PHYS 111

1st semester 1446

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Lecture 5

Chapter 26 Capacitance and Dielectric


nichicon
4700µF 35V
100µF 16V
VITAMIN Q
CAPACITOR
22 - 400 D.C.
35 V 470µF
189-505-5
193-10 8216
4.7µF 35V
103K
500V
561
10
151
15
22µF 35V

Capacitors

- Capacitors are devices that store electric charge.
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Makeup of a Capacitor

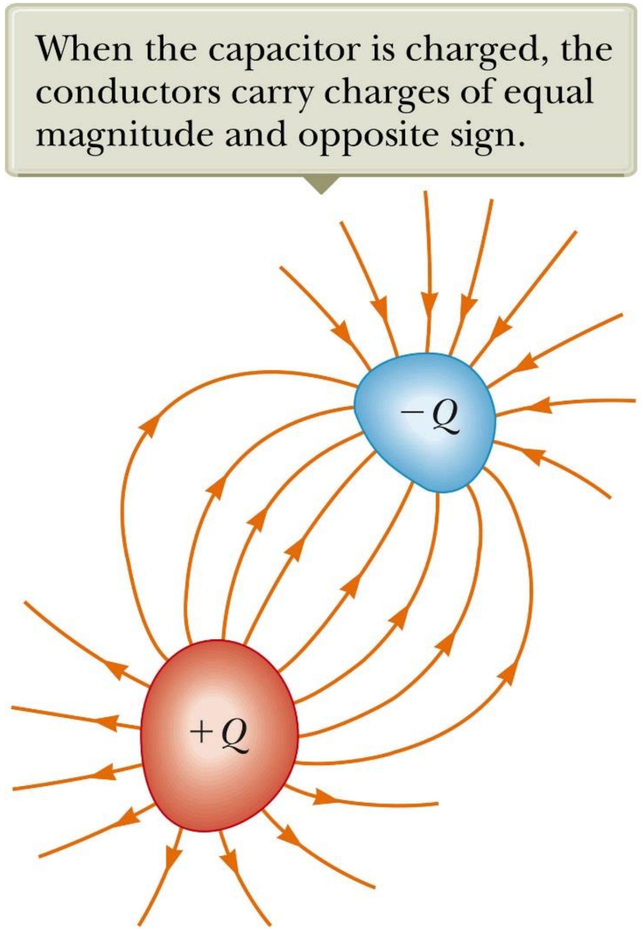
- A capacitor consists of two conductors.
  - These conductors are called plates.
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.
- A potential difference exists between the plates due to the charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$q = (4\pi\varepsilon_0 R)V$$

$$q = CV$$

$$C = 4\pi\varepsilon_0 R$$



Definition of Capacitance

- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F).
- The farad is a large unit, typically you will see microfarads (mF) and picofarads (pF).
- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant.
- The capacitance is a measure of the capacitor's ability to store charge .
    - The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

Parallel Plate Capacitor

•Each plate is connected to a terminal of the battery.

- The battery is a source of potential difference.

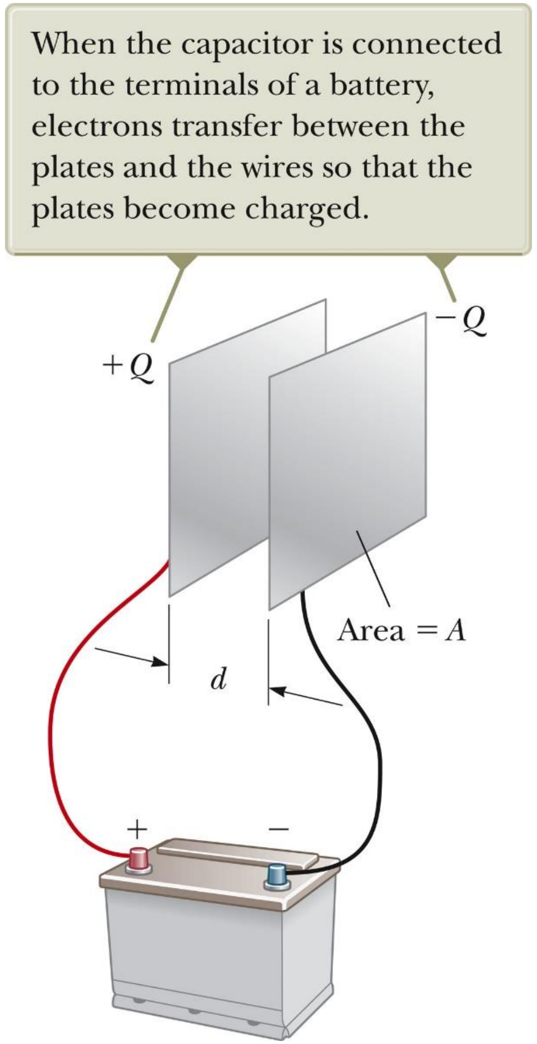
•If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.

$$V = Ed = \frac{Q}{A\varepsilon_0}d$$

$$C = \frac{Q}{V}$$

$$= \frac{Q}{\frac{Q}{A\varepsilon_0}d}$$

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$



Capacitance – Parallel Plates

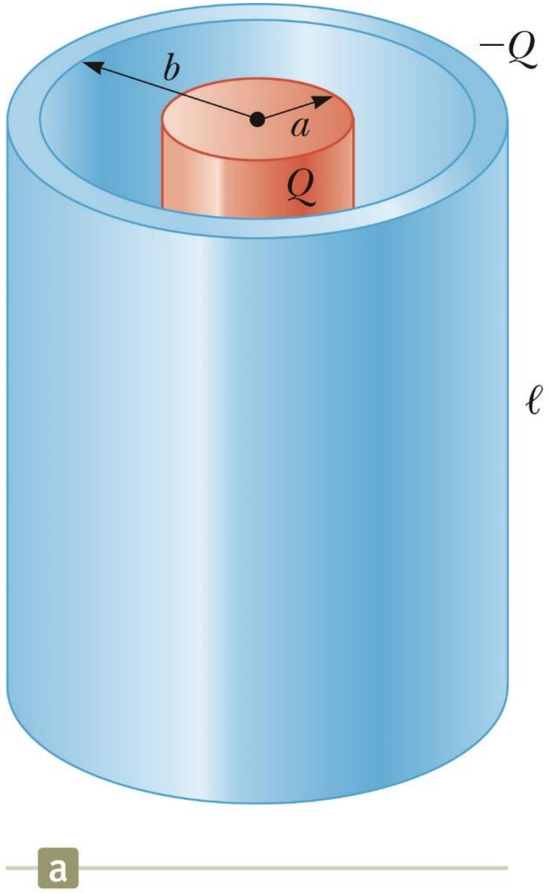
- The charge density on the plates is $\sigma = Q/A$.
  - A is the area of each plate, the area of each plate is equal
  - Q is the charge on each plate, equal with opposite signs
- The electric field is uniform between the plates and zero elsewhere.
- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

Capacitance of a Cylindrical Capacitor

- DV = -2$k_e \lambda$ ln (b/a)
- λ = Q/l
- The capacitance is

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)}$$



Capacitance of a Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b - a\right)}$$

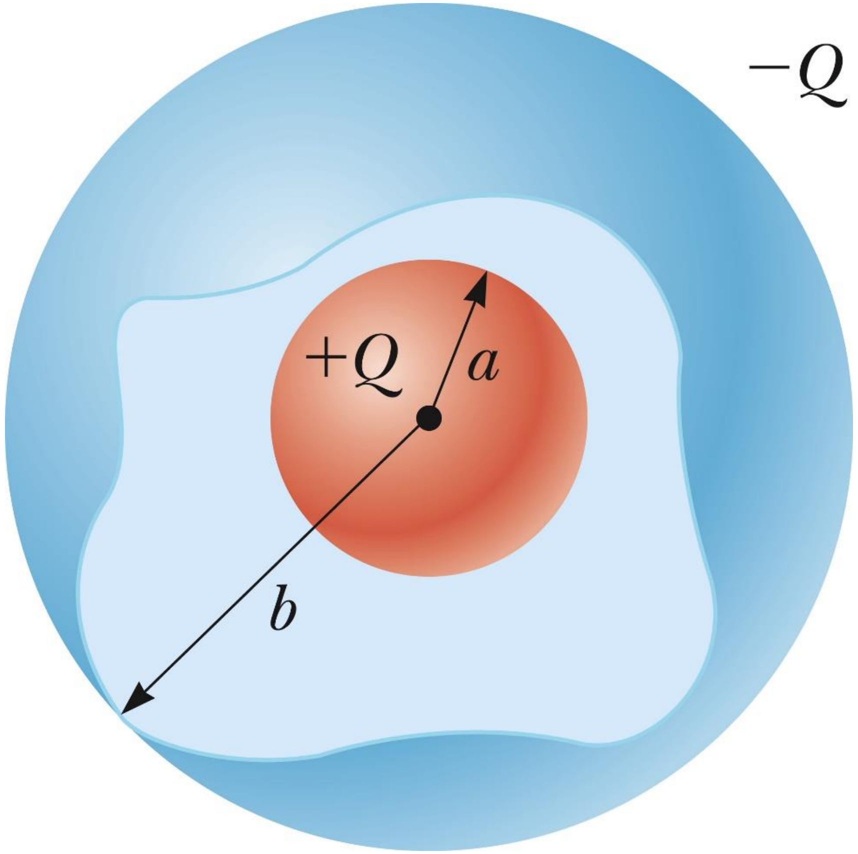


Table 26.2

Capacitance and Geometry		
Geometry	**Capacitance**	**Equation**
Isolated sphere of radius R (second spherical conductor assumed to have infinite radius)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \dfrac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \dfrac{\ell}{2k_e \ln(b/a)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \dfrac{ab}{k_e(b-a)}$	26.6

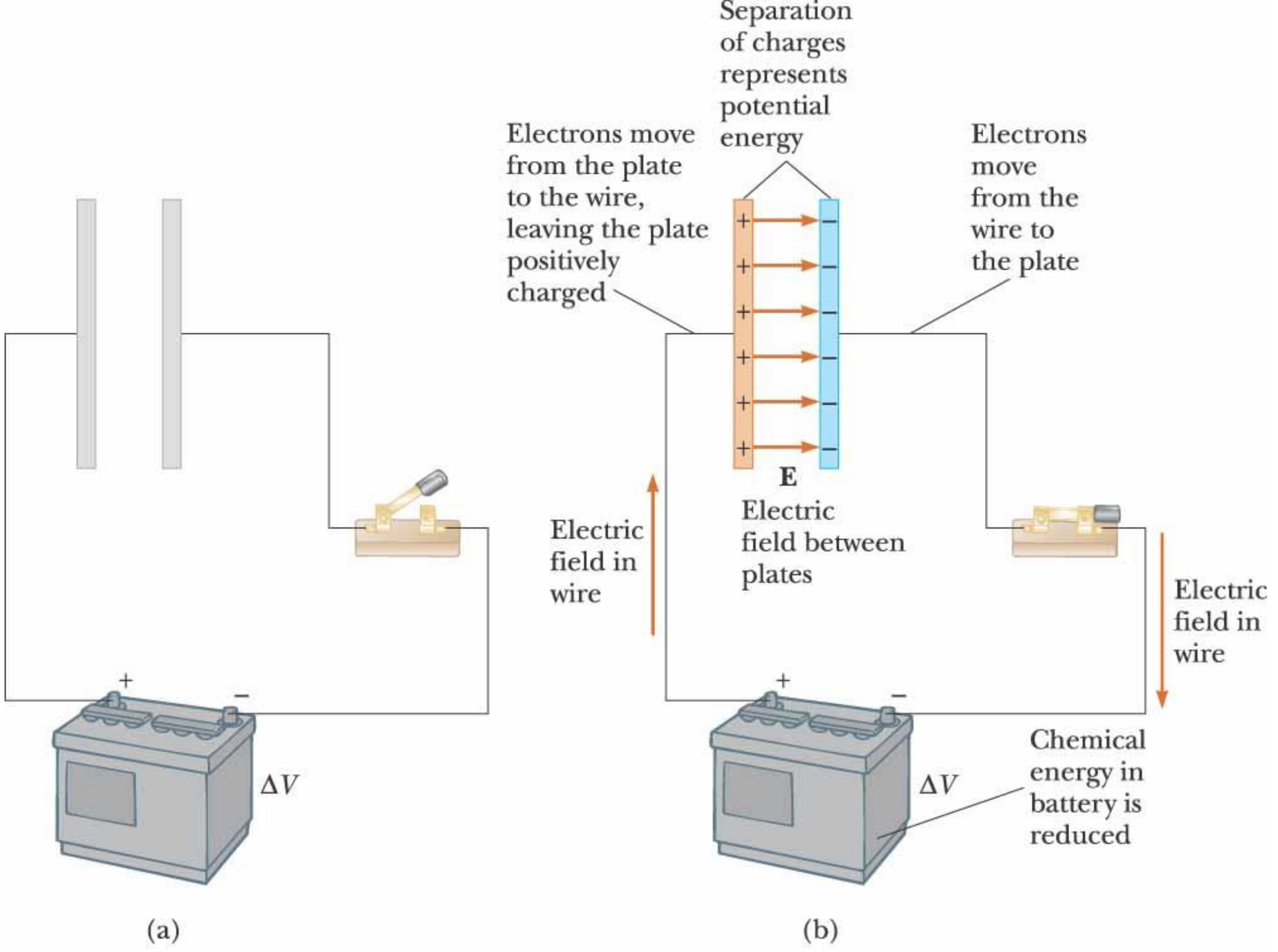
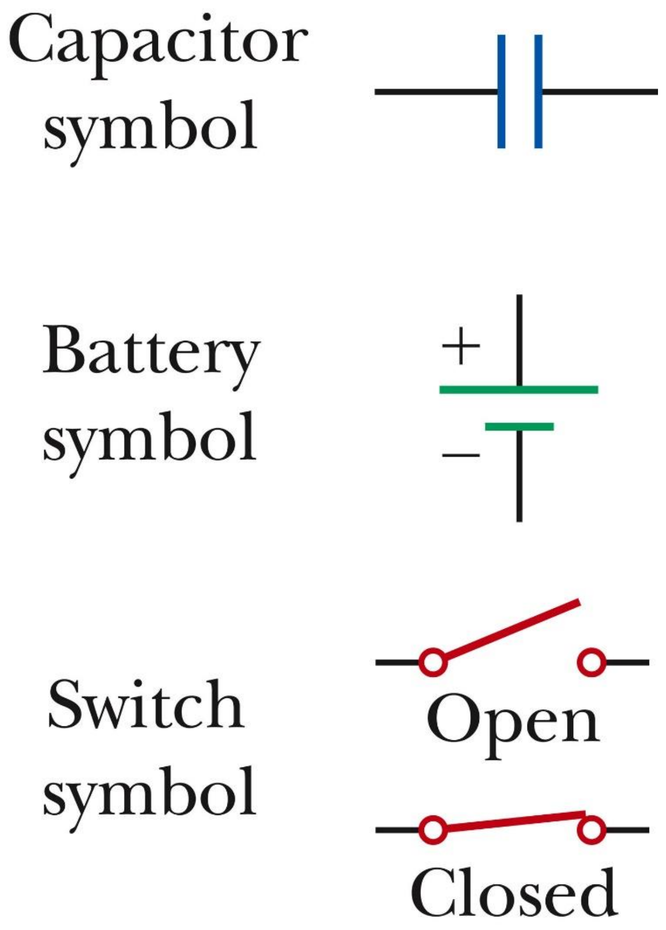

Separation
of charges
represents
potential
energy
Electrons move
from the plate
to the wire,
leaving the plate
positively
charged
Electrons
move
from the
wire to
the plate
E
Electric
field between
plates
Electric
field in
wire
Electric
field in
wire
+
−
ΔV
+
−
ΔV
Chemical
energy in
battery is
reduced
(a)
(b)

Fig 26-4, p.800

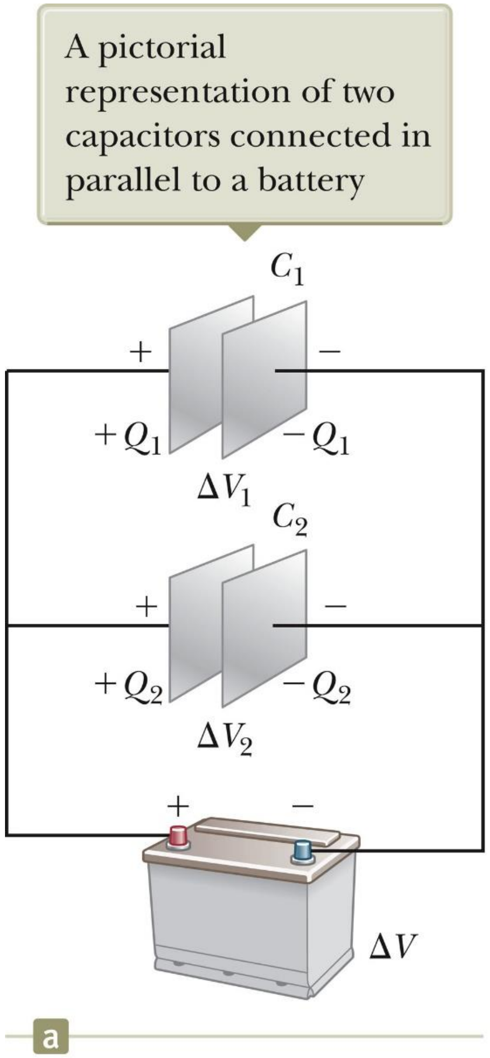
Circuit Symbols

- A circuit diagram is a simplified representation of an actual circuit.
- Circuit symbols are used to represent the various elements.
- Lines are used to represent wires.
- The battery's positive terminal is indicated by the longer line.



Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

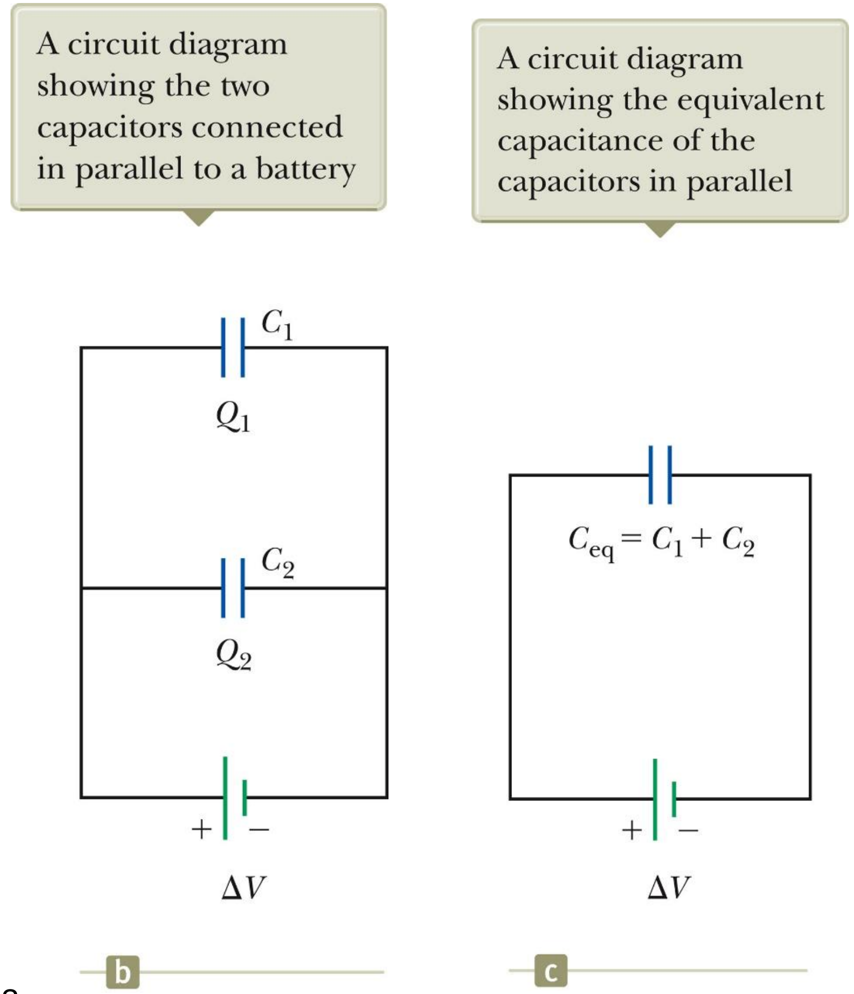


Capacitors in Parallel, 2

- The flow of charges ceases when the voltage across the capacitors equals that of the battery.
- The potential difference across the capacitors is the same.
  - And each is equal to the voltage of the battery
  - $DV_1 = DV_2 = DV$
    - DV is the battery terminal voltage
- The capacitors reach their maximum charge when the flow of charge ceases.
- The total charge is equal to the sum of the charges on the capacitors.
  - $Q_{\text{tot}} = Q_1 + Q_2$

Capacitors in Parallel, 3

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}.
  - The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors.

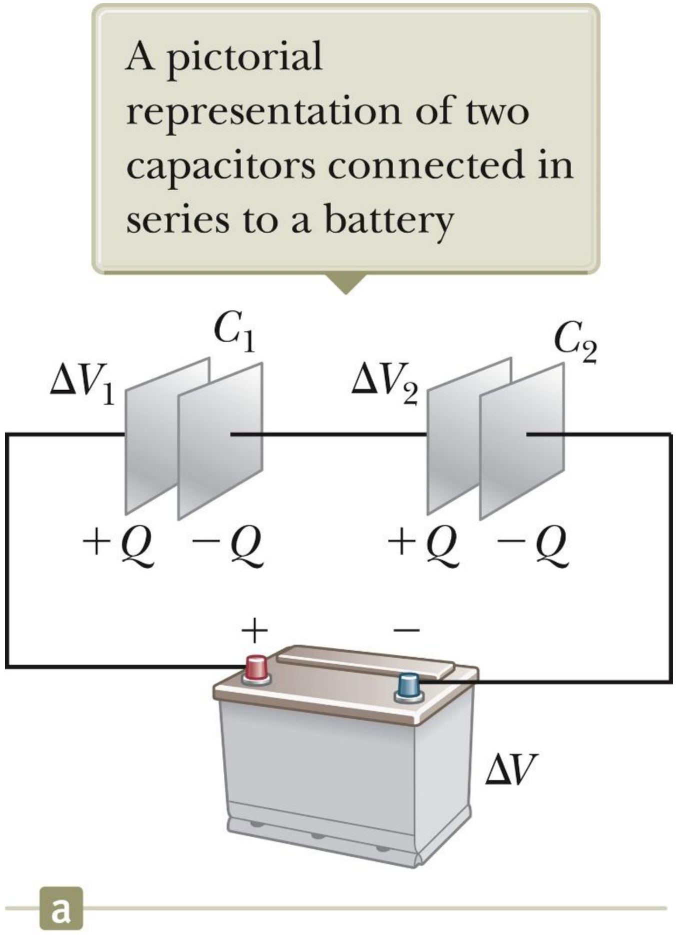


Capacitors in Parallel, final

- $C_{eq} = C_1 + C_2 + C_3 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.
  - Essentially, the areas are combined

Capacitors in Series

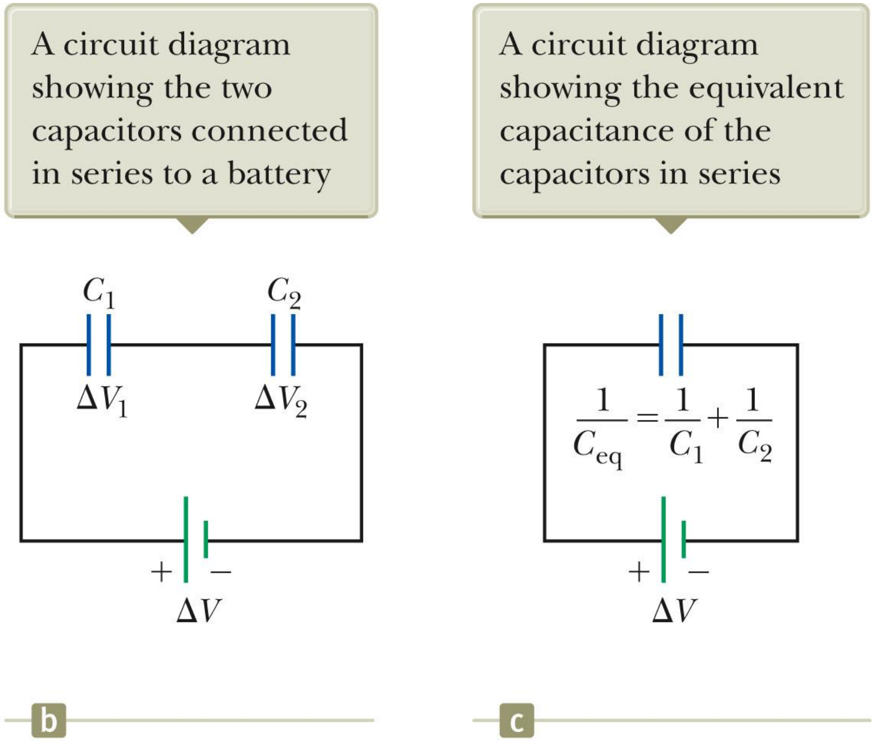
- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.
- All of the right plates gain charges of –Q and all the left plates have charges of $+Q$.



Capacitors in Series, cont.

- An equivalent capacitor can be found that performs the same function as the series combination.
- The charges are all the same.

$$Q_1 = Q_2 = Q$$



Capacitors in Series, final

- The potential differences add up to the battery voltage.

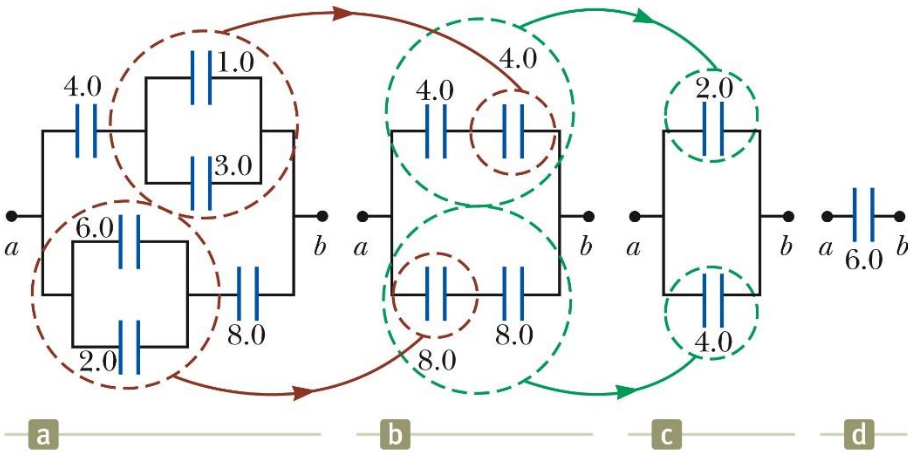
$$\Delta V_{tot} = DV_1 + DV_2 + \ldots$$

- The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.

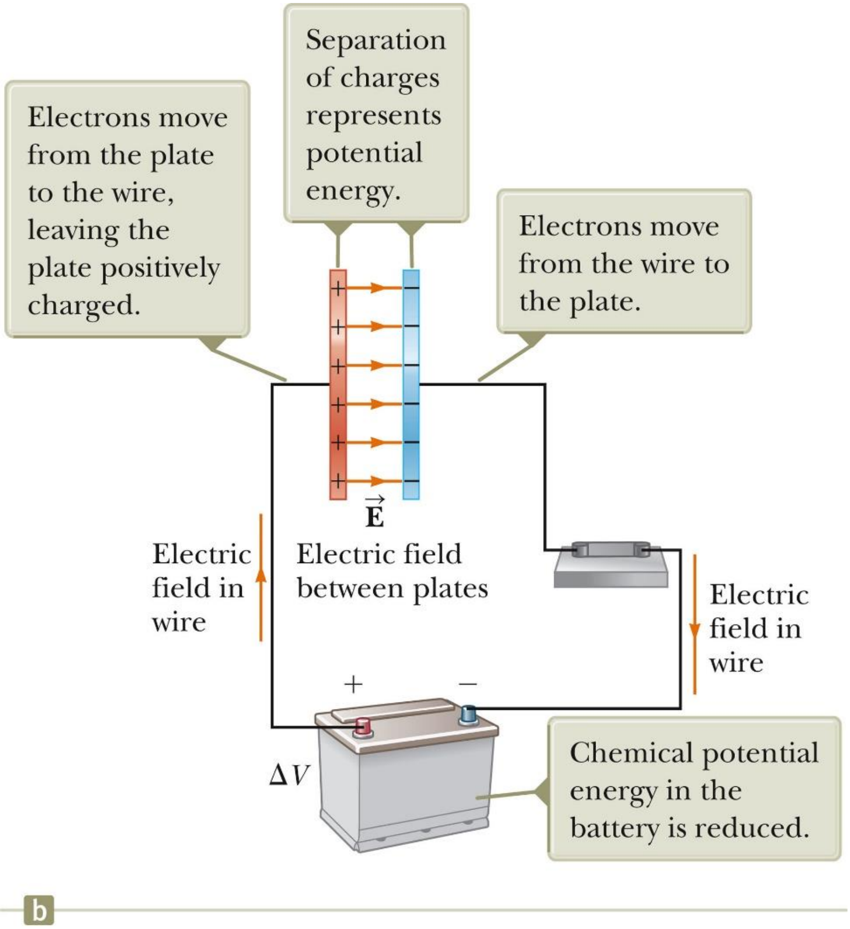
Equivalent Capacitance, Example



- The 1.0-mF and 3.0-mF capacitors are in parallel as are the 6.0-mF and 2.0-mF capacitors.
- These parallel combinations are in series with the capacitors next to them.
- The series combinations are in parallel and the final equivalent capacitance can be found.

Energy in a Capacitor – Overview

- Consider the circuit to be a system.
- Before the switch is closed, the energy is stored as chemical energy in the battery.
- When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.
- The electric potential energy is related to the separation of the positive and negative charges on the plates.
- A capacitor can be described as a device that stores energy as well as charge.



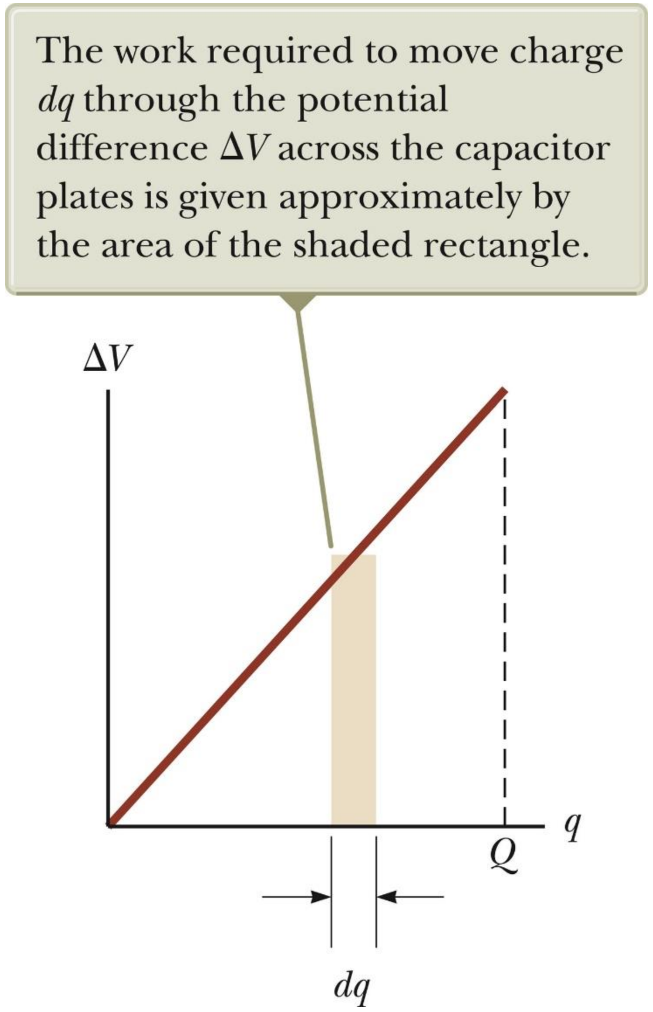
Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it.
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The work required is the area of the tan rectangle.
- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$



Energy, cont

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- This applies to a capacitor of any geometry.
- The energy stored increases as the charge increases and as the potential difference increases.
- In practice, there is a maximum voltage before discharge occurs between the plates.

Energy, final

- The energy can be considered to be stored in the electric field .
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = ½ (\varepsilon_o Ad)E^2$.
- It can also be expressed in terms of the energy density (energy per unit volume)

$u_E = ½\ e_o E^2$.

Some Uses of Capacitors

- Defibrillators
  - When cardiac fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern.
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

Capacitors with Dielectrics

- A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.
  - Dielectrics include rubber, glass, and waxed paper
- With a dielectric, the capacitance becomes $C = \kappa C_o$.
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.
  - κ is the dielectric constant of the material.
- If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.
- If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.

Dielectrics, cont

- For a parallel-plate capacitor, $C = \kappa\ (\varepsilon_o A)\ /\ d$
- In theory, *d* could be made very small to create a very large capacitance.
- In practice, there is a limit to *d*.
  - *d* is limited by the electric discharge that could occur though the dielectric medium separating the plates.
- For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material.

Dielectrics, final

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching.
    - This decreases d and increases C.

Some Dielectric Constants and Dielectric Strengths

TABLE 26.1 *Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature*

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

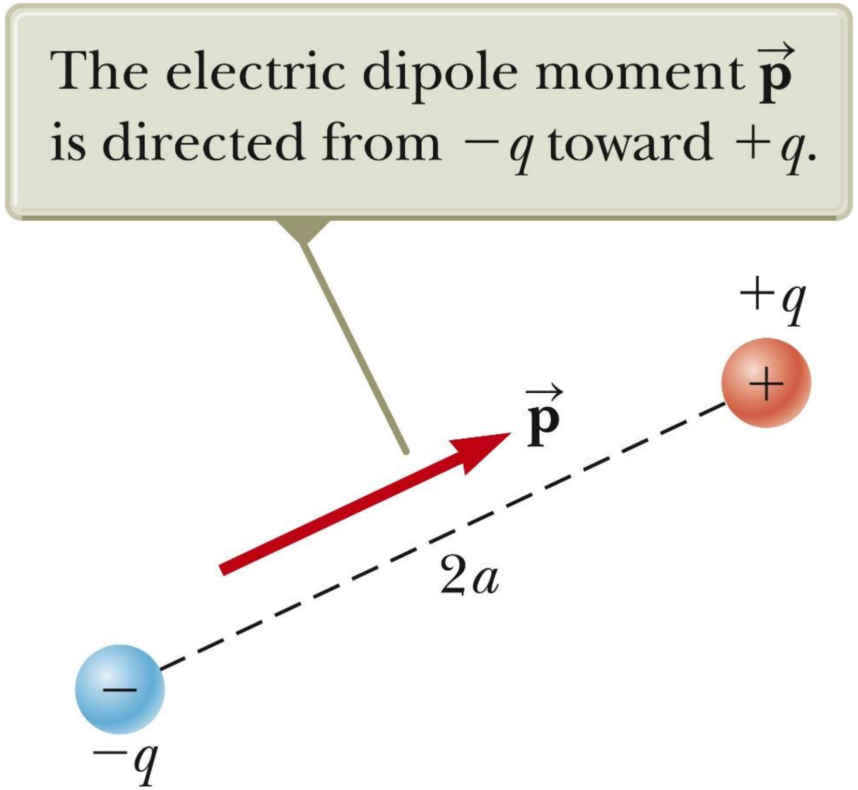
[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Electric Dipole

- An electric dipole consists of two charges of equal magnitude and opposite signs.
- The charges are separated by $2a$.
- The **electric dipole moment** ( )

$$\vec{\mathbf{p}}$$

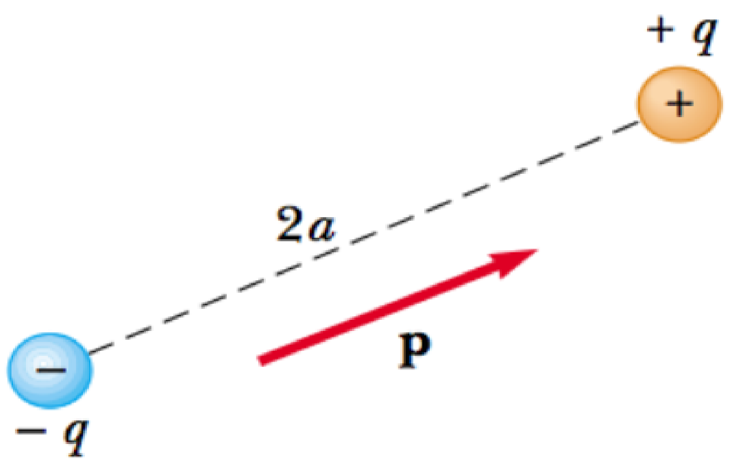
- is directed along the line joining the charges from $-q$ to $+q$.



26.6 Electric Dipole in an Electric Field

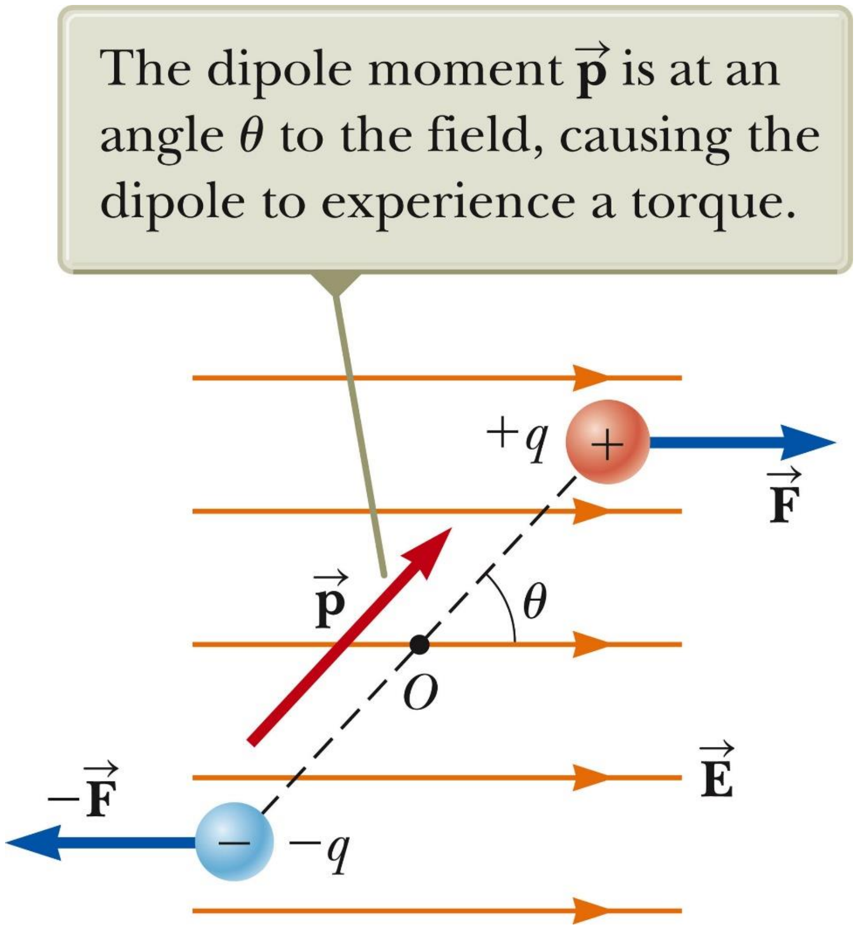
- The electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$,
- The **electric dipole moment** of this configuration is defined as the vector **p** directed from $-q$ toward $+q$ along the line joining the charges and having magnitude $2aq$:

$$p = 2aq$$



Electric Dipole, 3

- Each charge has a force of $F = Eq$ acting on it.
- The net force on the dipole is zero.
- The forces produce a net torque on the dipole.
- The dipole is a rigid object under a net torque.



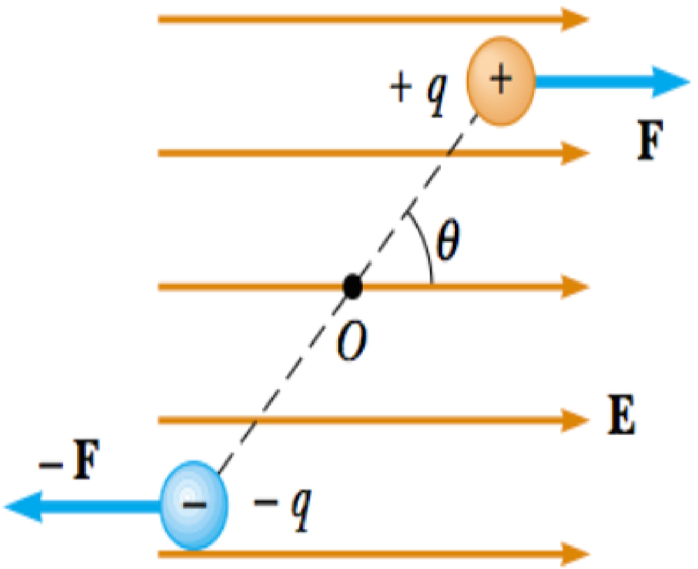
26.6 Electric Dipole in an Electric Field

- Now suppose that an electric dipole is placed in a uniform electric field **E**,
- The electric forces acting on the two charges are equal in magnitude ($F=qE$) and opposite in direction.
- The two forces produce a net **torque** on the dipole; as a result, the dipole rotates.
- This forces tend to produce a clockwise rotation.
- The magnitude of the net torque about O is

$$\tau = 2Fa\sin\theta$$

- Because $F=qE$ and $p=2aq$, we can express τ as $\tau = 2aqE\sin\theta = pE\sin\theta$
- **Torque on an electric dipole in an external electric field**

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$



A
G

Electric Dipole, final

- The magnitude of the torque is:

$$\tau = 2Fa \sin \theta = pE \sin \theta$$

- The torque can also be expressed as the cross product of the moment and the field:

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

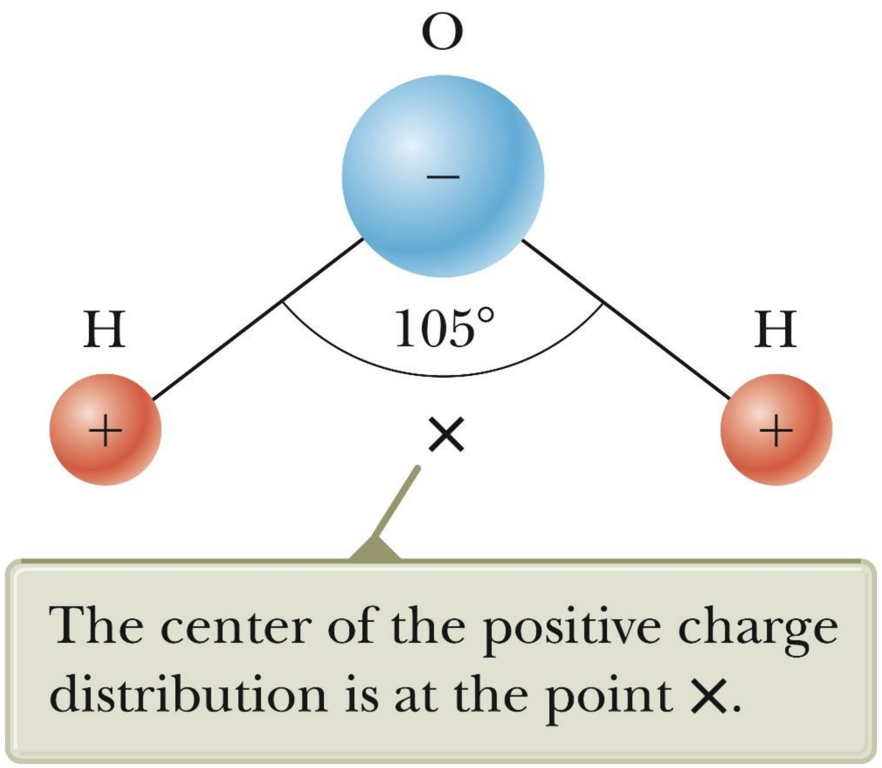
- The system of the dipole and the external electric field can be modeled as an isolated system for energy.
- The potential energy can be expressed as a function of the orientation of the dipole with the field:

$$U_f - U_i = pE(\cos \theta_i - \cos \theta_f) ® U = - pE \cos \theta$$

This expression can be written as a dot product. $U = \vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$

Water Molecules

- A water molecule is an example of a polar molecule.
- The center of the negative charge is near the center of the oxygen atom.
- The x is the center of the positive charge distribution.



Example 26.8 The H_2O Molecule

The water (H_2O) molecule has an electric dipole moment of 6.3×10^{-30} C·m. A sample contains 10^{21} water molecules, with the dipole moments all oriented in the direction of an electric field of magnitude 2.5×10^5 N/C. How much work is required to rotate the dipoles from this orientation ($\theta = 0°$) to one in which all the moments are perpendicular to the field ($\theta = 90°$)?

Solution The work required to rotate one molecule 90° is equal to the difference in potential energy between the 90° orientation and the 0° orientation. Using Equation 26.19, we obtain

$$\begin{aligned} W &= U_{90°} - U_{0°} = (-pE\cos 90°) - (-pE\cos 0°) \\ &= pE = (6.3 \times 10^{-30}\ \mathrm{C\cdot m})(2.5 \times 10^5\ \mathrm{N/C}) \\ &= 1.6 \times 10^{-24}\ \mathrm{J} \end{aligned}$$

Because there are 10^{21} molecules in the sample, the *total* work required is

$$W_{\text{total}} = (10^{21})(1.6 \times 10^{-24}\ \mathrm{J}) = \boxed{1.6 \times 10^{-3}\ \mathrm{J}}$$