PHYS 111

1st semester 1446

Prof. Omar Abd Elkader

Lecture 6

Chapter 27 Current and Resistance

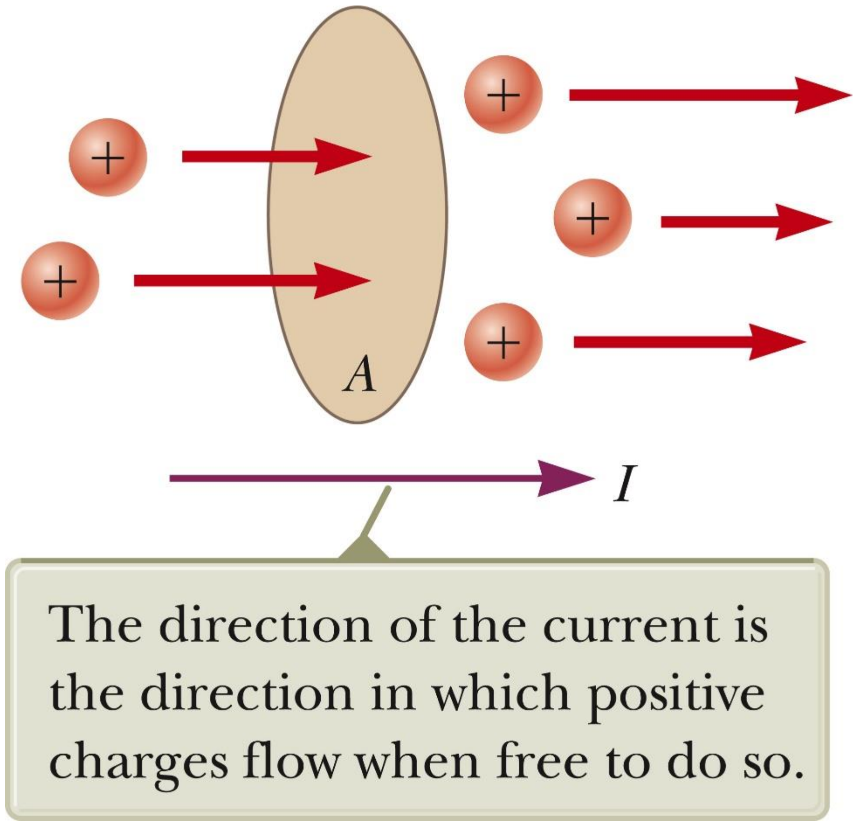
Electric Current

- **Electric current** is the rate of flow of charge through some region of space.
- The SI unit of current is the **ampere** (A).
  - 1 A = 1 C / s
- The symbol for electric current is *I*.

Average Electric Current

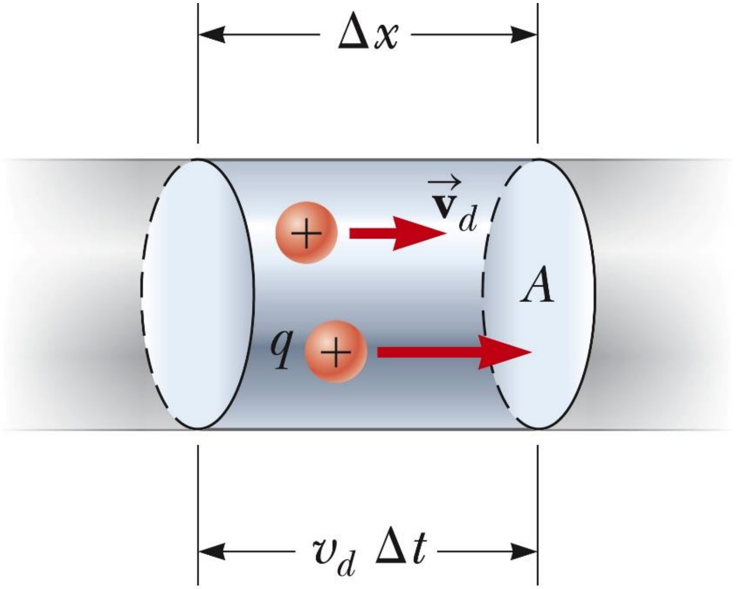
- Assume charges are moving perpendicular to a surface of area *A*.
- If ΔQ is the amount of charge that passes through *A* in time Δt, then the average current is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



Current and Drift Speed

- Charged particles move through a cylindrical conductor of cross-sectional area *A*.
- *n* is the number of mobile charge carriers per unit volume.
- $n\,A\,\Delta\,x$ is the total number of charge carriers in a segment.



Current and Drift Speed, cont

- The total charge is the number of carriers times the charge per carrier, *q*.
    - $\Delta Q = (n\,A\,\Delta x\,)q$
- Assume the carriers move with a velocity parallel to the axis of the cylinder such that they experience a displacement in the x-direction.
- If v_d is the speed at which the carriers move, then
    - $v_d = \Delta x\,/\,\Delta t$ and $Dx = v_d\,Dt$
- Rewritten: $\Delta Q = (n\,A\,v_d\,\Delta t)q$
- Finally, current, $I_{\text{ave}} = \Delta Q/\Delta t = n\,q\,v_d\,A$
- v_d is an average speed called the **drift speed.**

Current Density

- *J* is the **current density** of a conductor.
- It is defined as the current per unit area.
    - $J \equiv I / A = n q \mathbf{v}_d$
    - This expression is valid only if the current density is uniform and *A* is perpendicular to the direction of the current.
- *J* has SI units of A/m^2
- The current density is in the direction of the positive charge carriers.

Conductivity

- A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.
- For some materials, the current density is directly proportional to the field.
- The constant of proportionality, σ, is called the **conductivity** of the conductor.

Ohm’s Law

- **Ohm’s law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.
  - Most metals obey Ohm’s law
  - Mathematically, $J = \sigma E$
  - Materials that obey Ohm’s law are said to be *ohmic*
  - Not all materials follow Ohm’s law
    - Materials that do not obey Ohm’s law are said to be *nonohmic*.
- Ohm’s law is not a fundamental law of nature.
- Ohm’s law is an empirical relationship valid only for certain materials.

Resistance

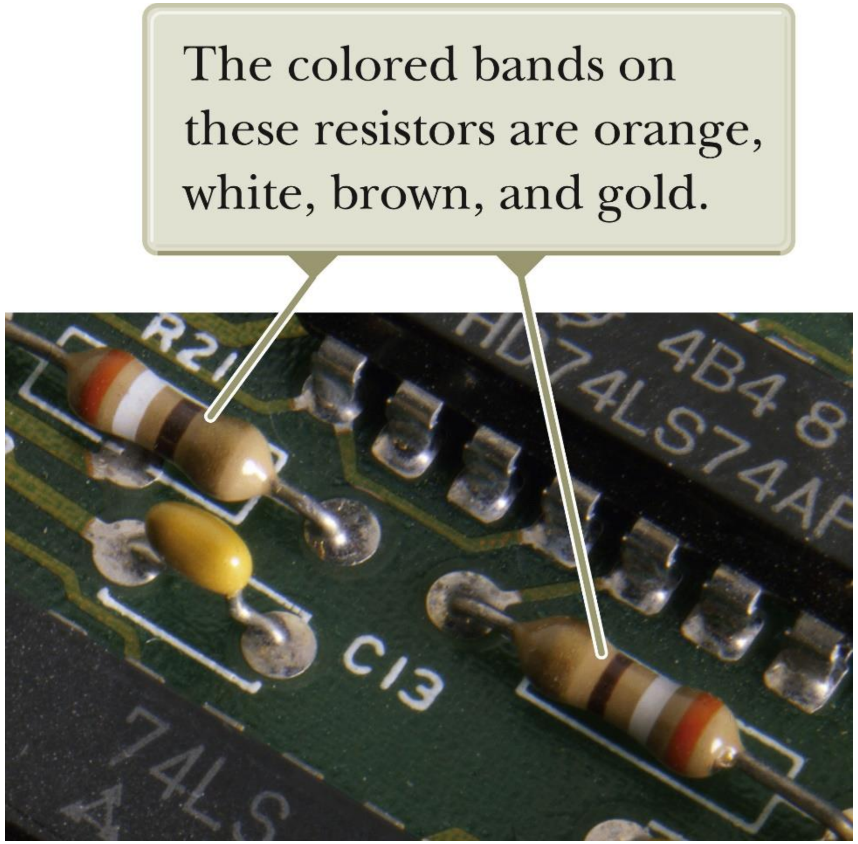
- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.
- The constant of proportionality is called the **resistance** of the conductor.

$$R \equiv \frac{\Delta V}{I}$$

- SI units of resistance are *ohms* (Ω).
  - 1 Ω = 1 V / A
- Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.

Resistors

- Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit.
- Stand-alone resistors are widely used.
    - Resistors can be built into integrated circuit chips.
- Values of resistors are normally indicated by colored bands.
    - The first two bands give the first two digits in the resistance value.
    - The third band represents the power of ten for the multiplier band.
    - The last band is the tolerance.

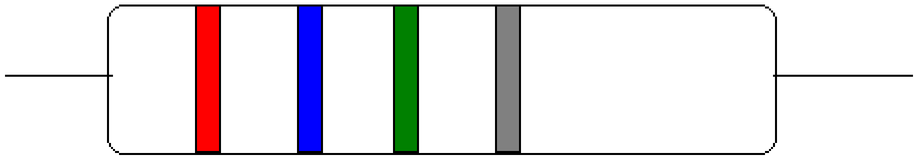


Resistor Color Codes

TABLE 27.1 *Color Coding for Resistors*

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

Resistor Color Code Example



- Red (=2) and blue (=6) give the first two digits: 26
- Green (=5) gives the power of ten in the multiplier: 10^5
- The value of the resistor then is $26 \times 10^5\ \Omega$ (or 2.6 MΩ)
- The tolerance is 10% (silver = 10%) or $2.6 \times 10^5\ \Omega$

Resistivity

- The inverse of the conductivity is the **resistivity**:
  - $\rho = 1 / \sigma$
- Resistivity has SI units of ohm-meters ($\Omega \cdot$ m)
- Resistance is also related to resistivity:

$$R = \rho \frac{\ell}{A}$$

Resistivity Values

TABLE 27.2 *Resistivities and Temperature Coefficients of Resistivity for Various Materials*

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

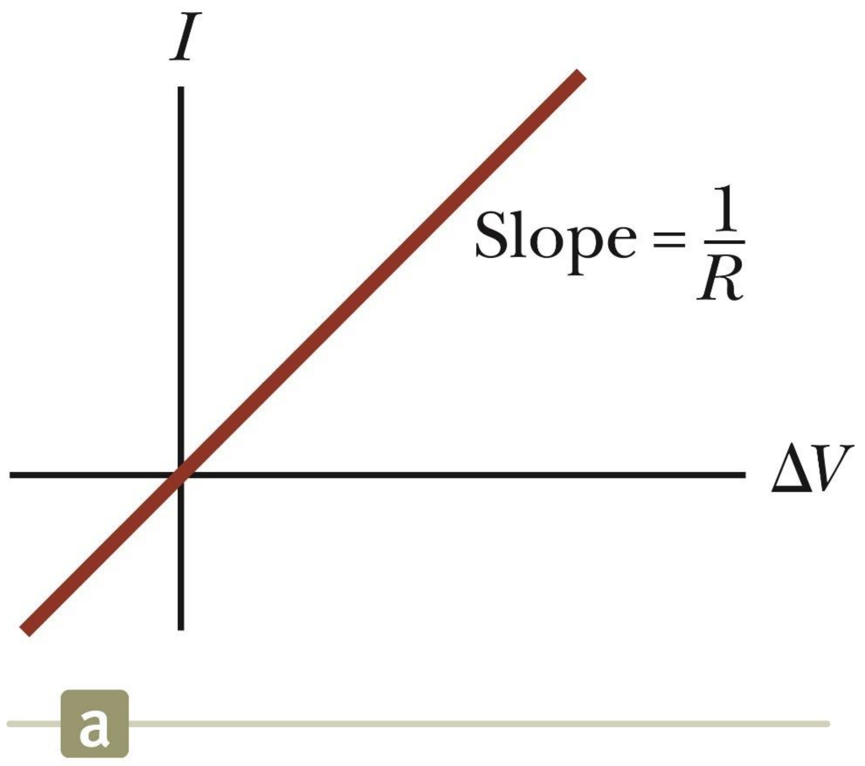
[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Resistivity, Summary

- Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.
  - Resistivity is a property of substances.
- The resistance of a material depends on its geometry and its resistivity.
  - Resistance is a property of an object.
- An ideal conductor would have zero resistivity.
- An ideal insulator would have infinite resistivity.

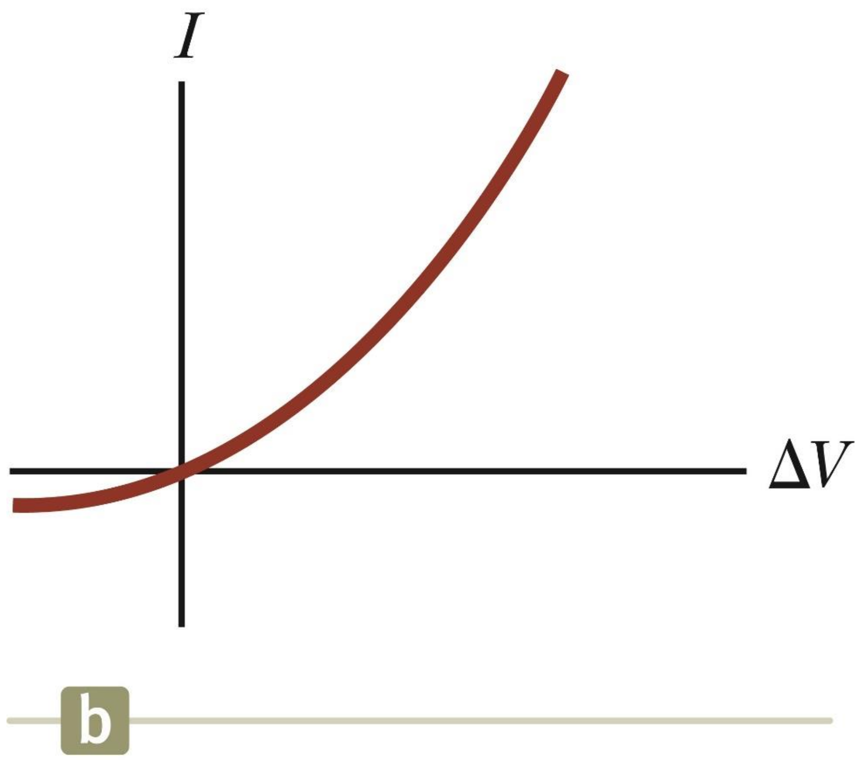
Ohmic Material, Graph

- An ohmic device
- The resistance is constant over a wide range of voltages.
- The relationship between current and voltage is linear.
- The slope is related to the resistance.



Nonohmic Material, Graph

- Nonohmic materials are those whose resistance changes with voltage or current.
- The current-voltage relationship is nonlinear.
- A junction diode is a common example of a nonohmic device.



Electrical Conduction – A Model

- Treat a conductor as a regular array of atoms plus a collection of free electrons.
  - The free electrons are often called conduction electrons.
  - These electrons become free when the atoms are bound in the solid.
- In the absence of an electric field, the motion of the conduction electrons is random.
  - Their speed is on the order of 10^6 m/s.

Conduction Model, 2

- When an electric field is applied, the conduction electrons are given a drift velocity.
- Assumptions:
  - The electron’s motion after a collision is independent of its motion before the collision.
  - The excess energy acquired by the electrons in the electric field is transferred to the atoms of the conductor when the electrons and atoms collide.
    - This causes the temperature of the conductor to increase.

Conduction Model – Calculating the Drift Velocity

- The force experienced by an electron is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

- From Newton's Second Law, the acceleration is

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m_e}$$

- Applying a motion equation

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t \text{ or } \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{q\vec{\mathbf{E}}}{m_e}t$$

  - Since the initial velocities are random, their average value is zero.

Conduction Model, 4

- Let τ be the average time interval between successive collisions.
- The average value of the final velocity is the drift velocity.

$$\vec{\mathbf{v}}_{f,avg} = \vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau$$

- This is also related to the current density: $J = nqv_d = (nq^2E / m_e)\ t$
- $J = n\ q\ (\ q\ E\ /\ m_e)\ t = n\ q^2\ E\ /\ m_e)\ t$
  - n is the number of charge carriers per unit volume.

Conduction Model, final

- Using Ohm's Law, expressions for the conductivity and resistivity of a conductor can be found: $\sigma = J / E$ and $J = n q^2 E / m_e) t$

$$\sigma = \frac{nq^2\tau}{m_e} \qquad \rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

- Note, according to this classical model, the conductivity and the resistivity do not depend on the strength of the field.
  - This feature is characteristic of a conductor obeying Ohm's Law.

Resistance and Temperature

- Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature.

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

- ρ_o is the resistivity at some reference temperature T_o
  - T_o is usually taken to be 20° C
  - α is the **temperature coefficient of resistivity**
    - SI units of α are $^oC^{-1}$
- The temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta T}$$

Temperature Variation of Resistance

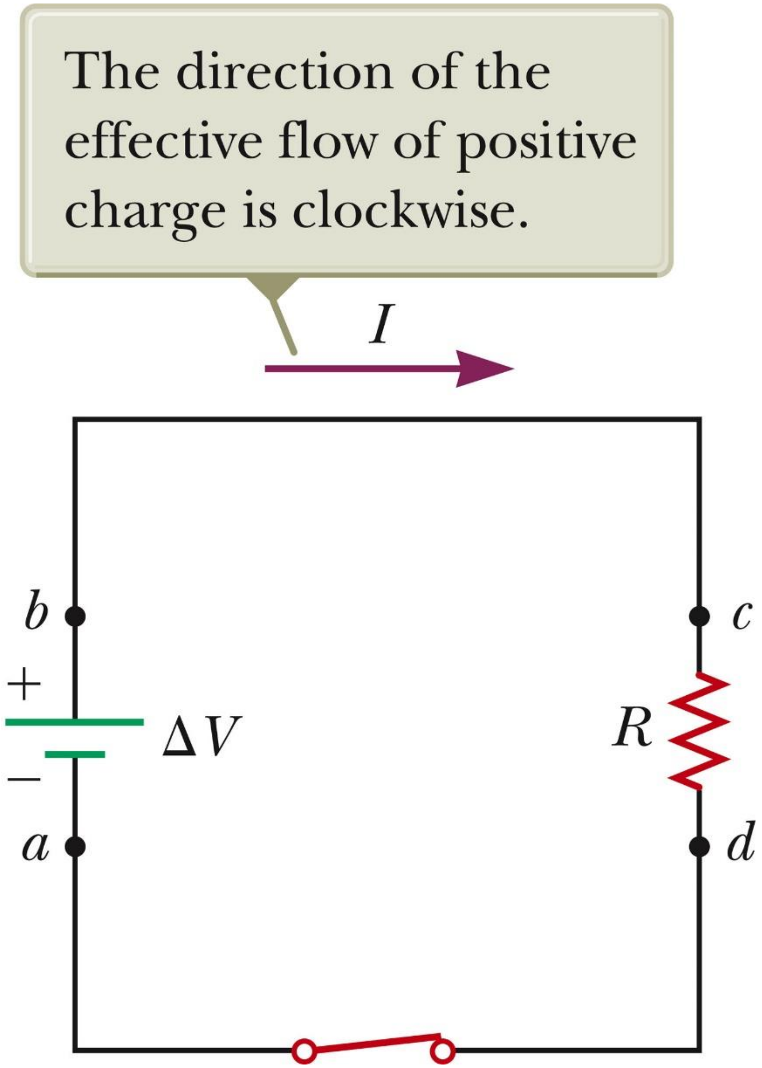
- Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance.

$$R = R_o[1 + \alpha(T - T_o)]$$

- Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

Electrical Power

- Assume a circuit as shown
- The entire circuit is the system.
- As a charge moves from *a* to *b*, the electric potential energy of the system increases by *Q is* DV.
    - The chemical energy in the battery must decrease by this same amount.
- This electric potential energy is transformed into internal energy in the resistor.
    - Corresponds to increased vibrational motion of the atoms in the resistor



Electric Power, 2

- The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.
- The resistor also emits thermal radiation.
- After some time interval, the resistor reaches a constant temperature.
  - The input of energy from the battery is balanced by the output of energy by heat and radiation.
- The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.
- The **power** is the rate at which the energy is delivered to the resistor.

Electric Power, final

- The power is given by the equation $P = I\,\Delta V$.
- Applying Ohm's Law, alternative expressions can be found: $R = \Delta V/I$

$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

- Units: *I* is in A, *R* is in Ω, Δ*V* is in V, and P is in W

28.1 Electromotive Force

A constant current can be maintained in a closed circuit through the use of a source of *emf, (such as a battery or generator)* that produces an electric field and thus may cause charges to move around a circuit.

One can think of a source of emf as a “charge pump.” When an electric potential difference exists between two points, the source moves charges “uphill” from the lower potential to the higher.
The emf describes the work done per unit charge, and hence the SI unit of emf is the volt.

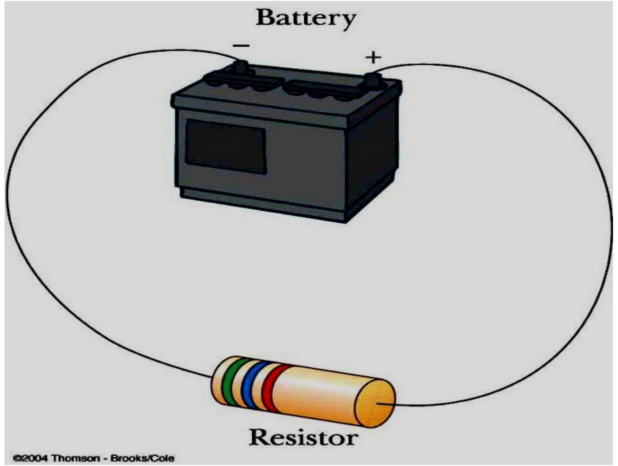
The emf of a battery $\mathcal{E}$ is the maximum possible voltage that the battery can provide between its terminals.

- assume that the connecting wires have no resistance.

- The positive terminal of the battery is at a higher potential than the negative terminal. If we neglect the internal resistance of the battery, the potential difference across it (called the terminal voltage) equals its emf.

- However, because a real battery always has some internal resistance *r, the terminal voltage is not equal to the emf for a battery in a* circuit in which there is a current.

A circuit consisting of a resistor connected to the terminals of a battery.



➢ As we pass from the negative terminal to the positive terminal, the potential *increases by an amount ε.*

➢ *As we move through the resistance r, the potential decreases by an amount Ir, where I is the current in the circuit.*

$$\Delta V = \varepsilon - Ir$$

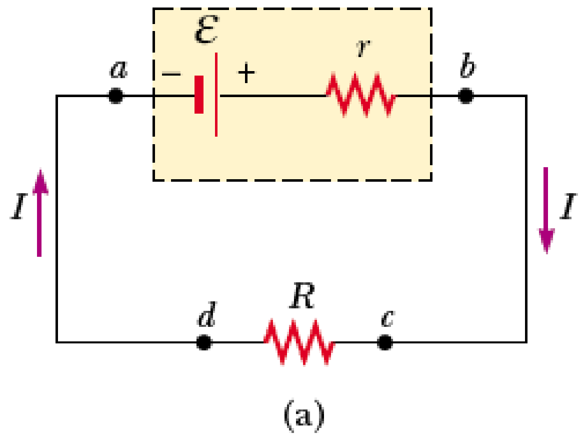


Figure 28.2 (a) Circuit diagram of a source of emf (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R.

➢ ε *:* is equivalent to the open-circuit voltage—that is, the *terminal voltage when the current is zero. The emf is the voltage labeled on a* battery,….

➢the terminal voltage *V must equal the potential difference across the external* resistance *R, often called the load resistance.*

the changes in electric potential as the circuit is traversed in the clockwise direction

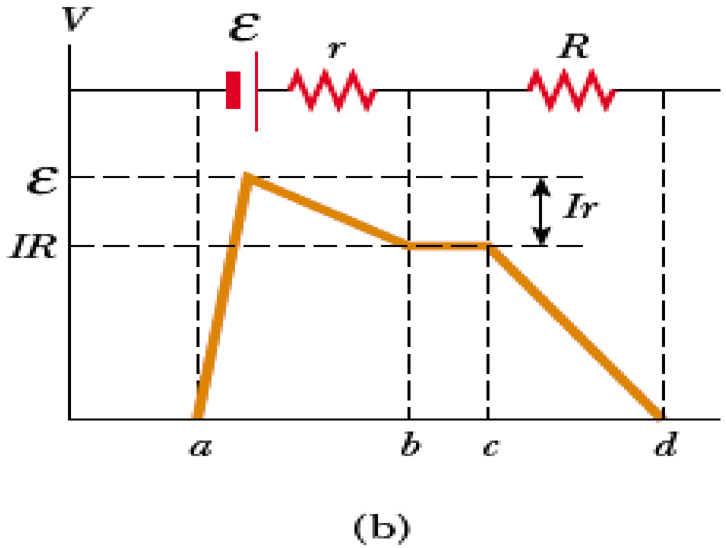


Figure 28.2 (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

- The resistor represents a *load on the battery because the battery must supply energy to* operate the device. The potential difference across the load resistance is

$$\Delta V = IR$$

$$\varepsilon = IR + Ir \qquad (28\text{-}2)$$

$$I = \frac{\varepsilon}{R + r} \qquad (28\text{-}3)$$

$$I\varepsilon = I^2R + I^2r \qquad (28\text{-}4)$$

- the total power output $I\varepsilon$ *of the battery is delivered to the external load resistance in the* amount I^2R *and to the internal resistance in the amount* I^2r.

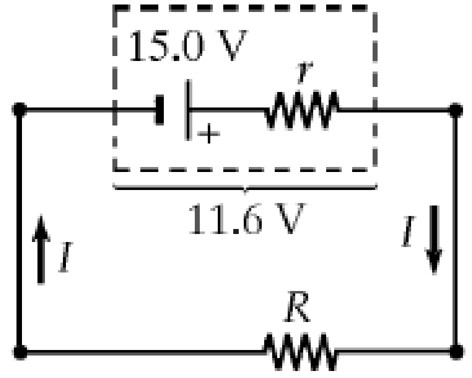
Example

* A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor *R. (a) What is the* value of *R? (b) What is the internal resistance of the* battery?

(a) $\mathcal{P} = \frac{(\Delta V)^2}{R}$

becomes $20.0\ \text{W} = \frac{(11.6\ \text{V})^2}{R}$

so $R = \boxed{6.73\ \Omega}$.



(b) $\Delta V = IR$

so $11.6\ \text{V} = I(6.73\ \Omega)$

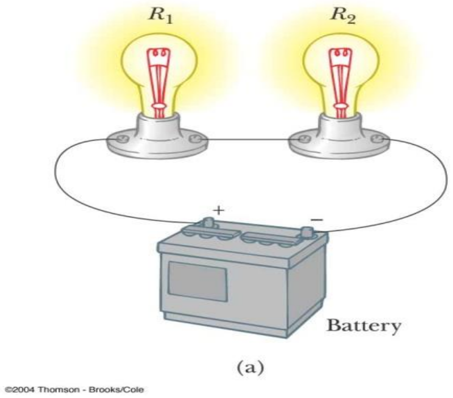
and $I = 1.72\ \text{A}$

$\varepsilon = IR + Ir$

so $15.0\ \text{V} = 11.6\ \text{V} + (1.72\ \text{A})r$

$r = \boxed{1.97\ \Omega}$.

28.2 Resistors in Series and Parallel



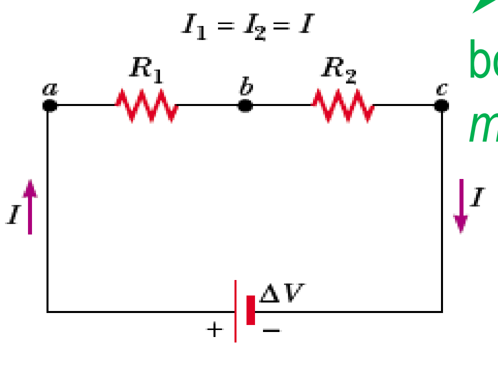
(a)

series connection

When two or more resistors are connected together as are the light- bulbs, they are said to be in *series.*

In a series connection, all the charges moving through one resistor must also pass through the second resistor.

- for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 *must also pass through* R_2

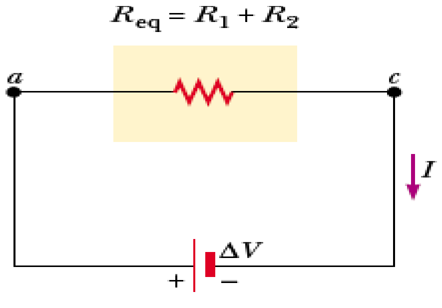


(b)

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$



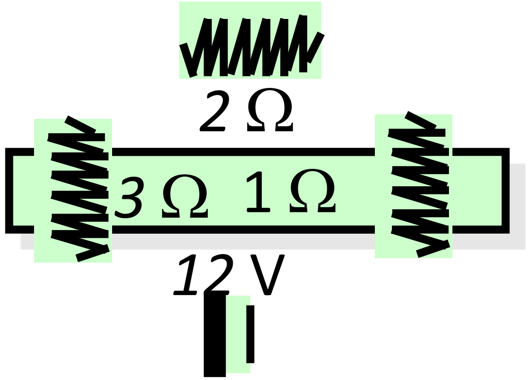
(c)

This relationship indicates that the equivalent resistance of a series connection of resistors is always greater than any individual resistance.

Example :

Find the equivalent resistance R_e. What is the current I in the circuit?

$$R_{eq} = R_1 + R_2 + R_3$$



Equivalent $R_{eq} = 3\ \Omega + 2\ \Omega + 1\ \Omega = 6\ \Omega$

The current is found from Ohm's law: $V = IR_e$

$$I = \frac{V}{R_e} = \frac{12\ V}{6\ \Omega}$$

$I = 2\ A$

Current I = 2 A same in each R.

$V_1 = IR_1$; $V_2 = IR_2$; $V_3 = IR_3$

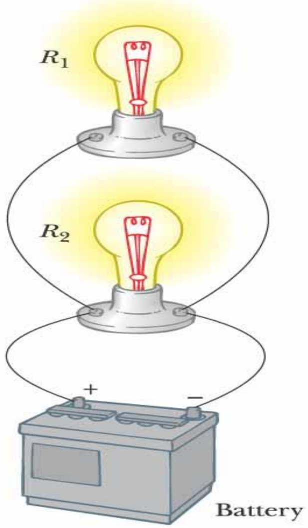
$V_1 = (2\ A)(1\ \Omega) = 2\ V$

$V_1 = (2\ A)(2\ \Omega) = 4\ V$

$V_1 = (2\ A)(3\ \Omega) = 6\ V$

$V_1 + V_2 + V_3 = V_{Total}$

$2\ V + 4\ V + 6\ V = 12\ V$



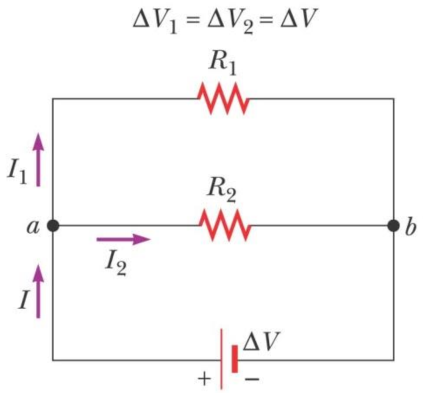
(a)

Now consider two resistors connected in *parallel,*

When the current *I reaches point a in Figure 28.5b, called a junction, it splits into* two parts, with *I1 going through* R_1 *and I 2 going through* R_2 . *A* ***junction is any*** point in a circuit where a current can split

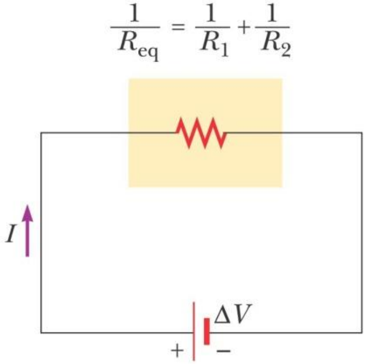
$$I = I_1 + I_2$$

when resistors are connected in parallel, the potential differences across the resistors is the same.



(b)

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

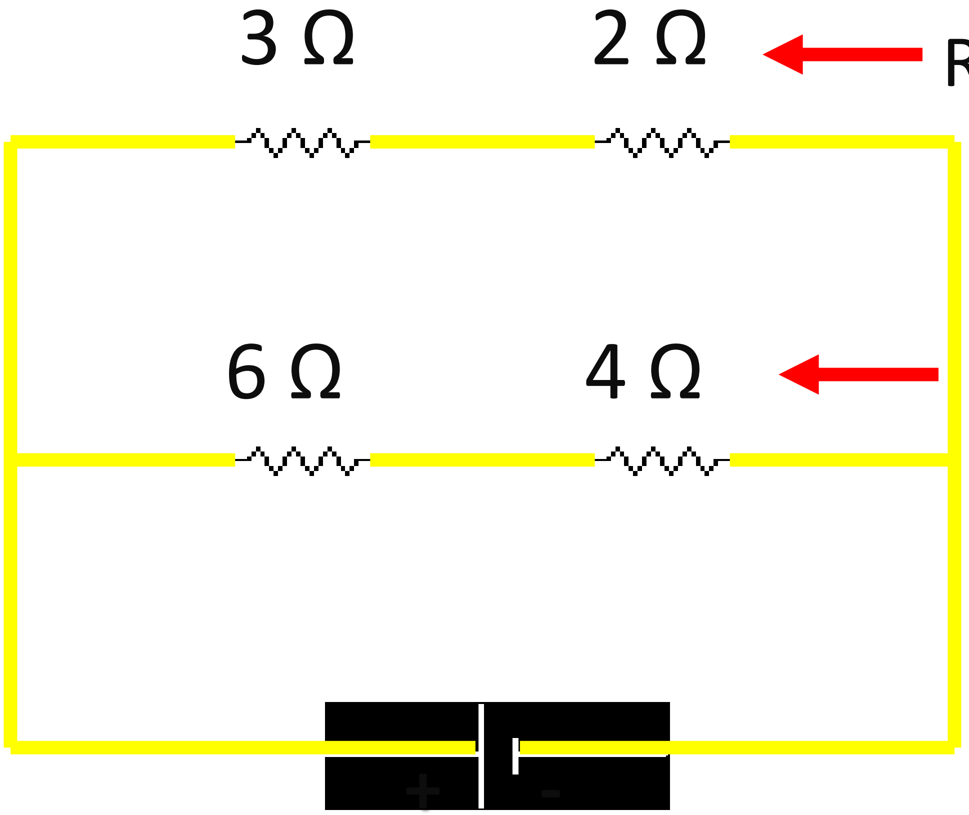


(c)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Calculate the total resistance in the circuit below



$R_{eq} = 3\ \Omega + 2\ \Omega = 5\ \Omega$

$R_{eq} = 6\ \Omega + 4\ \Omega = 10\ \Omega$

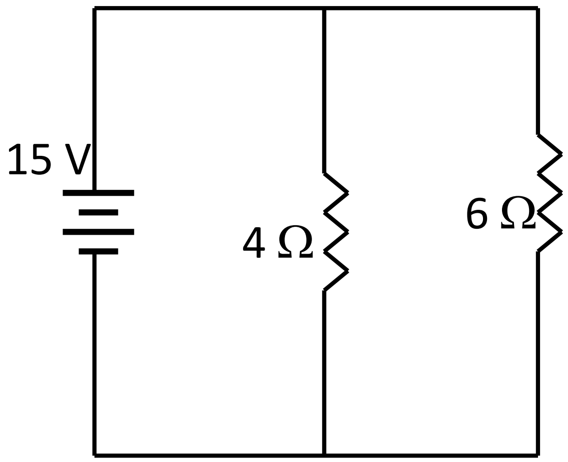
$R_{eq} = 3\ .33\ \Omega$

$1/Req = 2/10\ \Omega + 1/10\ \Omega = 3/10\ \Omega$

Parallel Example

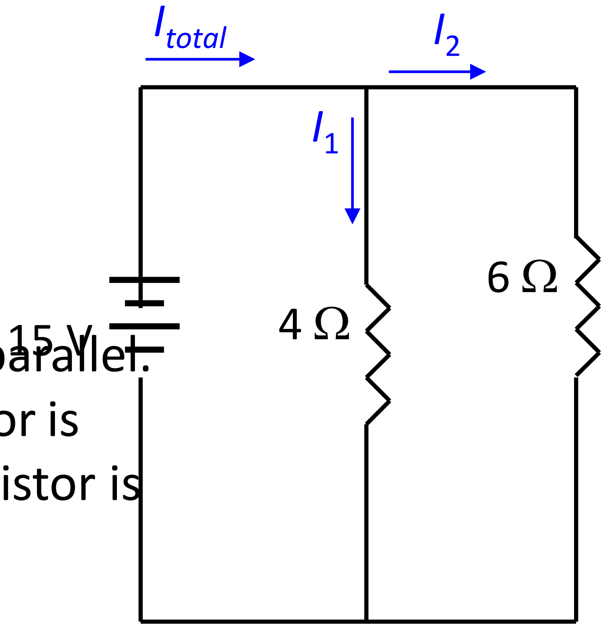
1. Find R_{eq} 2. Find I_{total}

3. Find the current through, and voltage drop across, each resistor.



1. $1/R_{eq} = 1/R_1 + 1/R_2 = 1/4 + 1/6$
   $= 6/24 + 4/24 = 5/12$
   $R_{eq} = 12/5 =$ **2.4 Ω**

2. $I_{total} = V / R_{eq}$
   $= 15 / (12/5)$
   $= 75/12 =$ **6.25 A**



3. The voltage drop across each resistor is the same in parallel. Each drop is 15 V. The current through the 4 Ω resistor is (15 V)/(4 Ω) = 3.75 A. The current through the 6 Ω resistor is (15 V)/(6 Ω) = 2.5 A. Check:

28.3 KIRCHHOFF'S RULES

- we can analyze simple circuits using the expression *V IR and the rules for series and parallel combinations of resistors.* Very often, however, it is not possible to reduce a circuit to a single loop.

- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff's rules:**

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \qquad (28.9)$$

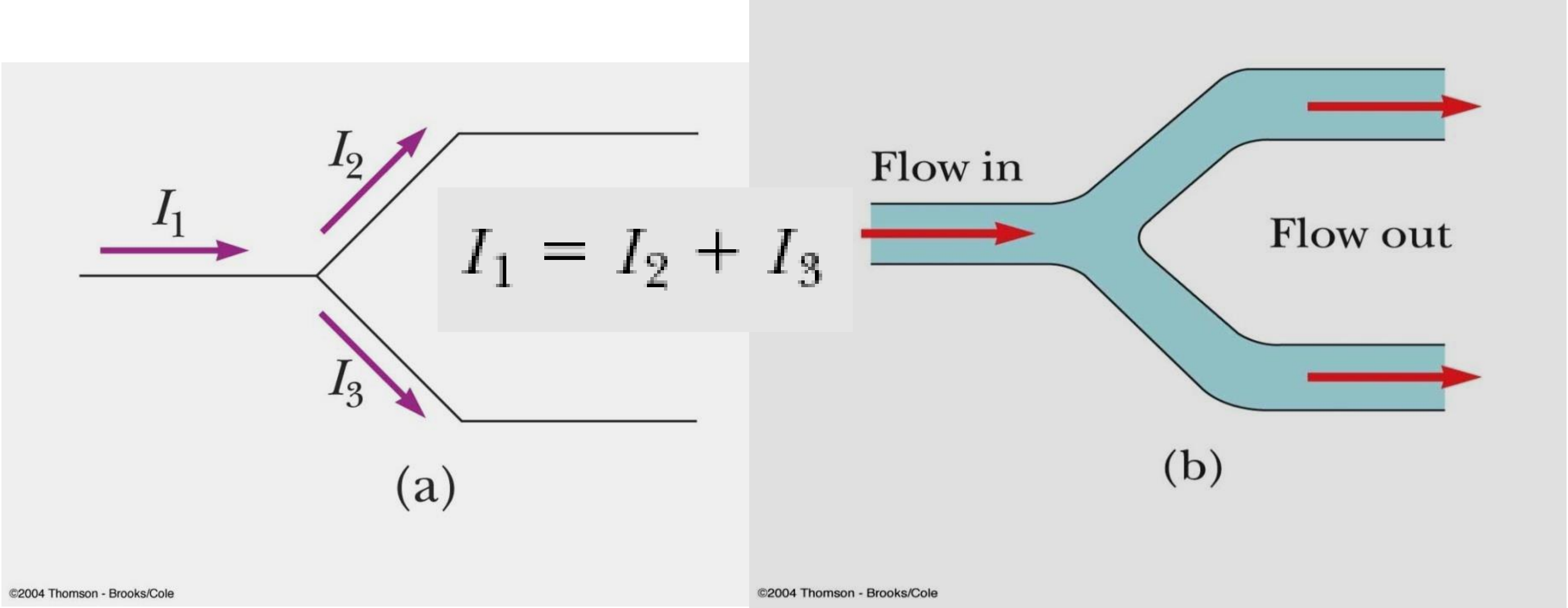
2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad (28.10)$$

Kirchhoff's first rule is a statement of conservation of electric charge.

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \qquad (28.9)$$



the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

Fig 28-14, p.870

Kirchhoff's second rule follows from the law of conservation of energy.

2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

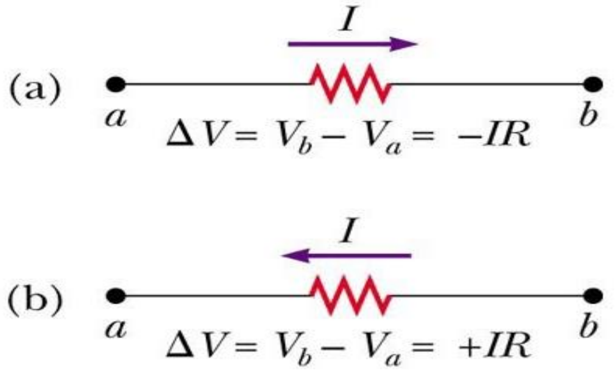
$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0 \tag{28.10}$$

Let us imagine moving a charge around the loop. When the charge returns to the starting point, the charge–circuit system must have the same energy as when the charge started from it.

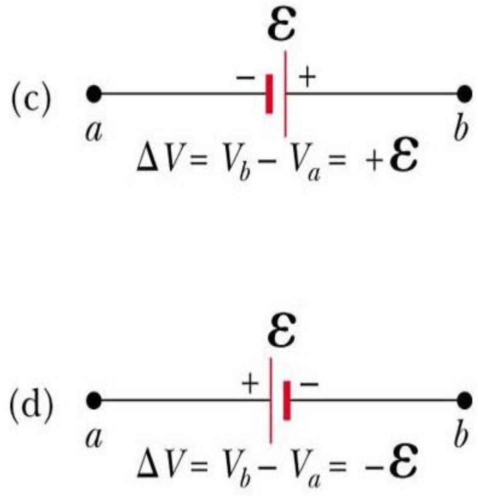
The sum of the increases in energy in some circuit elements must equal the sum of the decreases in energy in other elements.

The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases when-ever the charge passes through a battery from the negative terminal to the positive terminal.

- Traveling around the loop from a to b
- In a, the resistor is transversed in the direction of the current, the potential across the resistor is –IR
- In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is +IR

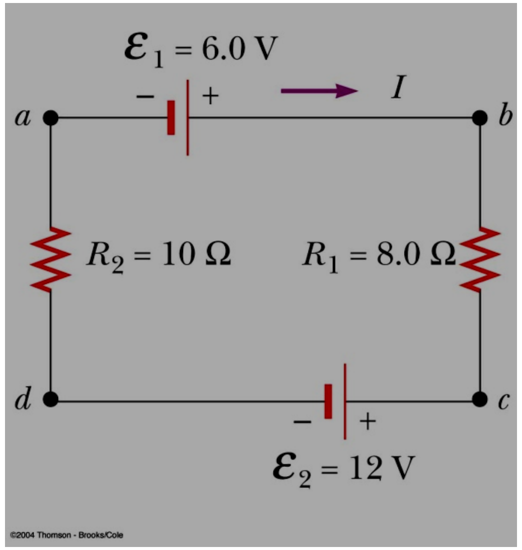


- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is -ε



EXAMPLE 28.7 A Single-Loop Circuit

(a) Find the current in the circuit. (Neglect the internal resistances of the batteries.)



Traversing the circuit in the clockwise direction, starting at *a, we* see that *a* → *b represents a potential change of* $+\varepsilon_1$*, b* → *c* represents a potential change of $-IR_1$*, c* → *d represents a potential* change of $-\varepsilon_2$, and *d* → *a represents a potential* change of $-IR_2$ *. Applying Kirchhoff's loop rule gives*

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

$$\mathcal{P}_1 = I^2R_1 = (0.33\ \text{A})^2(8.0\ \Omega) = 0.87\ \text{W}$$

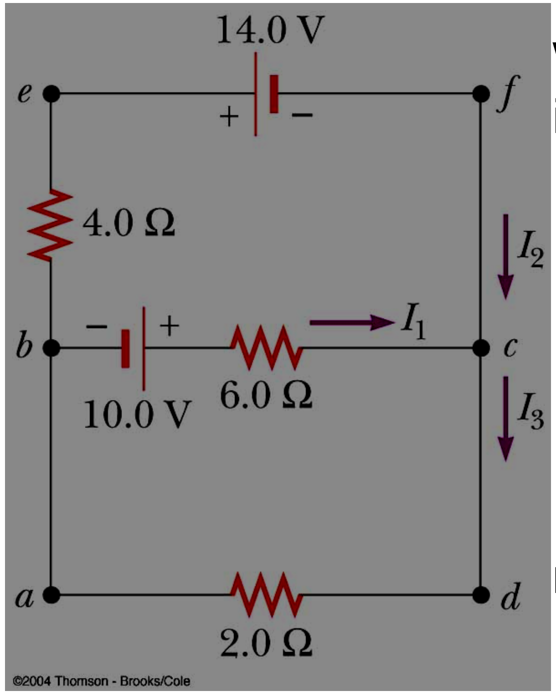
$$\mathcal{P}_2 = I^2R_2 = (0.33\ \text{A})^2(10\ \Omega) = 1.1\ \text{W}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0\ \text{W}$.

The 12-V battery delivers power $I\varepsilon_2$. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery.

EXAMPLE 28.8 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit



We arbitrarily choose the directions of the currents as labeled in Figure

$$(1) \quad I_1 + I_2 = I_3$$

$$(2) \quad abcda \quad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$$

Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)(I_1 + I_2) = 0$$

$$(4) \quad 10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \quad -12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)(2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = -3\text{ A}$$

$$I_3 = I_1 + I_2 = -1\text{ A}$$

The fact that I_2 *and* I_3 *are both negative indicates only that* the currents are opposite the direction we chose for them. However, the numerical values are correct.