PHYS 221

Electromagnetism (1)
2nd semester 1446

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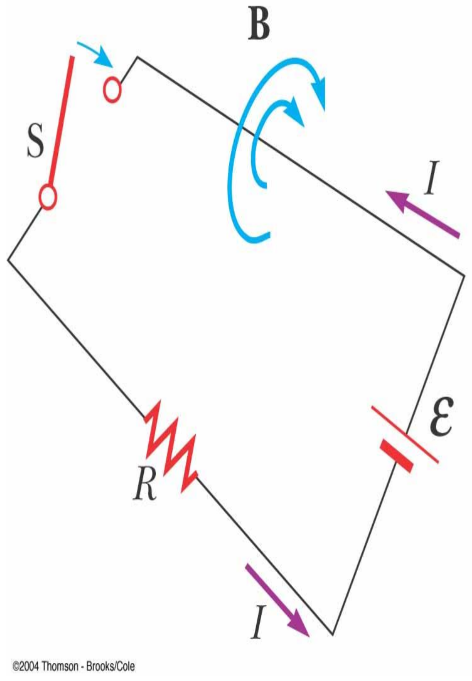
Lecture 9

Chapter 32 Inductance

32-1 Self-Inductance
32-3 Energy of a Magnetic Field

32-1 Self-Inductance



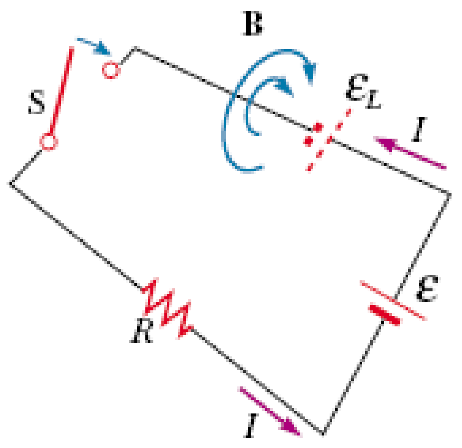
- **When the switch is closed, the current does not immediately reach its maximum value**
- **Faraday's law can be used to describe the effect**

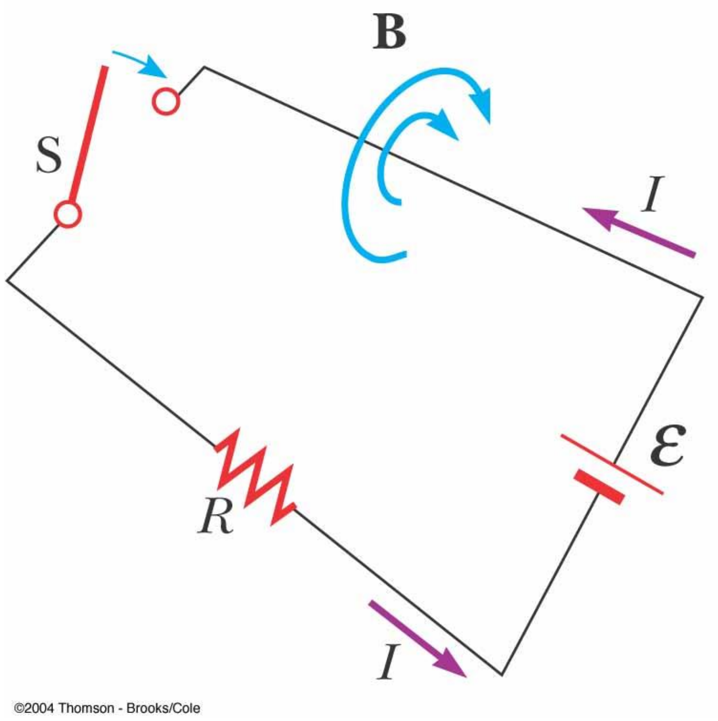
❑ **As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.**

❑ **The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field.**

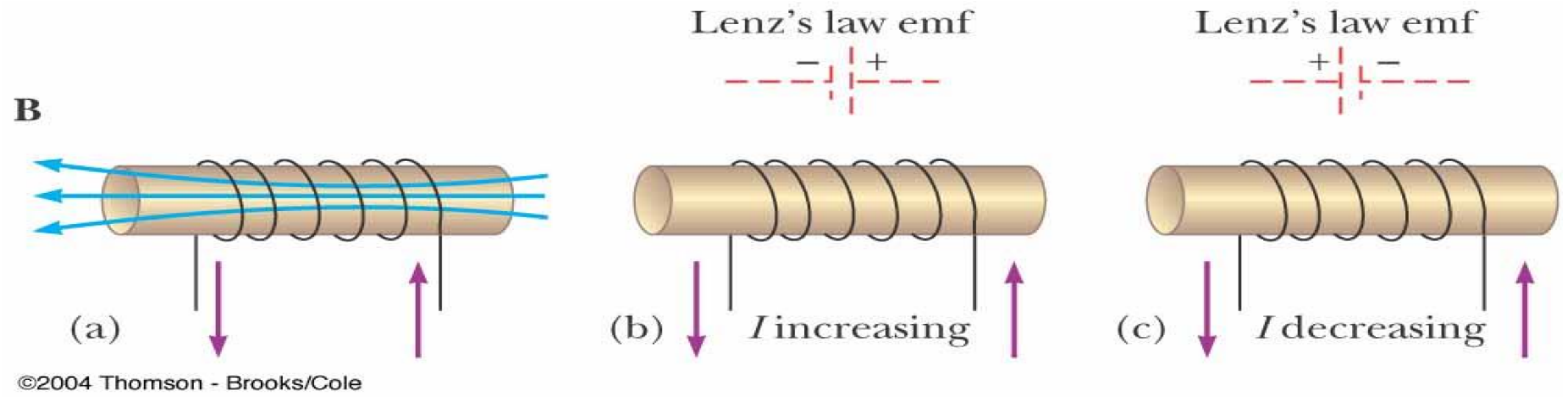
After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop.

As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop (back emf).



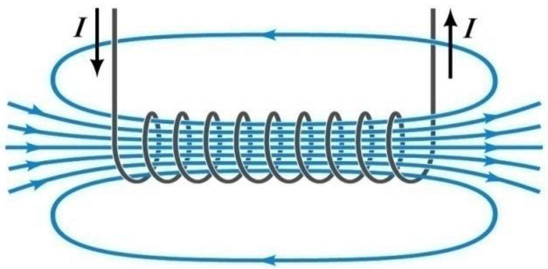


The emf set up in this case is called a self-induced emf.



- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)

$$B = \mu_o n I$$



$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d(\mu_o n I A)}{dt} = -N\mu_o n A\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Define: Self Inductance $L = \frac{N\Phi_B}{I}$

Inductance Units

$$L = \left[\frac{V}{A/s}\right] = [\Omega - s] = [\text{Henry}] = [H]$$

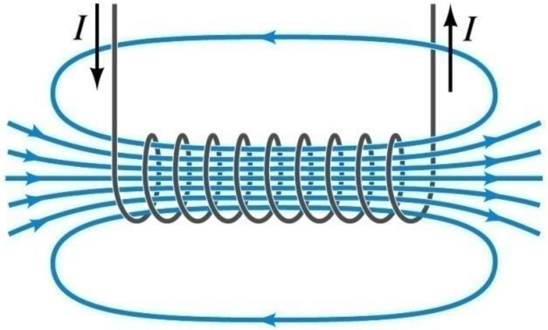
Inductance of a Solenoid

- **The magnetic flux through each turn is**

$$\Phi_B = BA = \left(\mu_o \frac{N}{\ell} I \right) A$$

- **Therefore, the inductance is**

$$L = \frac{N\Phi_B}{I} = \frac{\mu_o N^2 A}{\ell}$$



L = inductance of the solenoid
N = # of turns in solenoid
l = length of solenoid
A = cross sectional area of solenoid
n = # of turns per unit length

- **This shows that *L* depends on the <u>geometry</u> of the object**
- **The inductance is a measure of the opposition to a change in current**

Example 32-1 Inductance of a Solenoid

Find the inductance of a uniformly wound solenoid having N turns and length ℓ. Assume that ℓ is much longer than the radius of the windings and that the core of the solenoid is air.

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

where A is the cross-sectional area of the solenoid. Using this expression and Equation 32.2, we find that

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell} \qquad (32.4)$$

This result shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A \ell = \mu_0 n^2 V \qquad (32.5)$$

where $V = A\ell$ is the volume of the solenoid.

Example 32-2 calculating Inductance and emf

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm².

Solution Using Equation 32.4, we obtain

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= (4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})\ \frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m^2})}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

Example :

A solenoid with L=1x10^{-4} H is in a circuit and experiences a change in current from 0 to 10 A in 1.0 ms. Find the self-induced emf in the solenoid

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -10^{-4}\frac{10}{10^{-3}} = -1.0V$$

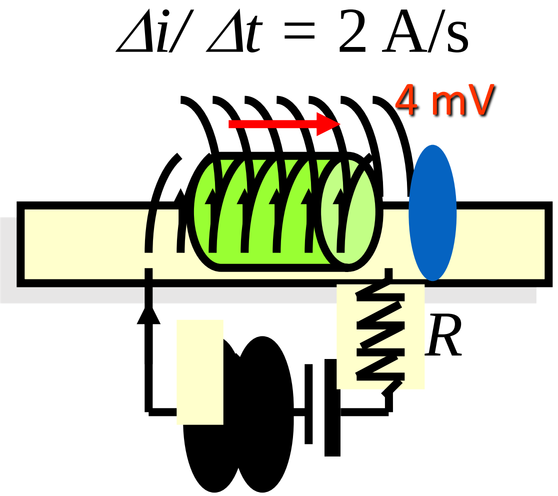
The emf opposes the current direction

Example :

Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then *i must be in* the process of decreasing.

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times10^{-4}\ \text{H}.$$

Example : A coil having 20 turns has an induced emf of 4 mV when the current is changing at the rate of 2 A/s. What is the inductance?



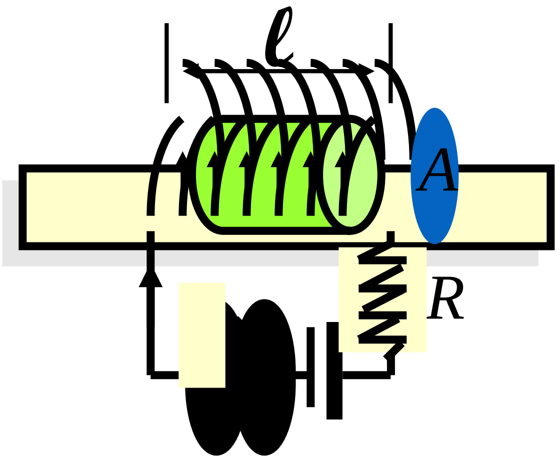
$$L = \frac{-(-0.004\ \text{V})}{2\ \text{A/s}}$$

L = 2.00 mH

Example A solenoid of area 0.002 m^2 and length 30 cm, has 100 turns. If the current increases from 0 to 2 A in 0.1 s, what is the inductance of the solenoid?

First we find the inductance of the solenoid:

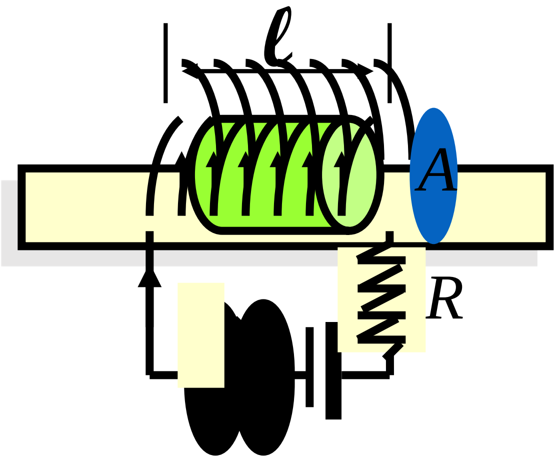
$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi \text{ x } 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})(100)^2(0.002 \text{ m}^2)}{0.300 \text{ m}}$$



L = 8.38 x 10^{-5} H

Note: L does NOT depend on current,.

Example (Cont.): If the current in the 83.8-μH solenoid increased from 0 to 2 A in 0.1 s, what is the induced emf?



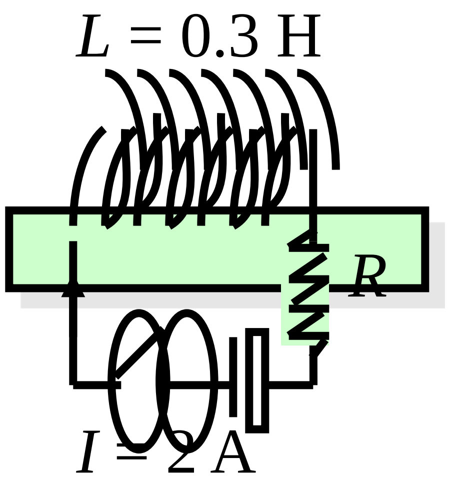
$L = 8.38 \times 10^{-5}$ H

$$\mathcal{E} = -L\frac{\Delta i}{\Delta t}$$

$$\mathcal{E} = \frac{-(8.38 \text{ x } 10^{-5}\text{H})(2 \text{ A - } 0)}{0.100 \text{ s}}$$

$$\mathcal{E} = -1.68 \text{ mV}$$

Example : What is the potential energy stored in a 0.3 H inductor if the current rises from 0 to a final value of 2 A?



$$U = \tfrac{1}{2}Li^2$$

$$U = \tfrac{1}{2}(0.3\text{ H})(2\text{ A})^2 = 0.600\text{ J}$$

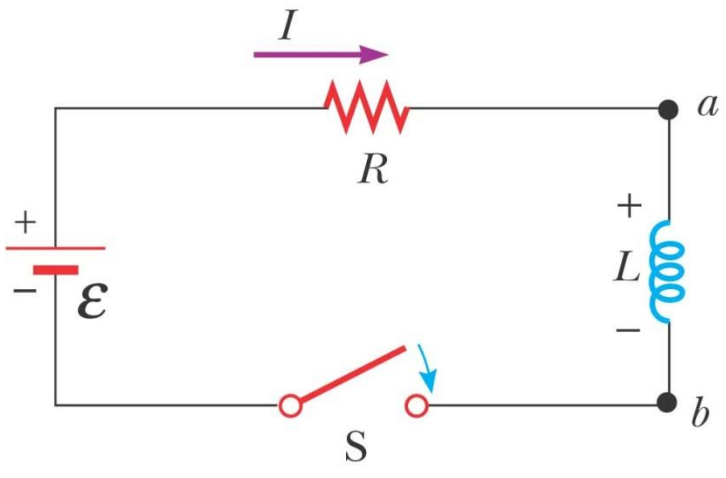
$$U = 0.600\text{ J}$$

This energy is equal to the work done in reaching the final current *I*; it is returned when the current decreases to zero.

32-3 Energy of a Magnetic Field

$$\varepsilon - L\frac{dI}{dt} - IR = 0 \qquad Kirchoff$$

$$I\varepsilon - LI\frac{dI}{dt} - I^2R = 0 \qquad power$$



- Iε is the rate at which energy is being supplied by the battery
- I^2R is the rate at which the energy is being delivered to the resistor
- LI (dI/dt) must be the rate at which the energy is being stored in the magnetic field

U denote the energy stored in the inductor at any time

$$\frac{dU_B}{dt} = LI\frac{dI}{dt}$$

$$dU_B = LI\,dI$$

$$U_B = \int_0^{U_B} dU_B = \int_0^{I} LIdI$$

$$U_B = \frac{1}{2} L I^2$$

Remember for a capacitor: $U = \frac{1}{2} CV^2$

$$\because L = \mu_0 n^2 A\ell \quad \because B = \mu_0 n I$$

$$\therefore U_B = \frac{1}{2} L I^2 = \frac{1}{2} (\mu_0 n^2 A\ell)(\frac{B}{\mu_0 n})^2$$

$A\ell$ is the volume of the solenoid

$$U_B = \frac{B^2}{2\mu_0}(A\ell)$$

Energy of a Magnetic Field

$$\text{energy density} = \frac{\text{energy}}{\text{volume}}$$

$$u_B = \frac{U_B}{A\ell} = \frac{B^2}{2\mu_0}$$

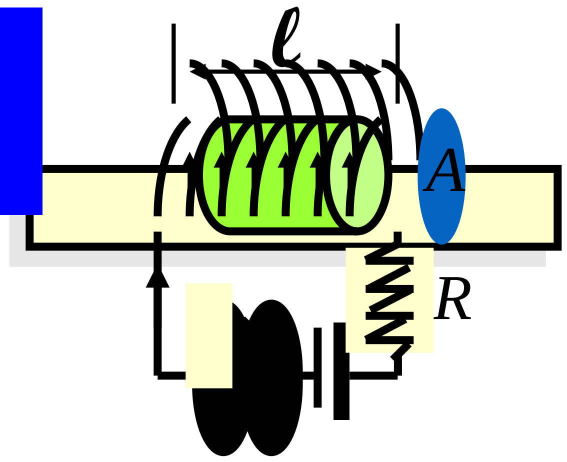
Energy Density in a coil

$$u = \frac{1}{2} \varepsilon_o E^2$$

Example 4: The final steady current in a solenoid of 40 turns and length 20 cm is 5 A. What is the energy density?

$$B = \frac{\mu_0 NI}{\ell} = \frac{(4\pi \text{ x } 10^{-7})(40)(5 \text{ A})}{0.200 \text{ m}}$$

B = 1.26 mT



$$u = \frac{B^2}{2\mu_0} = \frac{(1.26 \text{ x } 10^{-3}\text{T})^2}{2(4\pi \text{ x } 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})}$$

u = 0.268 J/m^3

Energy density is important for the study of electro-magnetic waves.

Example

a) Find The Inductance of a long solenoid length *L*=2m and radius=2cm with 2000 turns?

b) if current decreased from 4A to 0 in 2 microseconds what is magnitude and direction of the self induced emf ?

c) what is the energy stored in the solenoid at the beginning of the 2 microsecond interval?

d) How much electrical power is dissipated during this time?

a) Inductance value

$$L = \frac{\mu_o N^2 A}{l}$$

$$A = \pi r^2$$

$A = 1.257\text{x}10^{-3}\text{m}^2$

$L = (1.2566\text{x}10^{-6})(2000)^2(1.257\text{x}10^{-3})/2.0 =$

$= 3.159\text{x}10^{-3}$ H

b) $$\varepsilon = -L\frac{dI}{dt} = -L\frac{I_2 - I_1}{\Delta t}$$

emf = $(3.159\text{x}10^{-3})(4-0)/(2.0\text{x}10^{-6})$ = 6318V

in direction of current trying to stop field collapse by trying to maintain current

c) Energy?

$$U_1 = \frac{1}{2}LI_1^2$$

$U_1 = (1/2)\ (3.158\text{x}10^{-3})(4)^2 = 2.52\text{x}10^{-2}$ Joules

e) Power

$$P = \frac{\Delta U}{\Delta t} = \frac{U_1}{\Delta t}$$

P = $(2.52\text{x}10^{-2})/(2\text{x}10^{-6})$ = 12,632 W

A 10.0-mH inductor carries a current $I = I_{max} \sin \omega t$,

with I_{max} = 5.00 A and $\omega /2\pi$ = 60.0 Hz. What is the back emf as a function of time?

Ans: (18.8V)cos (377t)

An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

Ans: -0.421A/s

SUMMARY

When the current in a coil changes with time, an emf is induced in the coil according to Faraday's law. The **self-induced emf** is

$$\mathcal{E}_L = -L\frac{dI}{dt} \tag{32.1}$$

where L is the **inductance** of the coil. Inductance is a measure of how much opposition an electrical device offers to a change in current passing through the device. Inductance has the SI unit of **henry** (H), where $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$.

The inductance of any coil is

$$L = \frac{N\Phi_B}{I} \tag{32.2}$$

where Φ_B is the magnetic flux through the coil and N is the total number of turns. The inductance of a device depends on its geometry. For example, the inductance of an air-core solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} \tag{32.4}$$

where A is the cross-sectional area, and ℓ is the length of the solenoid.

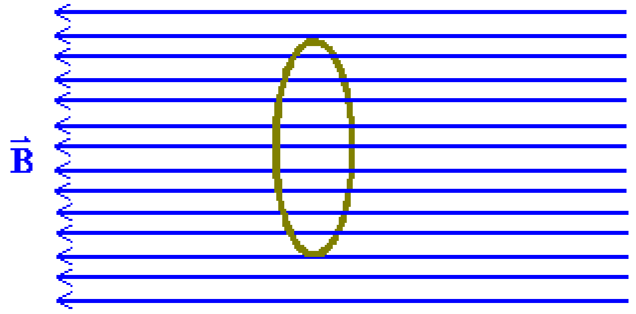
The energy stored in the magnetic field of an inductor carrying a current I is

$$U = \tfrac{1}{2} L I^2 \tag{32.12}$$

Summary

- Inductance (units, henry H) is given by $$L = \frac{N\Phi_B}{i}$$
- Inductance of a solenoid is: $$L = \frac{\mu_0 N^2 A}{l}$$ (depends only on geometry)
- EMF, in terms of inductance, is: $$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$$
- Energy in inductor: $$U_B = \frac{1}{2}Li^2$$ Energy in magnetic field

Preflight 16:



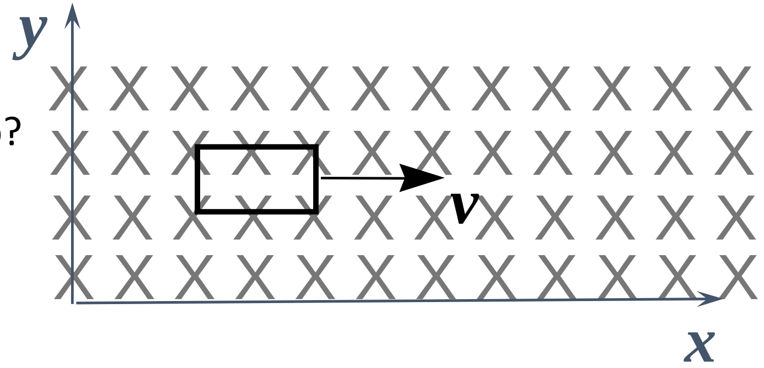
5) The ring is moving to the right. The magnetic field is uniform and constant in time. You are looking from right to left. What is the induced current?

a) zero
b) clockwise
c) counter-clockwise

6) The ring is stationary. The magnetic field is decreasing in time. What is the induced current?

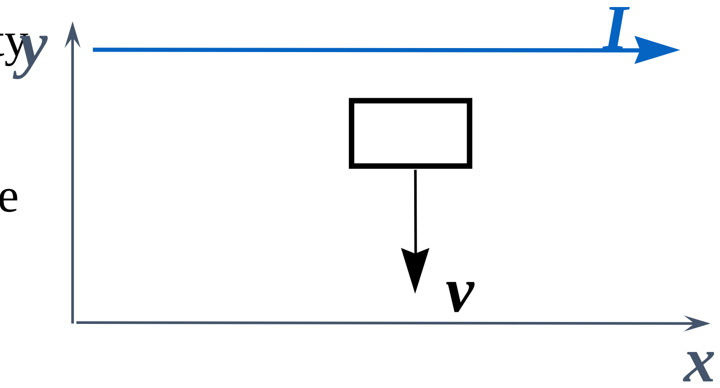
a) zero
b) clockwise
c) counter-clockwise

- A conducting rectangular loop moves with constant velocity v in the +x direction through a region of constant magnetic field B in the -z direction as shown.
  - What is the direction of the induced current in the loop?



(a) ccw (b) cw (c) no induced current

- A conducting rectangular loop moves with constant velocity v in the -y direction and a constant current I flows in the +x direction as shown.
  - What is the direction of the induced current in the loop?



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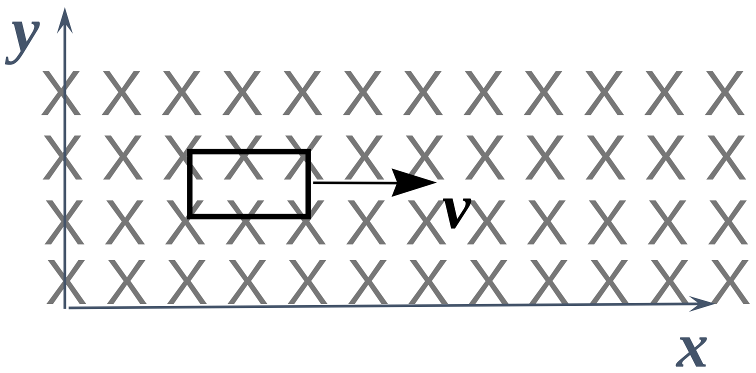


2A

(a) ccw

(b) cw

(c) no induced current

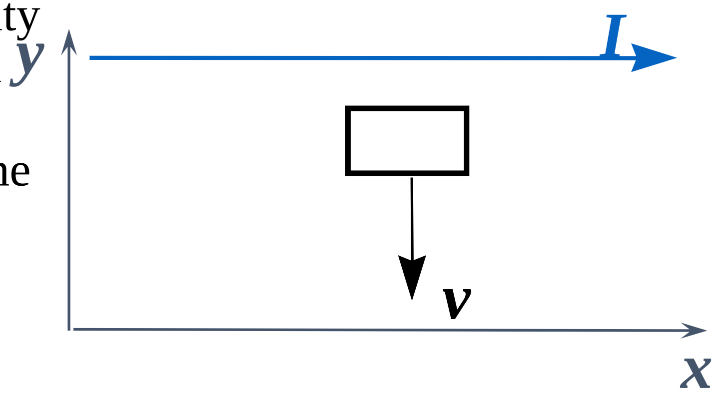
- There is a non-zero flux Φ_B passing through the loop since B is perpendicular to the area of the loop.

- Since the velocity of the loop and the magnetic field are CONSTANT, however, this flux DOES NOT CHANGE IN TIME.

- Therefore, there is NO emf induced in the loop; NO current will flow!!

• A conducting rectangular loop moves with constant velocity v in the -y direction and a constant current I flows in the +x direction as shown.

• What is the direction of the induced current in the loop?



2B

(a) ccw

(b) cw

(c) no induced current

• The flux through this loop DOES change in time since the loop is moving from a region of higher magnetic field to a region of lower field.

• Therefore, by Lenz' Law, an emf will be induced which will oppose the change in flux.

• Current is induced in the clockwise direction to restore the flux.