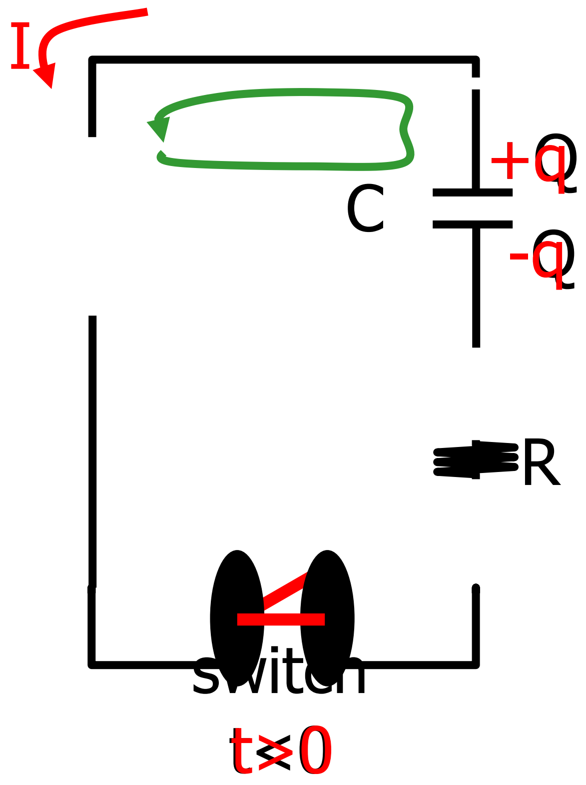


Apply Kirchoff's loop rule* (green loop) at the instant charge on C is q.

$$\frac{q}{C} - IR = 0$$

*Convention for capacitors is "like" batteries: positive if going across from - to +.

Math:

$$\frac{q}{C} - IR = 0$$

$$IR = \frac{q}{C}$$

$$I = -\frac{dq}{dt}$$

negative because charge decreases

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

More math:

$$\int_Q^q \frac{dq}{q} = -\int_0^t \frac{dt}{RC} = -\frac{1}{RC}\int_0^t dt$$

$$\ln(q)\Big|_Q^q = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$\boxed{q(t) = Q\,e^{-\frac{t}{RC}}}$$

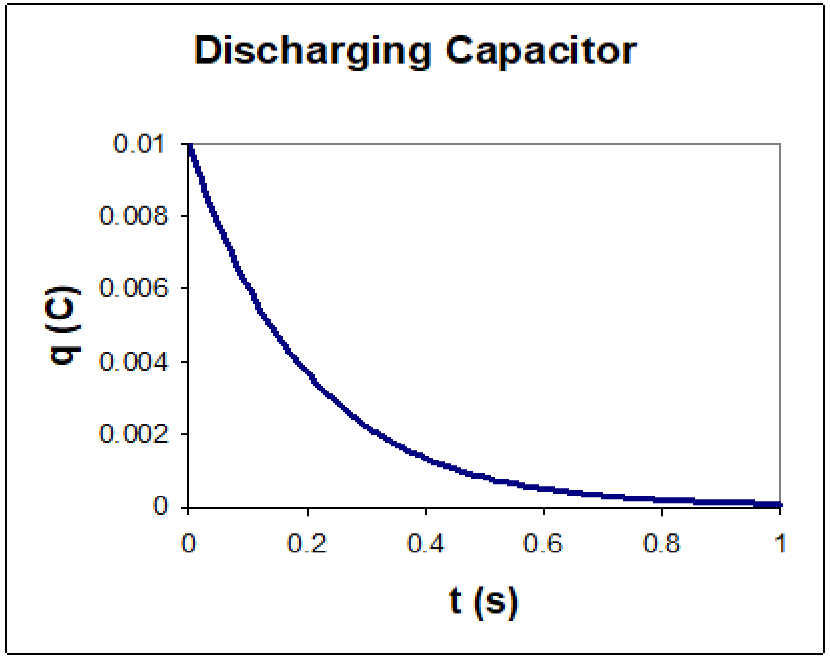
$$I(t) = -\frac{dq}{dt} = \frac{Q}{RC}\,e^{-\frac{t}{RC}} = \boxed{I_0\,e^{-\frac{t}{RC}}}$$

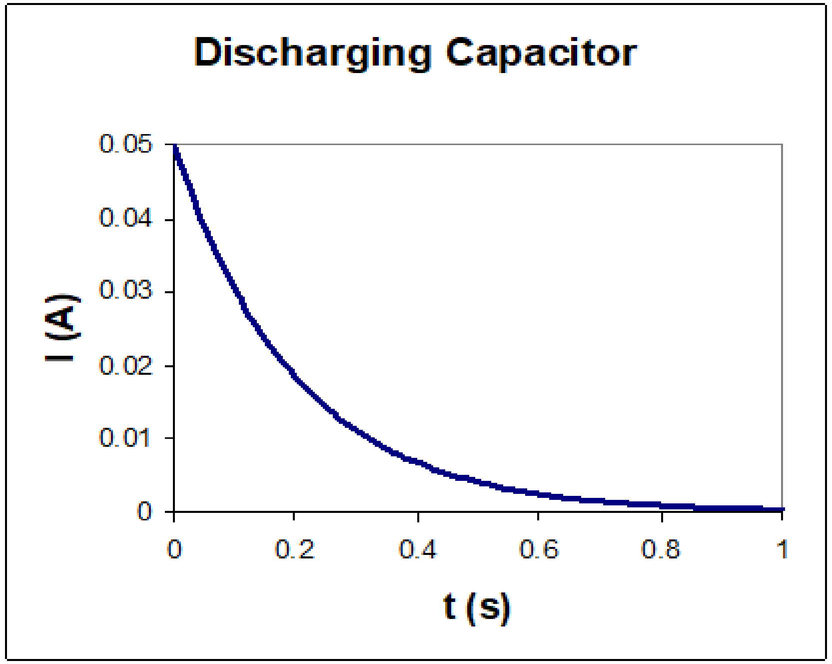
same equation
as for charging

Disharging a capacitor; summary:

$$q(t) = Q_0\, e^{-\frac{t}{RC}}$$

$$I(t) = I_0 e^{-\frac{t}{RC}}$$



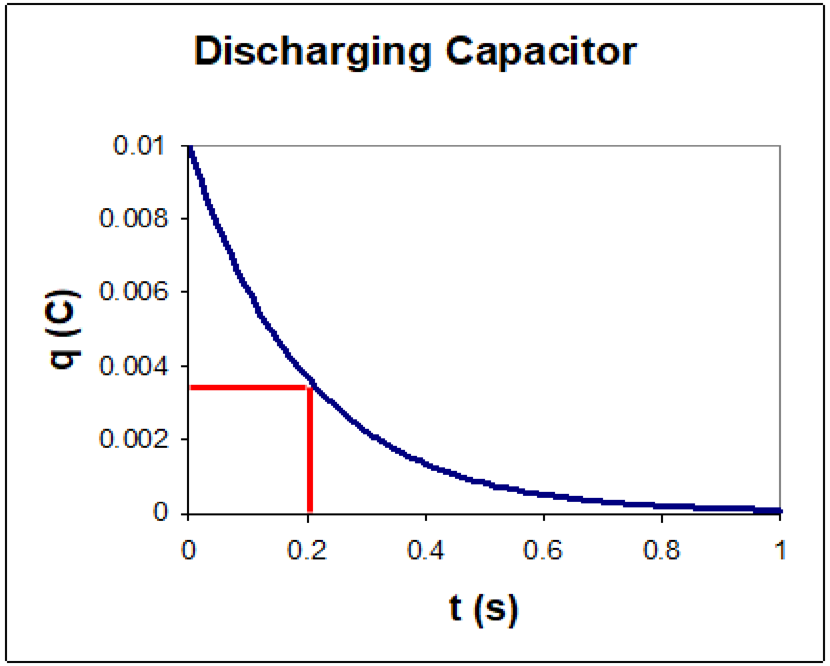


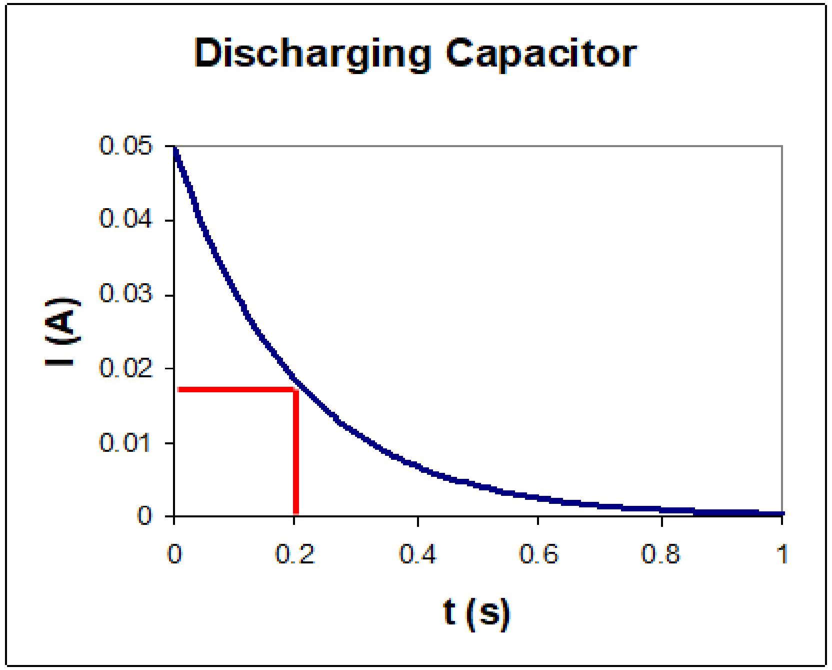
Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.

RC=0.2 s

In a time $t=RC$, the capacitor discharges to Qe^{-1} or 37% of its capacity…

…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





$RC=0.2$ s

Notes

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

This is for charging a capacitor. $\varepsilon/R = I_0 = I_{max}$ is the initial current, and depends on the charging emf and the resistor.

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

This is for discharging a capacitor. $I_0 = Q/RC$, and depends on how much charge Q the capacitor started with.

I_0 for charging is equal to I_0 for discharging only if the discharging capacitor was fully charged.

Notes

In a series RC circuit, the same current I flows through both the capacitor and the resistor. Sometimes this fact comes in handy.

In a series RC circuit, where a source of emf is present (so this is for capacitor charging problems)...

$$V_R + V_C = \Delta V$$

$$V_R = \Delta V - V_C = IR$$

V_c and I must be at the same instant in time for this to work.

$$I = \frac{\Delta V - V_C}{R}$$

Any technique that begins with a starting equation and is worked correctly is acceptable, but I don't recommend trying to memorize a bunch of special cases. Starting with $I(t) = dq(t)/dt$ always works.

Notes

In a discharging capacitor problem...

$$V_R = V_C = IR$$

$$I = \frac{V_C}{R}$$

So sometimes you can "get away" with using V = IR, where V is the potential difference across the capacitor (if the circuit has only a resistor and a capacitor).

Rather than hoping you get lucky and "get away" with using V = IR, I recommend you understand the physics of the circuit!