PHYS 221

Electromagnetism (1)
2nd semester 1446

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Lecture 10

Chapter 33
Alternating Current Circuits

33.1 AC Sources

- An AC circuit consists of a combination of circuit elements and a power source
- The power source provides an alternative voltage, Δv

The output of an AC power source is sinusoidal and varies with time according to the following equation:

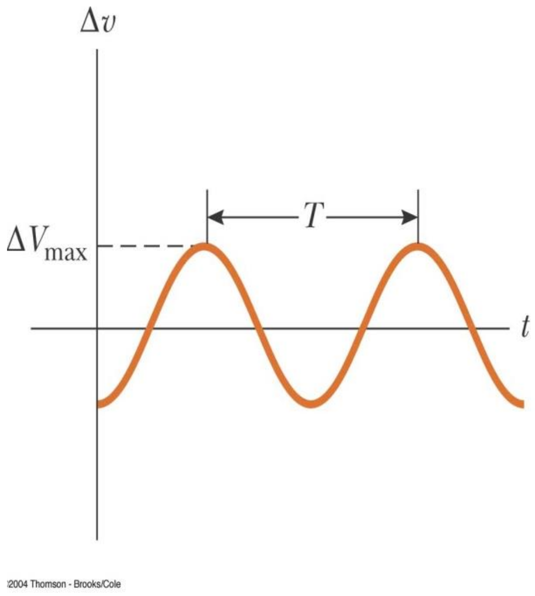
$$\Delta v = \Delta V_{max} \sin \omega t$$

Δv is the instantaneous voltage

ΔV_{max} is the maximum output voltage of the source

Also called the **voltage amplitude**

ω is the angular frequency of the AC voltage



The angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

f is the frequency of the source

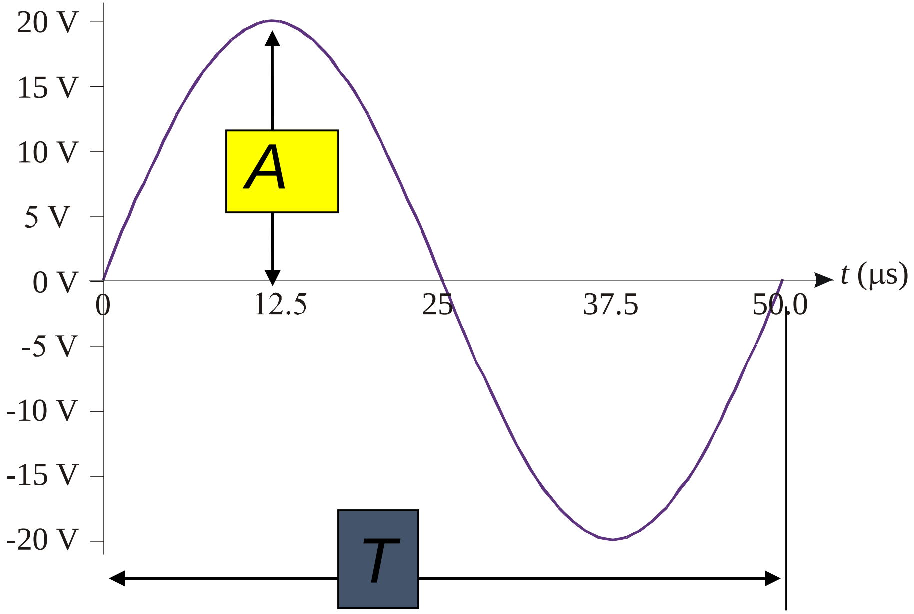
T is the period of the source

The voltage is positive during one half of the cycle and negative during the other half

- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time

Sine waves are characterized by the amplitude and period. The **amplitude** is the maximum value of a voltage or current; the **period** is the time interval for one complete cycle.

The amplitude (*A*) of this sine wave is 20 V



The period is 50.0 μs

If the period is 50 μs, the frequency is 0.02 MHz = 20 kHz.

Sine

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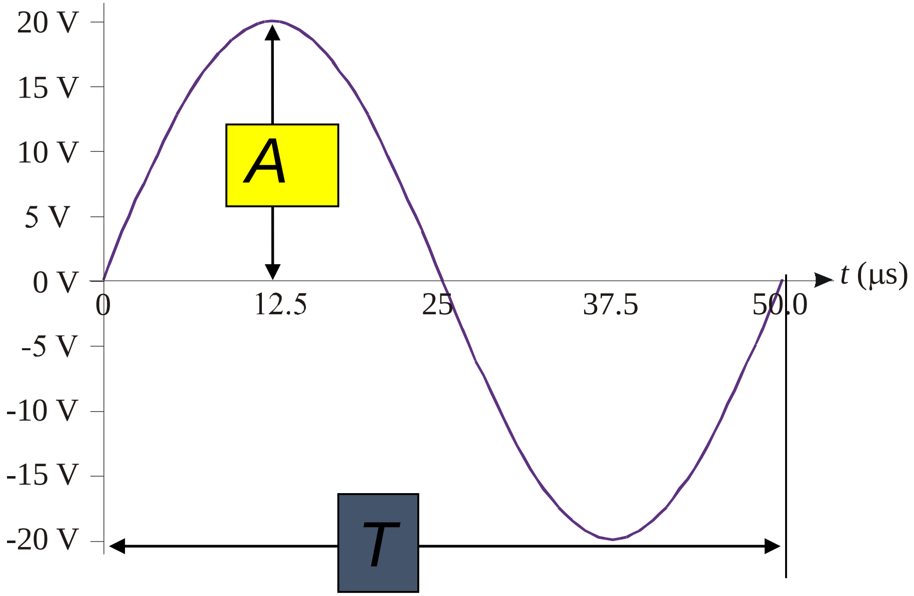
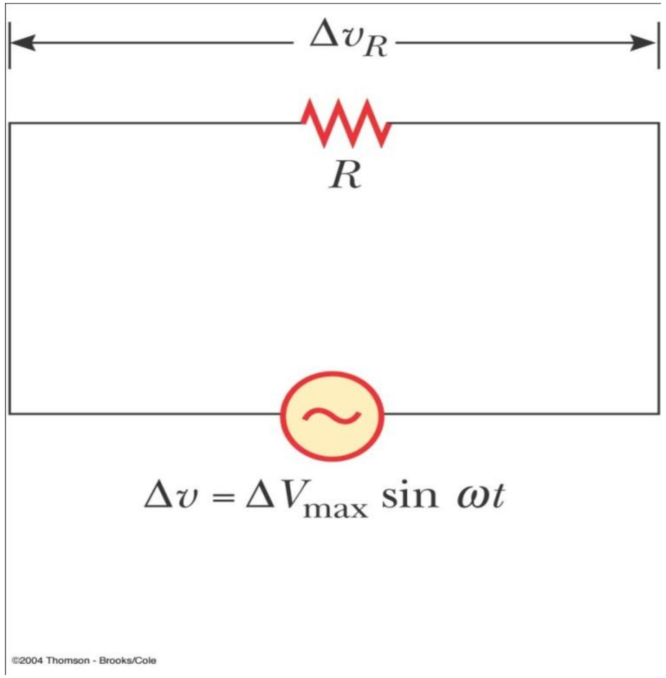


33.2 Resistors in an AC Circuit



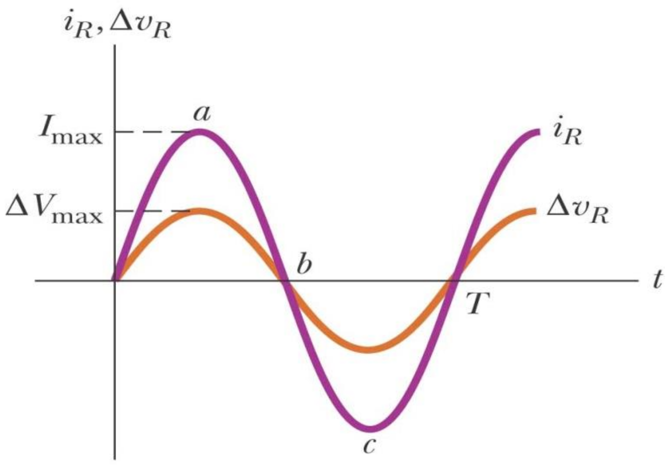
- Consider a circuit consisting of an AC source and a resistor
- The AC source is symbolized by 
- Δv_R = Dv= V_{max} sin *wt*
- Δv_R is the instantaneous voltage across the resistor

- The instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- The instantaneous voltage across the resistor is also given as $\Delta v_R = I_{max} R \sin \omega t$

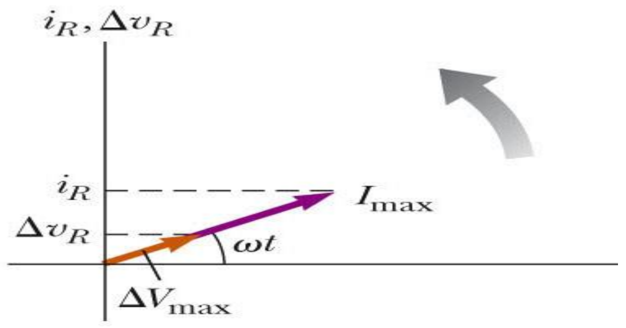
- The graph shows the current through and the voltage across the resistor
- The current and the voltage reach their maximum values at the same time
- The current and the voltage are said to be *in phase*
  - For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor
  - The direction of the current has no effect on the behavior of the resistor
  - Resistors behave essentially the same way in both DC and AC circuits



(a)

Phasor Diagram

- To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used
- A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents



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- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable
- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents

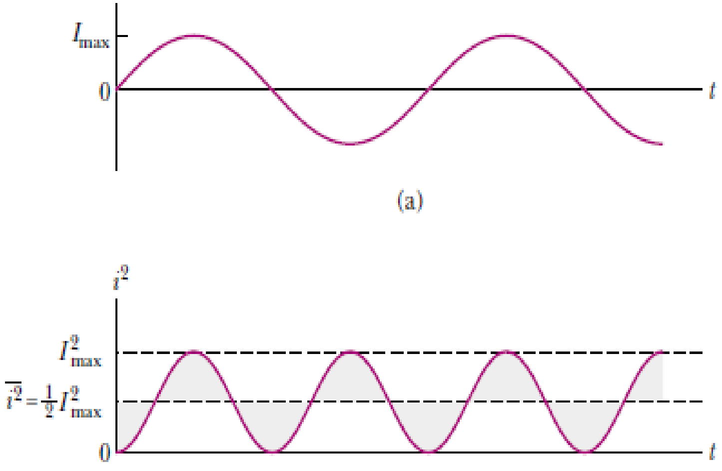
rms Current and Voltage

- The average current in one cycle is zero
- The rms current is the average of importance in an AC circuit
  - rms stands for root mean square

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

- Alternating voltages can also be
- discussed in terms of rms values

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$



(a)

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- Alternating voltages can also be discussed in terms of rms values

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

rms values are used when discussing alternating currents and voltages because

AC ammeters and voltmeters are designed to read rms values

Example : For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

$I_{rms} = 0.707\ i_{max}$

$V_{rms} = 0.707\ V_{max}$

$$I_{\max} = \frac{I_{rms}}{0.707} = \frac{10}{0.707} = 14.14A$$

$$V_{\max} = \frac{V_{rms}}{0.707} = \frac{120}{0.707} = 170V$$

Example

- A 60 W light bulb operates on a peak voltage of 156 V. Find the V_{rms}, I_{rms}, and resistance of the light bulb.

- V_{rms} = 110 V
- I_{rms} = 0.55 A
- R = 202 Ω

V_{rms} = 156 V / $\sqrt{2}$ = 110 V
I_{rms}: P = IV → 60 W = I (110V) → .55 A
$P=V^2/R$ → 60 W = $(110\ V)^2/R$ → R = $(110V)^2/60W$ → 202 Ω

Power

- The rate at which electrical energy is dissipated in the circuit is given by
  - $P = i^2 R$
    - *i* is the *instantaneous current*
    - The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value
    - The maximum current occurs for a small amount of time

- The average power delivered to a resistor that carries an alternating current is $P_{av} = I_{rms}^2 R$

Example:

Assume a sine wave with a peak value of 40 V is applied to a 100 Ω resistive load. What power is dissipated?

$V_{rms} = 0.707 \times V_p = 0.707 \times 40 \text{ V} = 28.3 \text{ V}$

$$P = \frac{V_{rms}^2}{R} = \frac{28.3 \text{ V}^2}{100\ \Omega} = 8 \text{ W}$$

Example 33.1 **What Is the rms Current?**

The voltage output of an AC source is given by the expression $\Delta v = (200\text{ V})\sin \omega t$. Find the rms current in the circuit when this source is connected to a 100-Ω resistor.

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{200\text{ V}}{\sqrt{2}} = 141\text{ V}$$

$$I_{rms} = \frac{\Delta V_{rms}}{R} = \frac{141\text{ V}}{100\ \Omega} = \boxed{1.41\text{ A}}$$

1- A heater takes 10 A rms from the 230 V rms mains. What is its power?

A) 1630 W
B) 2300 W
C) 3250 W
D) 4600 W

2- The voltage output of an AC source is given by the expression :
v = (200 V) sin wt. Find the rms current in the circuit when this source is connected to a 100-Ωresistor.

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_{\max}/\sqrt{2}}{R} = \frac{200/\sqrt{2}}{100}$$

$$I_{rms} = 2 \times 0.707 = 1.414A$$

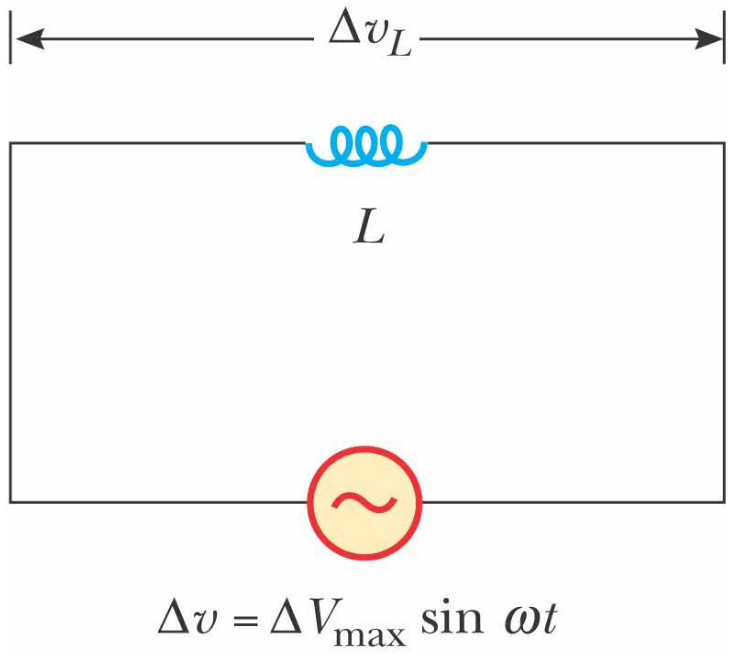
33-3 Inductors in an AC Circuit

- Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\, sin\,\omega t$$



Current in an Inductor

- The equation obtained from Kirchhoff's loop rule can be solved for the current

$$\sin\left(\omega t-\frac{\pi}{2}\right)=-\cos\omega t$$

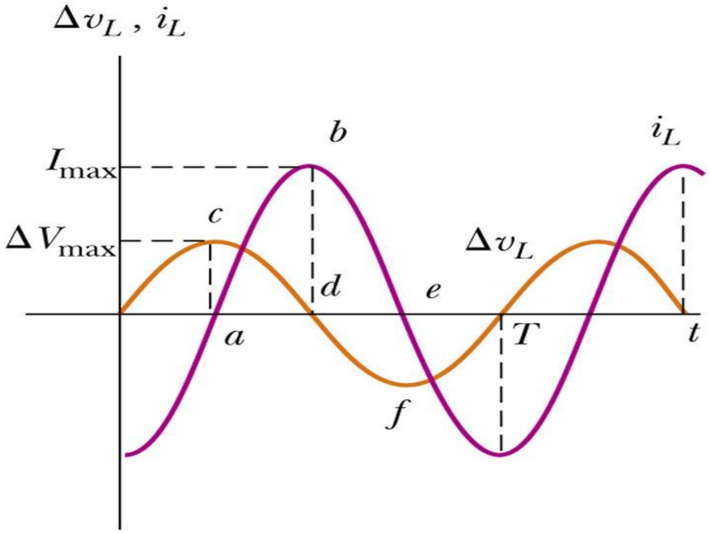
$$i_L=\frac{\Delta V_{max}}{L}\int \sin\ \omega t\ dt=-\frac{\Delta V_{max}}{\omega L}\cos\omega t$$

$$i_L=\frac{\Delta V_{max}}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \qquad I_{max}=\frac{\Delta V_{max}}{\omega L}$$

- This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by (π/2) rad = 90°

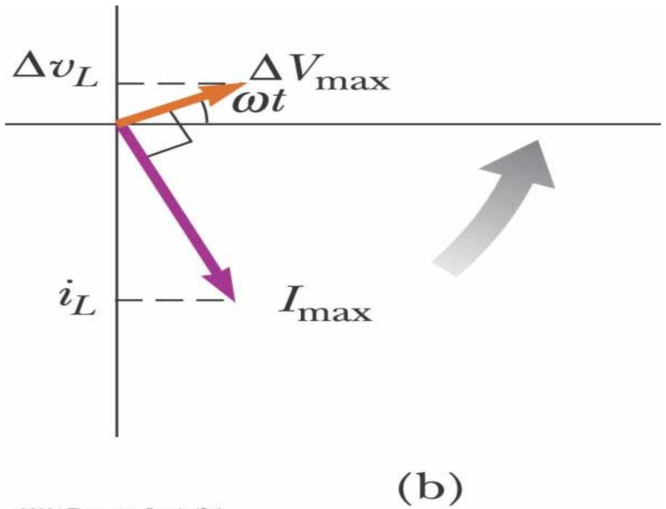
Phase Relationship of Inductors in an AC Circuit

- The current is a maximum when the voltage across the inductor is zero
  - The current is momentarily not changing
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$)



Phasor Diagram for an Inductor

- The phasors are at 90º with respect to each other
- This represents the phase difference between the current and voltage
- Specifically, the current always lags behind the voltage by 90º



Inductive Reactance

- The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance
- Because ω depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies
- The factor X_L is the **inductive reactance** and is given by:

  $X_L = \omega L$

Inductive Reactance, cont.

- Current can be expressed in terms of the inductive reactance

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L} \; or \; I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

- As the frequency increases, the inductive reactance increases
  - This is consistent with Faraday's Law:
    - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current

Example 33.2 A Purely Inductive AC Circuit

In a purely inductive AC circuit, *L = 25.0 mH* and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

$$X_L = \omega L = 2\pi f L = 2\pi(60.0\ \text{Hz})(25.0 \times 10^{-3}\ \text{H})$$

$$= 9.42\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{150\ \text{V}}{9.42\ \Omega} = 15.9\ \text{A}$$

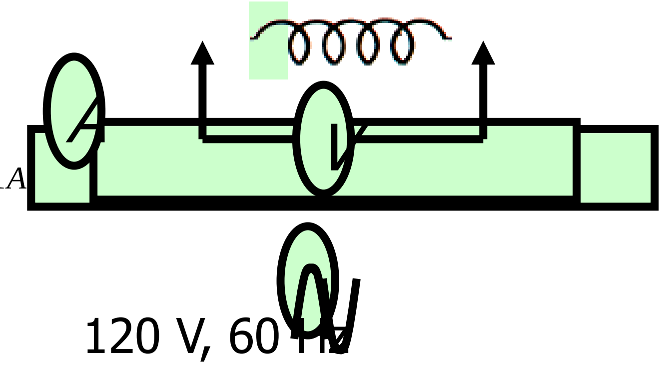
What if the frequency increases to 6.00 kHz? What happens to the rms current in the circuit?

$$X_L = 2\pi(6.00 \times 10^3\ \text{Hz})(25.0 \times 10^{-3}\ \text{H}) = 942\ \Omega$$

$$I_{\text{rms}} = \frac{150\ \text{V}}{942\ \Omega} = 0.159\ \text{A}$$

Example : A coil having an inductance of 0.6 H is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current(I_{rms}) through the coil?

Reactance: $X_L = 2\pi f L$



$$X_L = 2\pi(60\text{ Hz})(0.6\text{ H})$$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{120}{226} = 0.531A$$

$$X_L = 226\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{120}{226} = 0.531A$$

i_{eff} = *0.531* A

Show that the peak current is I_{max} = 0.750 A

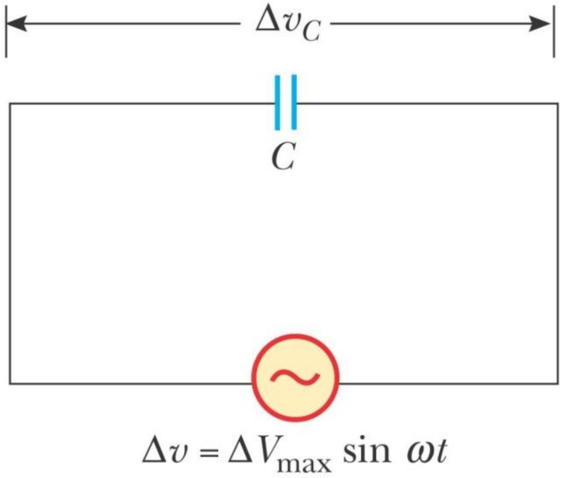
33.4 Capacitors in an AC Circuit

- The circuit contains a capacitor and an AC source
- Kirchhoff's loop rule gives:

  $\Delta v + \Delta v_c = 0$ and so

  $\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$

  - Δv_c is the instantaneous voltage across the capacitor



$q = C V$ [V=Vmax sin(wt)]

- The charge is $q = C\,\Delta V_{max} \sin \omega t$

  Take derivative: $i_c = dq/dt = C\,dV/dt$

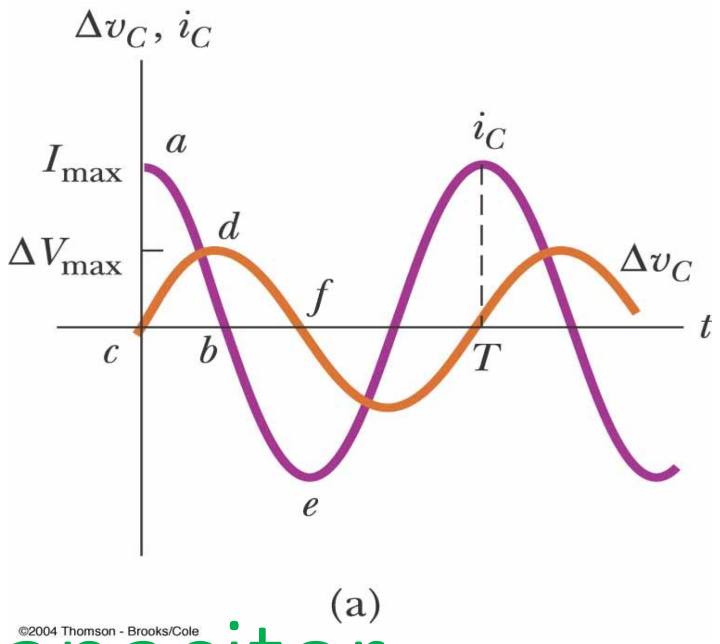
- The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

$$\text{or } i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

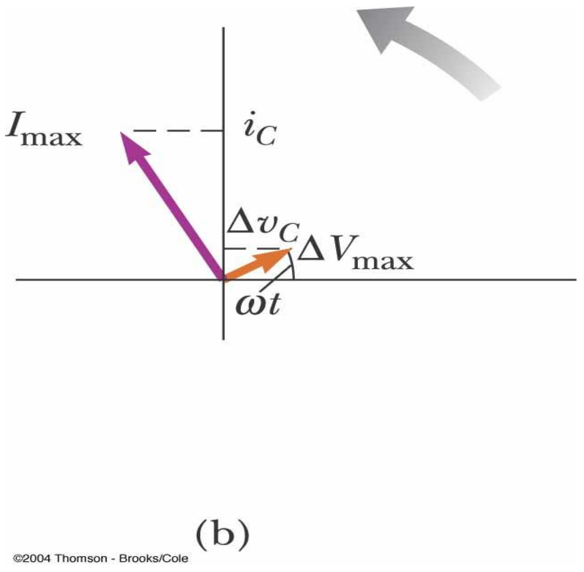
- The current is $\pi/2$ rad = 90° out of phase with the voltage

- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value
- The current leads the voltage by 90°



Phasor Diagram for Capacitor

- The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°



Capacitive Reactance

- The maximum current in the circuit occurs at cos ωt = 1 which gives

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

- The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

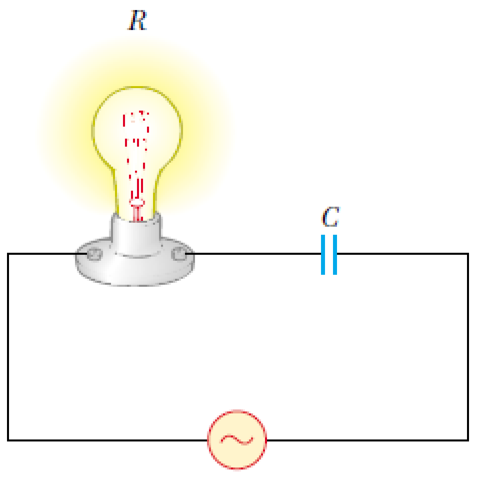
Voltage Across a Capacitor

- The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$
- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases
- As the frequency approaches zero, X_C approaches infinity and the current approaches zero
  - This would act like a DC voltage and the capacitor would act as an open circuit

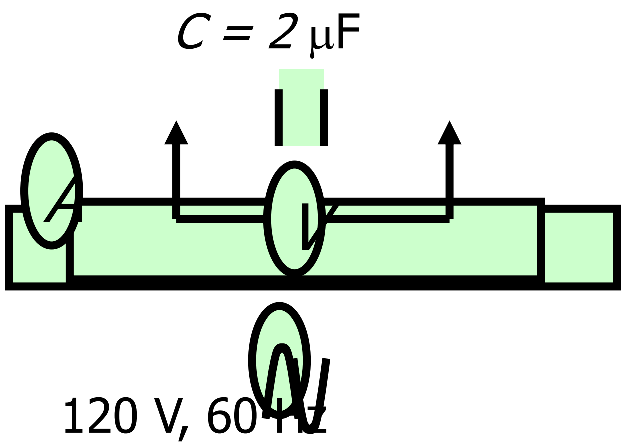
Quick Quiz

Consider the AC circuit in the Figure .The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulbwill glow the brightest at;

a)high frequencies
b)low frequencies
c)The brightness will be same at all frequencies.



Example :A 2-μF capacitor is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?



Reactance:

$$X_C = \frac{1}{2\pi(60\,\text{Hz})(2 \text{ x } 10^{-6}\text{F})}$$

X_C = 1330 Ω

$$i_{eff} = \frac{V_{eff}}{X_C} = \frac{120\text{V}}{1330\ \Omega}$$

i_{eff} = 90.5 mA

Show that the peak current is i_{max} = 128 mA

Example

What are the peak and rms currents of a circuit if

$C = 1.0\ \mu F$ and $V_{rms} = 120\ V$. The frequency is 60.0 Hz.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\ Hz)(1.0 \times 10^{-6} F)}$$

$X_C = 2700\ \Omega$

$V_{rms} = V_{max}/\sqrt{2}$

$V_{max} = V_{rms}\sqrt{2}$

$V_{max} = (120\ V)(\sqrt{2}) = 170\ V$

$I_{max} = V_{max}/X_C$

$I_{max} = 170\ V/2700\ \Omega$

$I_{max} = 63\ mA$

$I_{rms} = V_{rms}/X_C$

$I_{rms} = 120\ V/2700\ \Omega = 44\ mA$

Example A Purely Capacitive AC Circuit

An 8.00μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = 332\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\ \text{V}}{332\ \Omega} = 0.452\ \text{A}$$

What if the frequency is doubled? What happens to the rms current in the circuit?

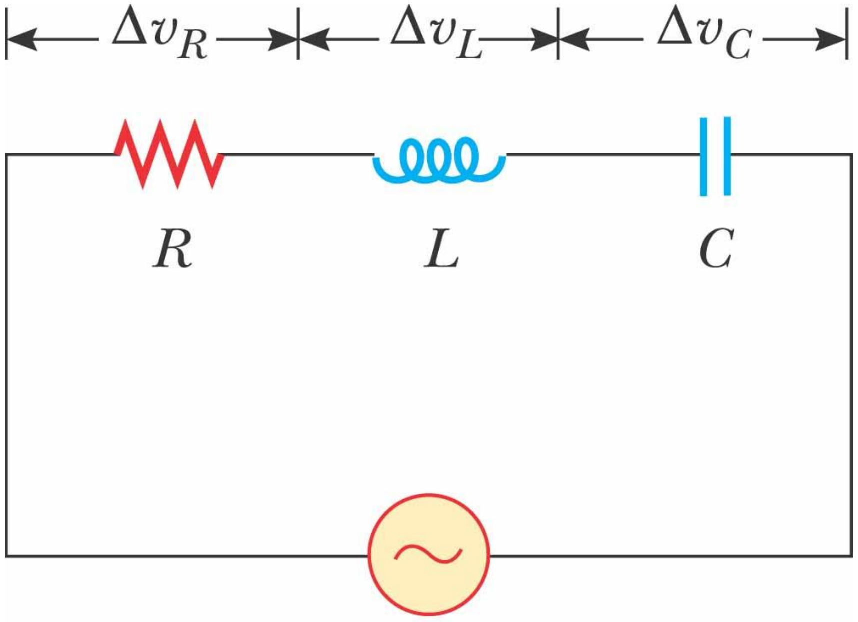
If the frequency increases, the capacitive reactance decreases—just the opposite as in the case of an inductor. The decrease in capacitive reactance results in an increase in the current. Let us calculate the new capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = 166\ \Omega$$

$$I_{\text{rms}} = \frac{150\ \text{V}}{166\ \Omega} = 0.904\ \text{A}$$

The *RLC* Series Circuit

- The resistor, inductor, and capacitor can be combined in a circuit
- The current and the voltage in the circuit vary sinusoidally with time



(a)

$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

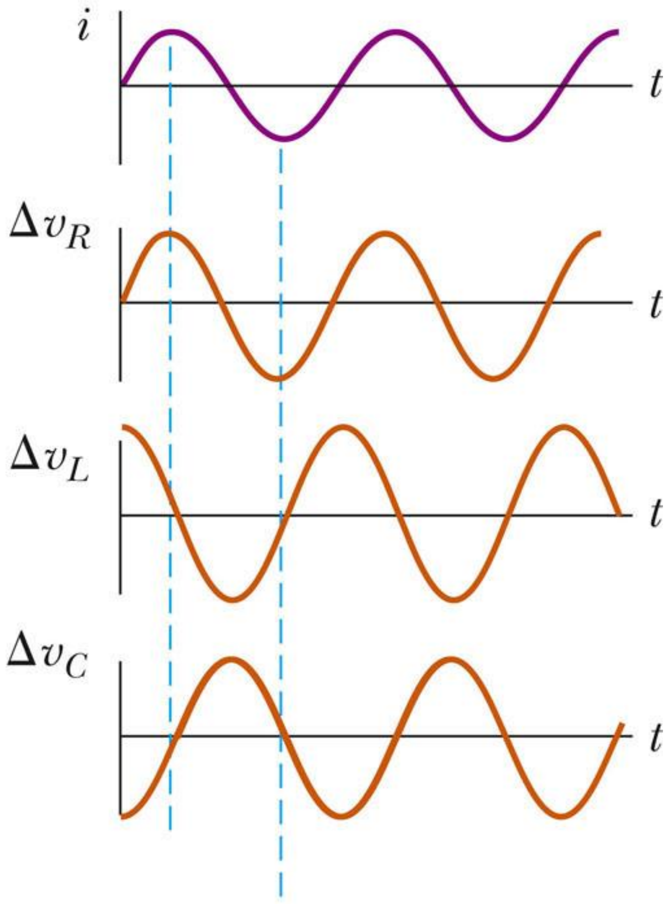
$$\Delta V_R = I_{max} R \qquad \Delta V_L = I_{max} X_L \qquad \Delta V_C = I_{max} X_C$$

The *RLC* Series Circuit, cont.

- The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$
- The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$
  - φ is the **phase angle** between the current and the applied voltage
- Since the elements are in series, the current at all points in the circuit has the same amplitude and phase

i and *v* Phase Relationships – Graphical View

- The instantaneous voltage across the resistor is in phase with the current
- The instantaneous voltage across the inductor leads the current by 90°
- The instantaneous voltage across the capacitor lags the current by 90°



(b)

i and *v* Phase Relationships – Equations

- The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

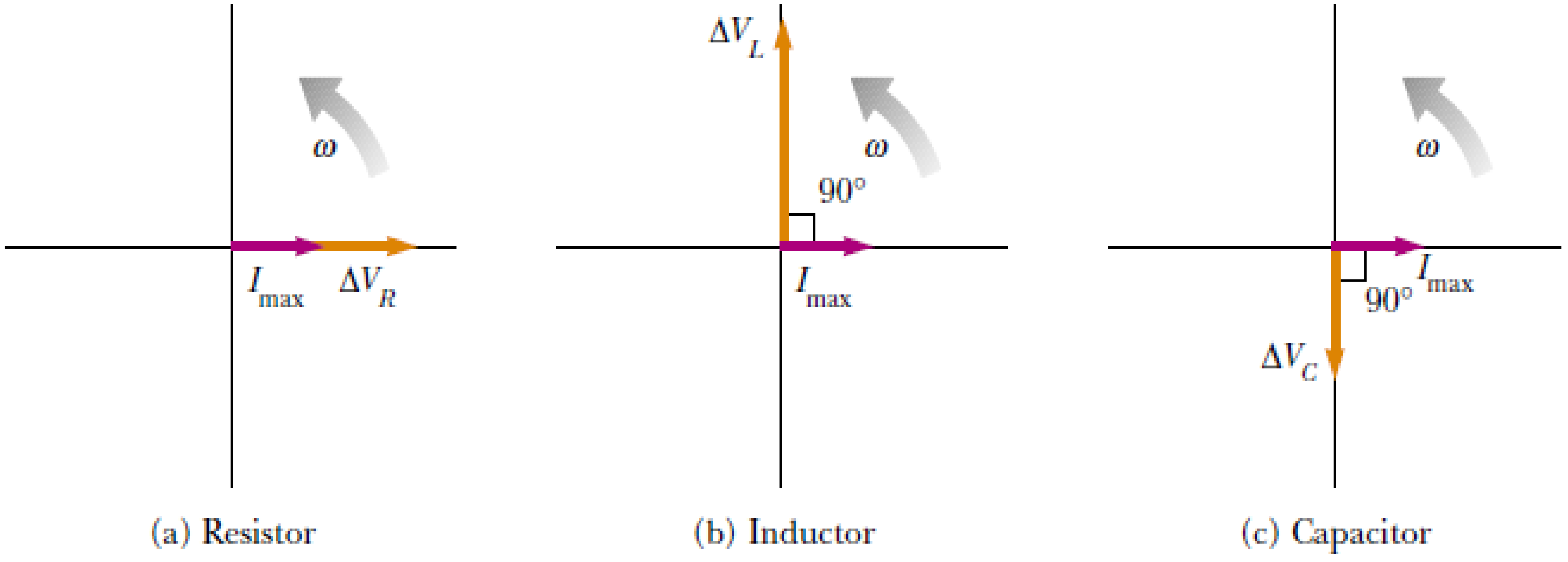
$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_L$
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_C$
- The sum of these voltages must equal the voltage from the AC source
- Because of the different phase relationships with the current, they cannot be added directly

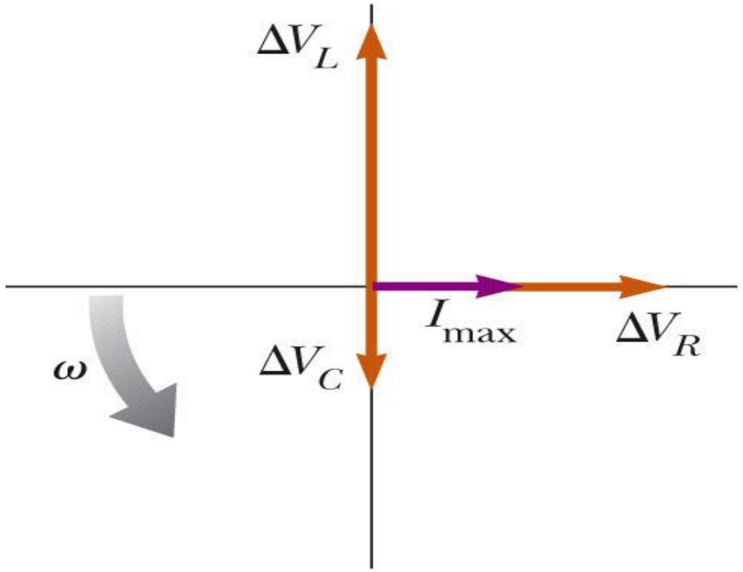
Phasor Diagrams



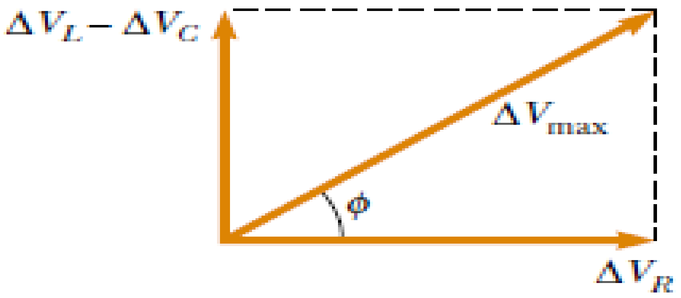
- To account for the different phases of the voltage drops, vector techniques are used
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown

Resulting Phasor Diagram

- The individual phasor diagrams can be combined
- Here a single phasor I_{max} is used to represent the current in each element
    - In series, the current is the same in each element



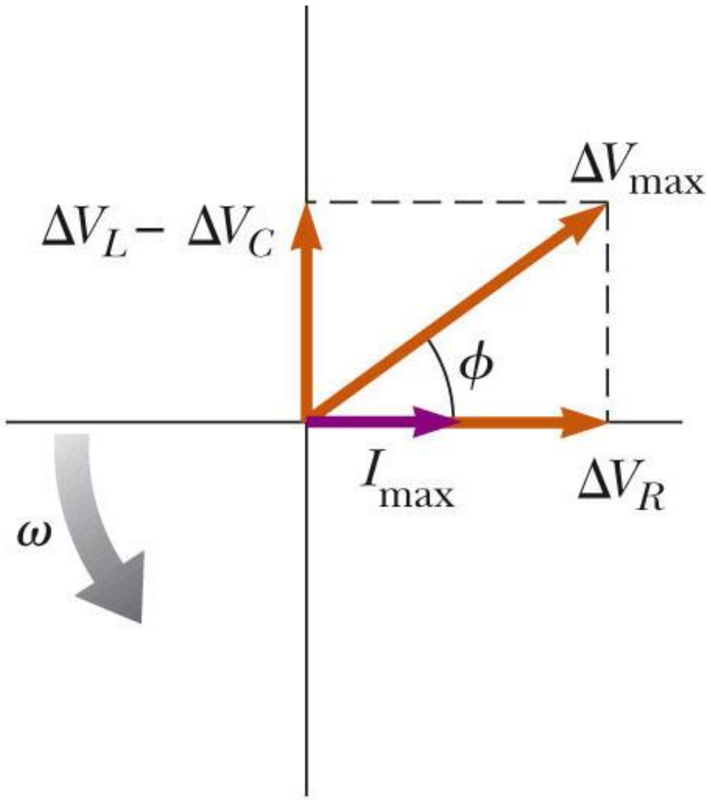
(a)



(b)

Vector Addition of the Phasor Diagram

- Vector addition is used to combine the voltage phasors
- ΔV_L and ΔV_C are in opposite directions, so they can be combined
- Their resultant is perpendicular to ΔV_R



(b)

Total Voltage in RLC Circuits

- From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + \left(I_{max} X_L - I_{max} X_C\right)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Impedance

- The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

- Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

  - Impedance has units of ohms

Phase Angle

- The right triangle in the phasor diagram can be used to find the phase angle, φ

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- The phase angle can be positive or negative and determines the nature of the circuit

Determining the Nature of the Circuit

- If ϕ is positive
  - $X_L > X_C$ (which occurs at high frequencies)
  - The current lags the applied voltage
  - The circuit is *more inductive than capacitive*
- If ϕ is negative
  - $X_L < X_C$ (which occurs at low frequencies)
  - The current leads the applied voltage
  - The circuit is *more capacitive than inductive*
- If ϕ is zero
  - $X_L = X_C$
  - The circuit is *purely resistive*

Power in an AC Circuit

- The average power delivered by the AC source is converted to internal energy in the resistor
  - $\wp_{av} = \frac{1}{2} I_{max} \Delta V_{max} \cos \phi = I_{rms} \Delta V_{rms} \cos \phi$
  - $\cos \phi$ is called the power factor of the circuit
- We can also find the average power in terms of R
  - $\wp_{av} = I^2_{rms} R$

Power in an AC Circuit, cont.

- The average power delivered by the source is converted to internal energy in the resistor
- No power losses are associated with pure capacitors and pure inductors in an AC circuit
  - In a capacitor, during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor
  - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit

Example 33.5 Analyzing a Series *RLC* Circuit

A series *RLC* AC circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ \text{s}^{-1}$, and $\Delta V_{max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution The reactances are $X_L = \omega L = 471\ \Omega$ and $X_C = 1/\omega C = 758\ \Omega$.
The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = 513\ \Omega$$

(B) Find the maximum current in the circuit.

Solution

$$I_{max} = \frac{\Delta V_{max}}{Z} = \frac{150\ \text{V}}{513\ \Omega} = 0.292\ \text{A}$$

(C) Find the phase angle between the current and voltage.

Solution

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right)$$

$$= \boxed{-34.0°}$$

Because the capacitive reactance is larger than the inductive reactance, the circuit is more capacitive than inductive. In this case, the phase angle ϕ is negative and the current leads the applied voltage.

(D) Find both the maximum voltage and the instantaneous voltage across each element.

Solution The maximum voltages are

$$\Delta V_R = I_{\text{max}} R = (0.292\ \text{A})(425\ \Omega) = \boxed{124\ \text{V}}$$

$$\Delta V_L = I_{\text{max}} X_L = (0.292\ \text{A})(471\ \Omega) = \boxed{138\ \text{V}}$$

$$\Delta V_C = I_{\text{max}} X_C = (0.292\ \text{A})(758\ \Omega) = \boxed{221\ \text{V}}$$

the instantaneous voltages across the three elements as

$$\Delta v_R = (124\text{ V}) \sin 377t$$

$$\Delta v_L = (138\text{ V}) \cos 377t$$

$$\Delta v_C = (-221\text{ V}) \cos 377t$$

Example 33.6 Average Power in an *RLC* Series Circuit

Calculate the average power delivered to the series RLC circuit described in Example 33.5.

Solution First, let us calculate the rms voltage and rms current, using the values of ΔV_{max} and I_{max} from Example 33.5:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{150\ V}{\sqrt{2}} = 106\ V$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{0.292\ A}{\sqrt{2}} = 0.206\ A$$

Because $\phi = -34.0°$, the power factor is $\cos(-34.0°) = 0.829$; hence, the average power delivered is

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi = (0.206\ A)(106\ V)(0.829)$$

$$= 18.1\ W$$

Example

A circuit has R=25.0 Ω, L = 30.0 mH, and C = 12.0 μF. Calculate the impedance of the circuit if they are connected to a 90.0 V ac(rms), 500 Hz source. Also calculate the phase angle.

$X_L = 2\pi fL$

$X_L = 2\pi(500\ \text{Hz})(0.030\ \text{H}) = 94.2\ \Omega$

$X_C = 1/2\pi fC$

$X_C = 1/2\pi(500\ \text{Hz})(12 \times 10^{-6}\text{F}) = 26.5\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(25.0\Omega)^2 + (94.2\ \Omega - 26.5\ \Omega)^2}$$

$$Z = 72.2\ \Omega$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{94.2\ \Omega - 26.5\ \Omega}{25.0\ \Omega}$$

$$\phi = 69.7^o$$

Calculate the rms current

$V = IZ$

$I_{rms} = V_{rms}/Z = 90.0\ V/\ 72.2\ \Omega$

$I_{rms} = 1.25\ A$

Calculate the voltage drop across each element

$(V_R)_{rms} = I_{rms}R = (1.25\ A)(25.0\ \Omega) = 31.2\ V$

$(V_L)_{rms} = I_{rms}X_L = (1.25\ A)(94.2\ \Omega) = 118\ V$

$(V_C)_{rms} = I_{rms}X_C = (1.25\ A)(26.5\ \Omega) = 33.1\ V$

Voltages do not add to 90.0 V (out of phase)

Calculate the power loss in the circuit

$P_{ave} = I_{rms}V_{rms}\cos\phi$

$P_{ave} = (1.25\ A)(90.0\ V)\cos(69.7^o)$

$P_{ave} = 39.0\ W$

33.7 Resonance in a Series RLC Circuit

- *Resonance* occurs at the angular frequency ω_o where the current has its maximum value
  - To achieve maximum current, the impedance must have a minimum value
  - This occurs **when $X_L = X_C$**
  - *The Resonance* frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

  - Solving for the angular frequency gives

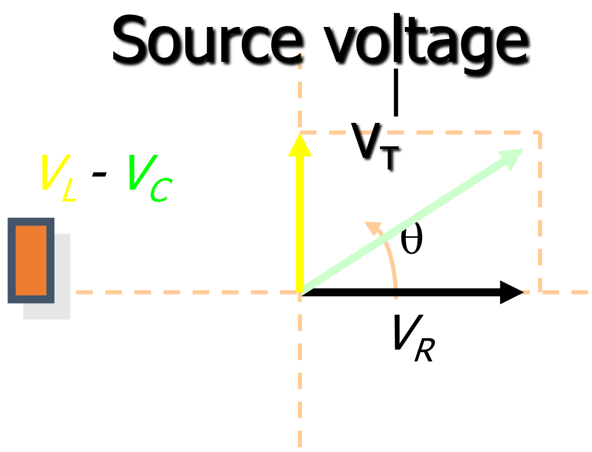
$$\omega_o = 1/\sqrt{LC}$$

$$I_{\max} = \frac{V_{\max}}{Z} = \frac{V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = X_C \qquad Z = R$$

$$I_{\max} = \frac{V_{\max}}{R}$$

Calculating Total Source Voltage



Treating as vectors, we find:

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

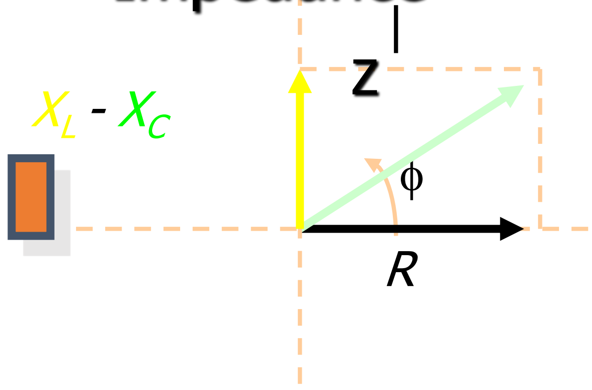
Now recall that: $V_R = iR$; $V_L = iX_L$; and $V_C = iV_C$

Substitution into the above voltage equation gives:

$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in an AC Circuit

Impedance



$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Z is defined:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

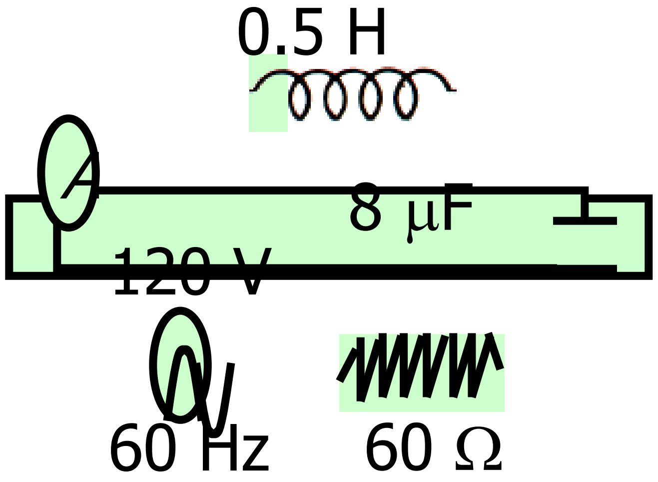
Ohm's law for ac current and impedance:

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

The impedance is the combined opposition to ac current consisting of both resistance and reactance.

Example : A 60-Ω resistor, a 0.5 H inductor, and an 8-μF capacitor are connected in series with a 120-V, 60 Hz ac source. Calculate the impedance for this circuit.

$$X_L = 2\pi fL \quad \text{and} \quad X_C = \frac{1}{2\pi fC}$$



$$X_L = 2\pi(60\text{Hz})(0.6\text{ H}) = 226\,\Omega$$

$$X_C = \frac{1}{2\pi(60\text{Hz})(8 \text{ x } 10^{-6}\text{F})} = 332\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(60\,\Omega)^2 + (226\,\Omega - 332\,\Omega)^2}$$

Thus, the impedance is:

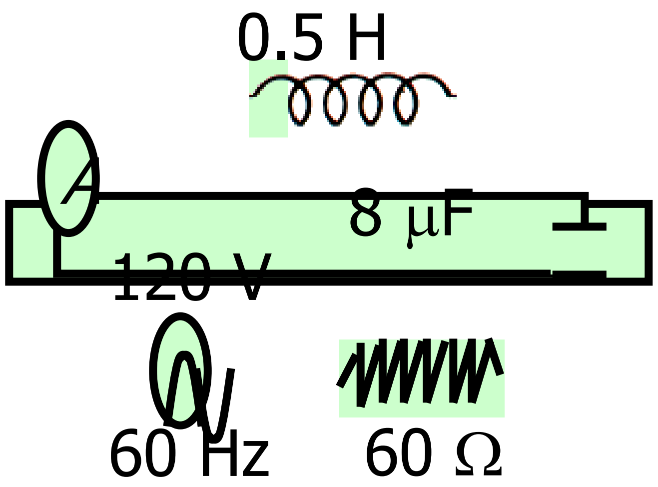
$Z = 122\ \Omega$

Example : Find the effective current (I_{rms}) and the phase angle for the previous example.

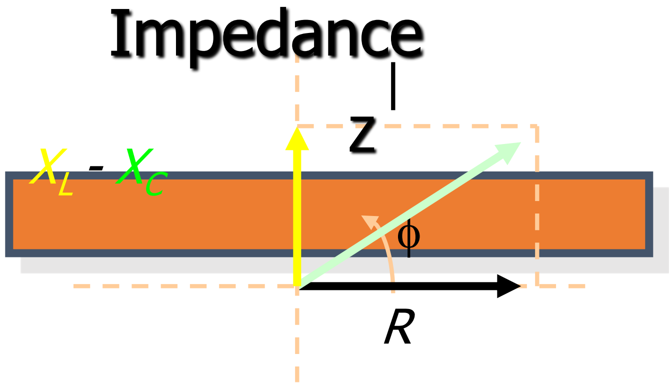
$X_L = 226\ \Omega;\ X_C = 332\ \Omega;$ $R = 60\ \Omega;\ Z = 122\ \Omega$

$$i_{eff} = \frac{V_T}{Z} = \frac{120\text{ V}}{122\ \Omega}$$

$I_{rms} = 0.985$ A



Next we find the phase angle:

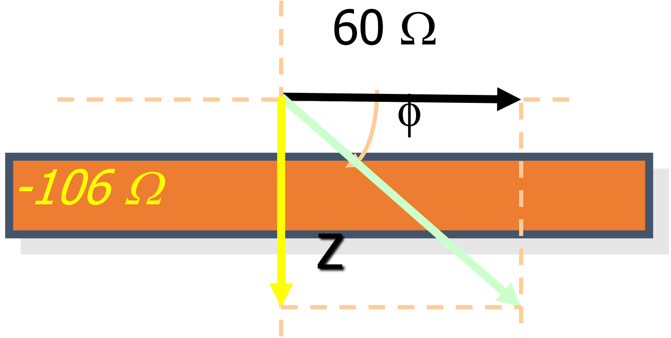


$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Example :Find the phase angle ϕ for the previous example.



$X_L - X_C$ = 226 – 332 = -106 Ω

R = 60 Ω

$$\tan\phi = \frac{X_L - X_C}{R}$$

ϕ = -60.5^0

The negative phase angle means that the ac voltage lags the current by 60.5^0. This is known as a capacitive circuit.

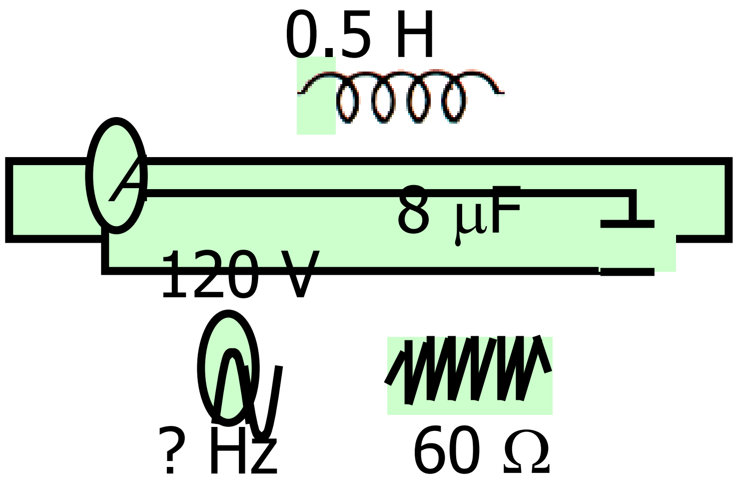
Example : Find the resonant frequency for the previous circuit example: L = .5 H, C = 8 μF

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{(0.5\,\text{H})(8 \text{ x } 10^{-6}\text{F})}}$$

Resonant f_r = 79.6 Hz

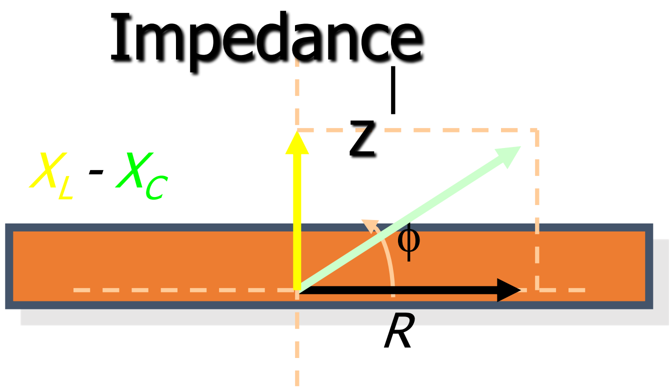
Resonance $X_L = X_C$



At resonant frequency, there is zero reactance (only resistance) and the circuit has a phase angle of zero.

Power in an AC Circuit

No power is consumed by inductance or capacitance. Thus power is a function of the component of the impedance along resistance:



P lost in *R* only

In terms of ac voltage:

$P = I_{rms} V_{rms} \cos \phi$

In terms of the resistance R:

$P = i_{rms}^2 R$

The fraction Cos ϕ is known as the power factor.

Example : What is the average power loss for the previous example: V = 120 V, ϕ = -60.5^0, I_{rms} = 90.5 A, and R = 60Ω .

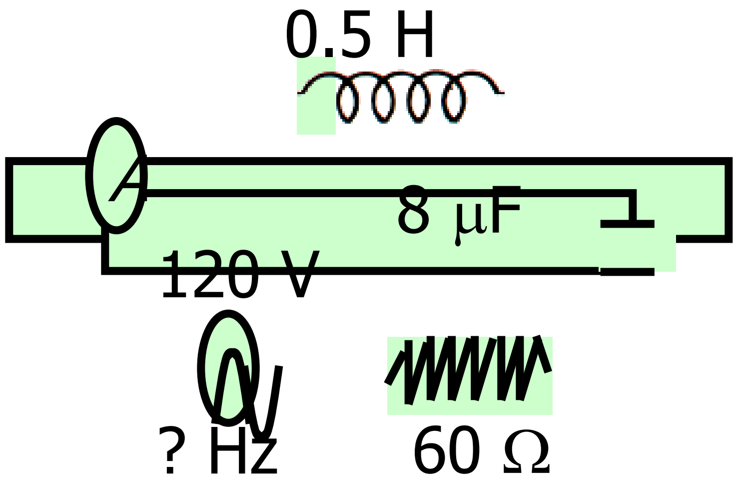
$P = i^2R$ = (0.0905 A)^2(60 Ω)

Average P = 0.491 W

The power factor is: Cos 60.5^0

Cos ϕ = 0.492 or 49.2%

Resonance $X_L = X_C$



The higher the power factor, the more efficient is the circuit in its use of ac power.

Example:

A radio tunes in a station at 980 kHz at a capacitance of 3 μF. What is the inductance of the circuit?

$X_{c} = 1/2\pi f c$

$X_{C} = 1/2x3.14x980x10^{3}x3x10^{-6}$

ANS: 8.8×10^{-9} H or 8.8 pH