PHYS 111

1ST semester 1446

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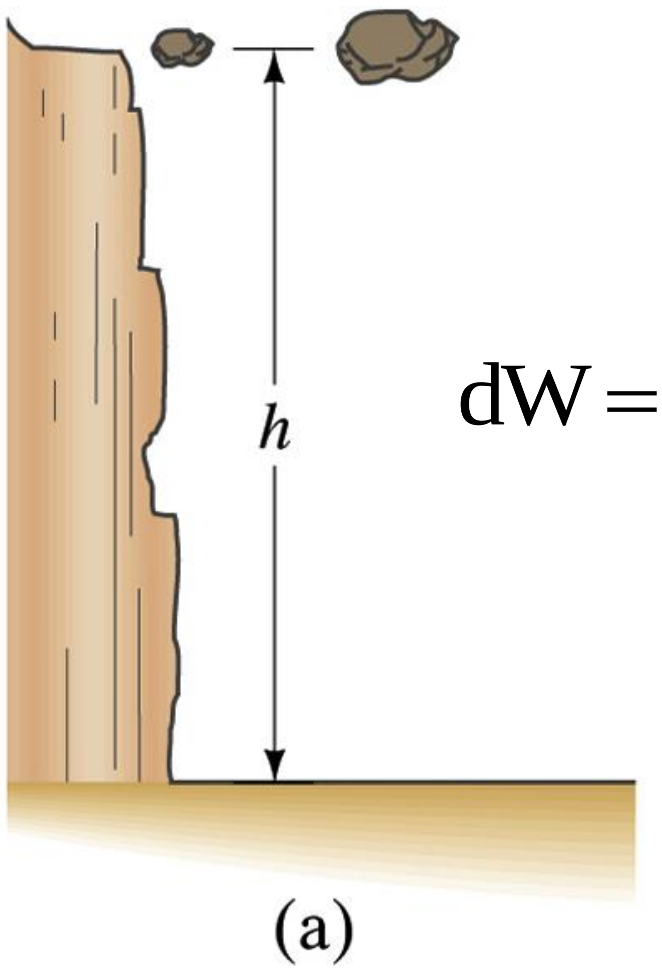
Lecture 4

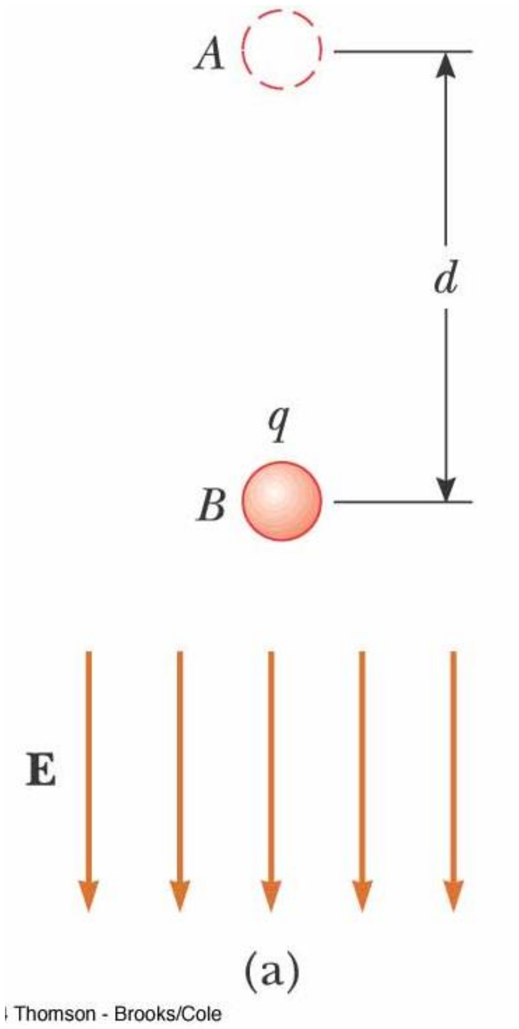
Chapter 25 Electric Potential

25-1 Potential difference and electric Potential
25-2 Potential Difference and electric field
25-3 Electric Potential and Potential energy due to point charges

25-1 Potential difference and electric Potential



$$dW = -dU$$



Work and Potential Energy

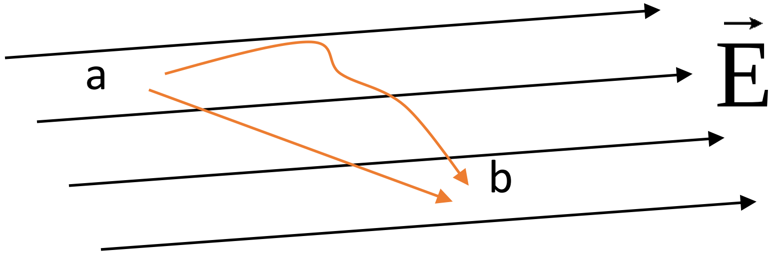
$$dW = \vec{F} \cdot d\vec{s}$$

Electric Field Definition:

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}$$

$$dW = q\vec{E} \cdot d\vec{s}$$

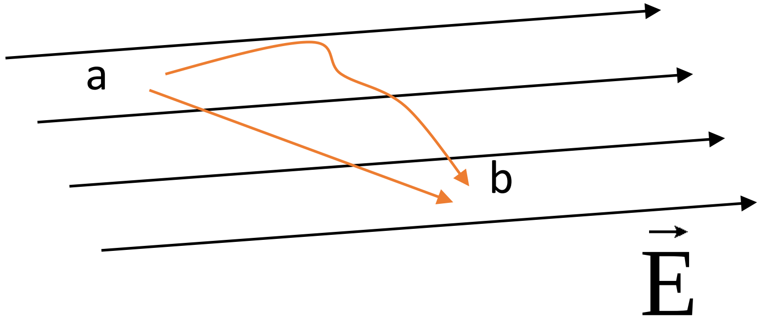
Work Energy Theorem



$$dW = -dU = q\vec{E} \cdot d\vec{s}$$

$$\int_a^b dU = -q\int_a^b \vec{E} \cdot d\vec{s}$$

Electric Potential Difference



$$\int_a^b dU = -q\int_a^b \vec{E}\cdot d\vec{s}$$

$$U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s}$$

$$\frac{U_b - U_a}{q} = -\int_a^b \vec{E}\cdot d\vec{s}$$

Definition: $$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E}\cdot d\vec{s}$$

Conventions for the potential "zero point"

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

"Potential"

Choice 1: $V_a=0$

$$V_b - V_a \equiv \frac{U_b - U_a}{q} \quad (V_a \to 0,\ U_a \to 0)$$

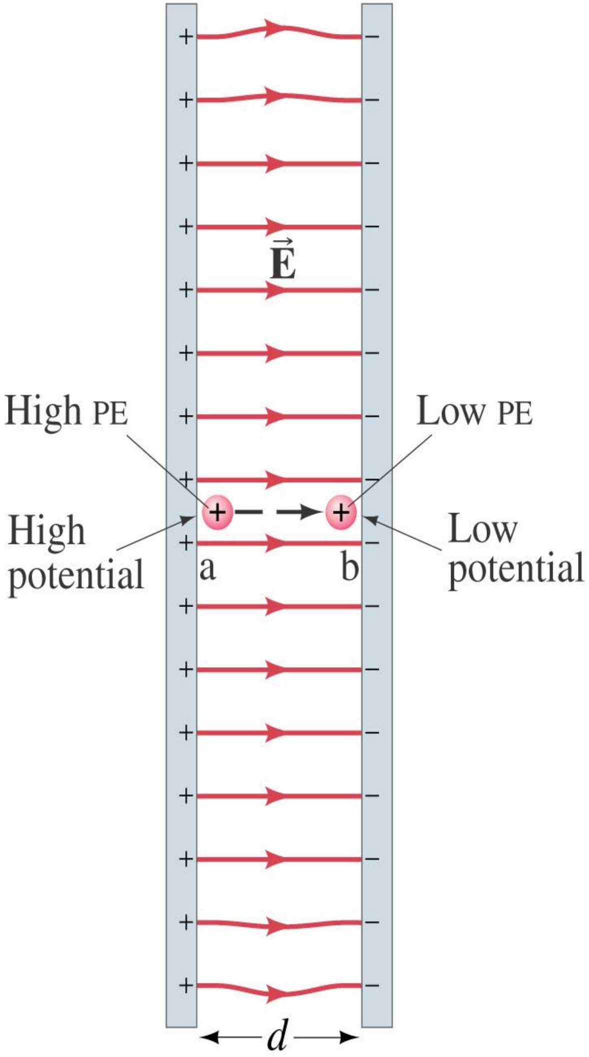
$$V_b = \frac{U_b}{q}$$

Choice 2: $V_\infty = 0$

$$V_{b\infty} = V_b - V_\infty \equiv \frac{U_b - U_\infty}{q} = -\int_\infty^b \vec{E}\cdot d\vec{s} \quad (V_\infty \to 0,\ U_\infty \to 0)$$

$$V_b = \frac{U_b}{q} = -\int_\infty^b \vec{E}\cdot d\vec{s}$$

25-2 Potential Difference and electric field



When a force is "conservative" ie gravitational and the electrostatic force a potential energy can be defined

Change in electric potential energy is negative of work done by electric force:

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$PE_b - PE_a = -qEd$$

$$\Delta V = -\int E\, ds = -Ed$$

•The change in potential energy is directly related to the change in voltage.

$$\Delta U = q\Delta V$$

$$\Delta V = \Delta U/q$$

- ΔU: change in electrical potential energy (J)
- q: charge moved (C)
- ΔV: potential difference (V)

•All charges will spontaneously go to lower potential energies if they are allowed to move.

Units of Potential Difference

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

$$\left[\frac{\text{Joules}}{\text{Coulomb}}\right] = \left[\frac{J}{C}\right] = \text{Volt} = V$$

Because of this, potential difference is often referred to as “voltage”

In addition, 1 N/C = 1 V/m - we can interpret the electric field as a measure of the rate of change with position of the electric potential.

So what is an electron Volt (eV)?

Electron-Volts

- Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt
- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt

  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

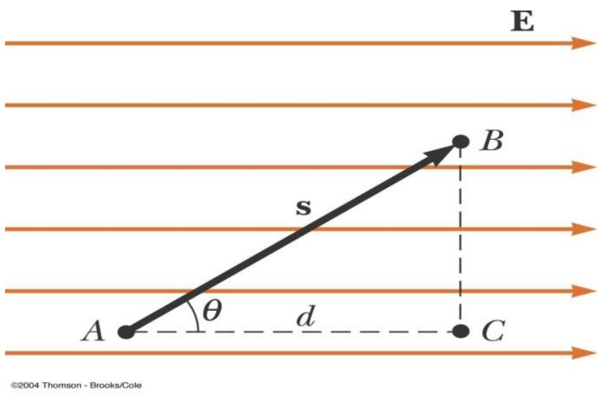
- Since all charges try to **decrease UE, and** $\Delta U_E = q\Delta V$, this means that *spontaneous* movement of charges result in **negative** ΔU.
- $\mathbf{\Delta V = \Delta U / q}$
- Positive charges like to DECREASE their potential ($\Delta V < 0$)
- Negative charges like to INCREASE their potential. ($\Delta V > 0$)

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\,.\,\vec{ds} = -\vec{E}\,.\int_A^B \vec{ds} = -\vec{E}\,.\,\vec{d} = E\, s\cos\theta$$

$V_B - V_A = V_C - V_A$

$V_B = V_C$

A uniform electric field directed along the positive *x* axis. Point *B* is at a lower electric potential than point A. Points *B* and *C* are at the *same* electric potential.



Example

If a 9 V battery has a charge of 46 C how much chemical energy does the battery have?

$$E = V \times Q = 9\ V \times 46C = 414\ \text{Joules}$$

Example

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m$$

$$\Delta V = 600V$$

$$E = ?$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = 113{,}207.55 N / C$$

$$q_{e^-} = 1.6x10^{-19} C$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19} C}$$

$$F_e = 1.81 \times 10^{-14} \text{ N}$$

Example

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_{p^+} = 1.6x10^{-19}C$$

$$m_{p^+} = 1.67x10^{-27}kg$$

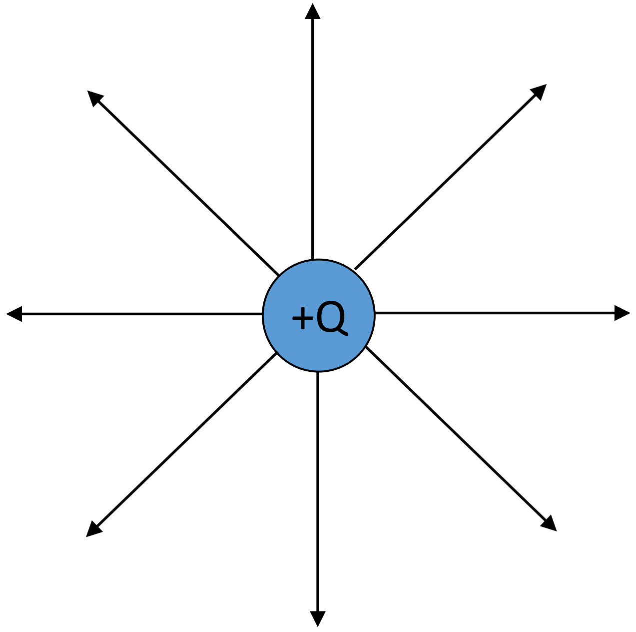
$$V = 120V$$

$$v = ?$$

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = 1.52\times10^5 \text{ m/s}$$

25-3 Electric Potential and **Potential energy** due to point charges

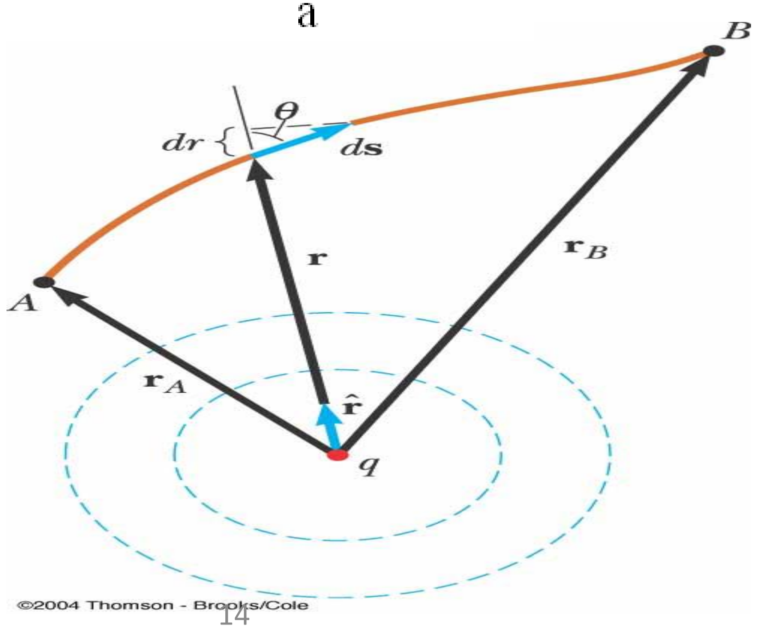


$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$V_{ba} = V_b - V_a = -\frac{kq}{r^2}\int_a^b \hat{r} \cdot d\vec{s}$$

d***s*** for a point charge



Recall the convention for the potential "zero point"

$$V_{ba} = V_b - V_a = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

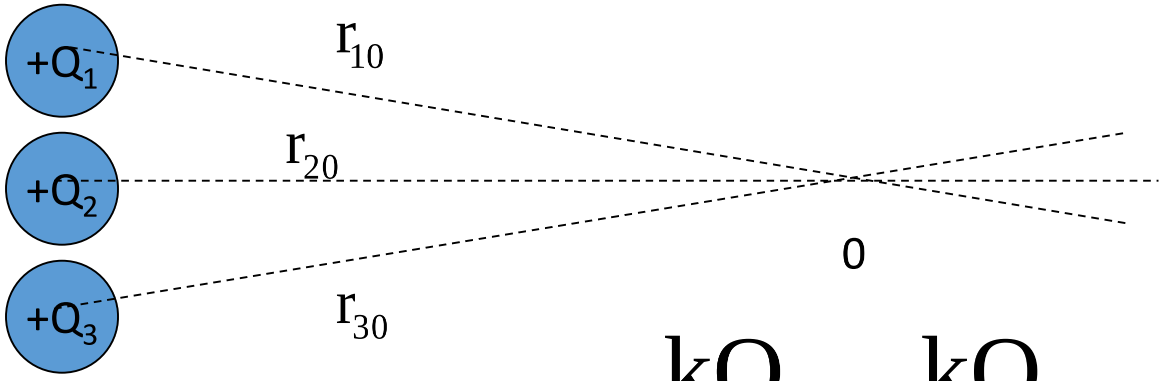
$$V_\infty = 0 \qquad\qquad V_{b\infty} = V_b - V_\infty = kq\left(\frac{1}{r_b} - \frac{1}{\infty}\right)$$

$$V(r) = \frac{kq}{r}$$

Equipotential surfaces are concentric spheres

Superposition of potentials

$$V_0 = V_1 + V_2 + V_3 + \ldots$$



$$V_0 = \frac{kQ_1}{r_{10}} + \frac{kQ_2}{r_{20}} + \frac{kQ_3}{r_{30}} + \ldots$$

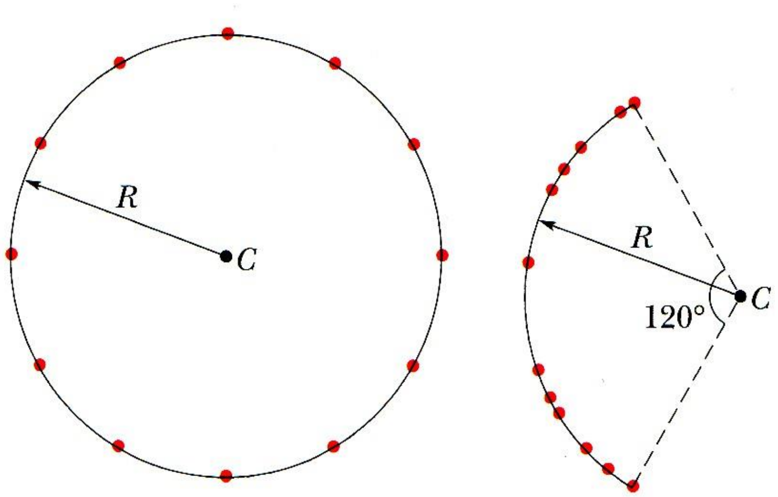
$$V_0 = \sum_{i=1}^{N} \frac{kQ_i}{r_{i0}}$$

Example: (a) In figure a, 12 electrons are equally spaced and fixed around a circle of radius *R*. Relative to *V*=0 at infinity, what are the electric potential and electric field at the center *C* of the circle due to these electrons? (b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° are (figure b), what then is the potential at *C*?

Solution:

$$(a):\quad V = -K\frac{12e}{R} \qquad \mathbf{E} = 0$$

$$(b):\quad V = -K\frac{12e}{R}$$



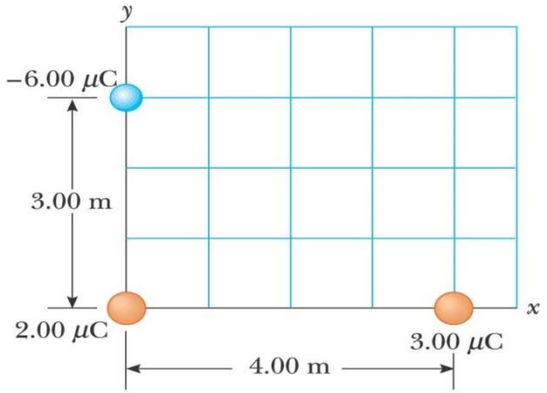
(a) (b)

Potential due to a group of point charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i}^{n}\frac{q_i}{r_i}$$

Example (25.3)

(a) The electric potential at *P* due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge q_3 = 3.00 C is brought from infinity to a position near the other charges.



(b)

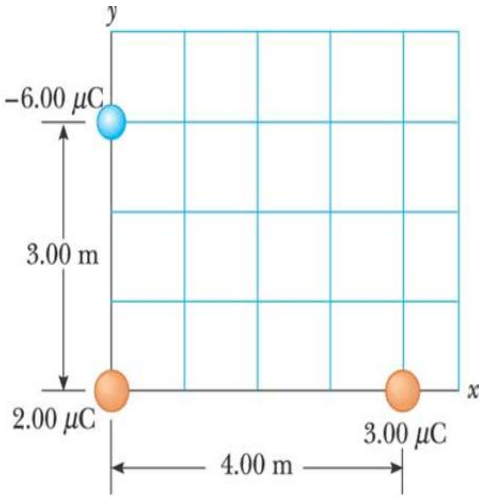
Solution For two charges, the sum

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})$$

$$\times\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} - \frac{6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= -6.29 \times 10^3\ \mathrm{V}$$



(b)

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P

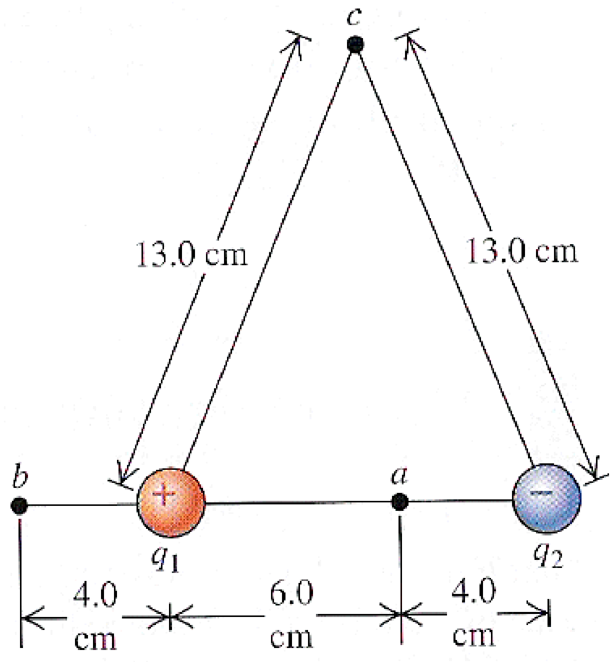
Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^{3}\ \mathrm{V})$$
$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$
$$= (8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})$$
$$\times \left(\frac{(2.00 \times 10^{-6}\ \mathrm{C})(-6.00 \times 10^{-6}\ \mathrm{C})}{3.00\ \mathrm{m}} + \frac{(2.00 \times 10^{-6}\ \mathrm{C})(3.00 \times 10^{-6}\ \mathrm{C})}{4.00\ \mathrm{m}} + \frac{(3.00 \times 10^{-6}\ \mathrm{C})(-6.00 \times 10^{-6}\ \mathrm{C})}{5.00\ \mathrm{m}} \right)$$
$$= -5.48 \times 10^{-2}\ \mathrm{J}$$

Example

An electric dipole consists of two charges q_1 = +12nC and q_2 = -12nC, placed 10 cm apart as shown in the figure. Compute the potential at points a,b, and c.



$$V_a = k\sum\left(\frac{q_1}{r_a} + \frac{q_2}{r_a}\right)$$

$$V_a = 8.99x10^9\left(\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04}\right)$$

$$V_a = -899 \text{ V}$$

$$V_b = k\sum\left(\frac{q_1}{r_b} + \frac{q_2}{r_b}\right)$$

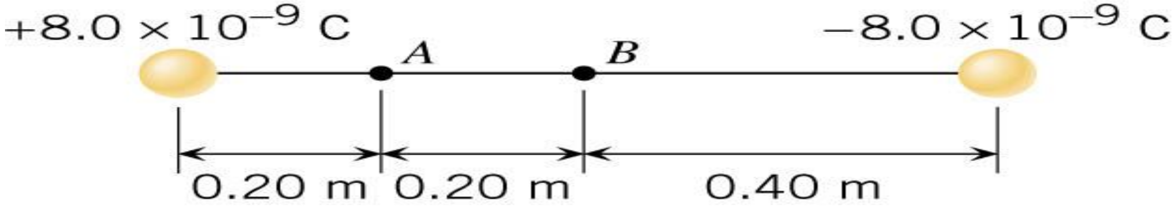
$$V_b = 8.99x10^9\left(\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14}\right)$$

$$V_b = 1926.4\ V$$

$$V_c = 0 \text{ V}$$

Example The Total Electric Potential

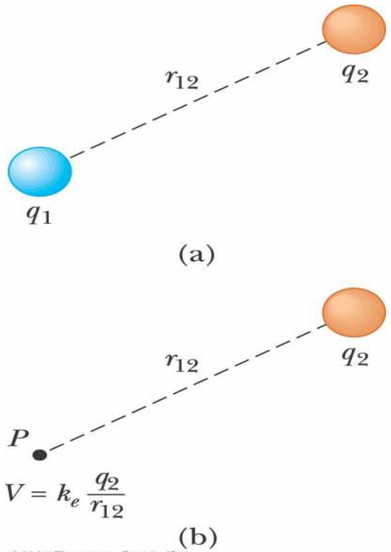
At locations A and B, find the total electric potential.



$$V_A = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.20\,\mathrm{m}} + \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.60\,\mathrm{m}} = +240\ \mathrm{V}$$

$$V_B = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\,\mathrm{m}} + \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\,\mathrm{m}} = 0\ \mathrm{V}$$

(a) If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by keq_1q_2/r_{12} . (b) If charge q_1 is removed, a potential keq_2/r_{12} exists at point *P* due to charge q_2 .

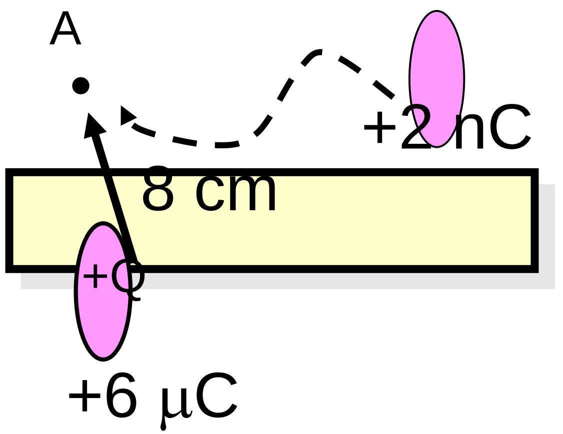


$$U = V q_1$$

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

Example 1. What is the potential energy if a +2 nC charge moves from ∞ to point A, 8 cm away from a +6 μC charge?

The P.E. will be positive at point A, because the field can do + work if q is released.



Potential Energy:

$$U = \frac{(9 \times 10^9 \frac{Nm^2}{C^2})(+6 \times 10^{-6} C)(+2 \times 10^{-9} C)}{(0.08 \text{ m})}$$

U = 1.35 mJ

Positive potential energy