PHYS 221

Electromagnetism (1)
2^{nd} semester 1446

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Lecture 7

Chapter 30
Sources of the Magnetic Field

30.1 The Biot–Savart Law

30.2 The Magnetic Force Between Two Parallel Conductors

30.3 Ampere's Law

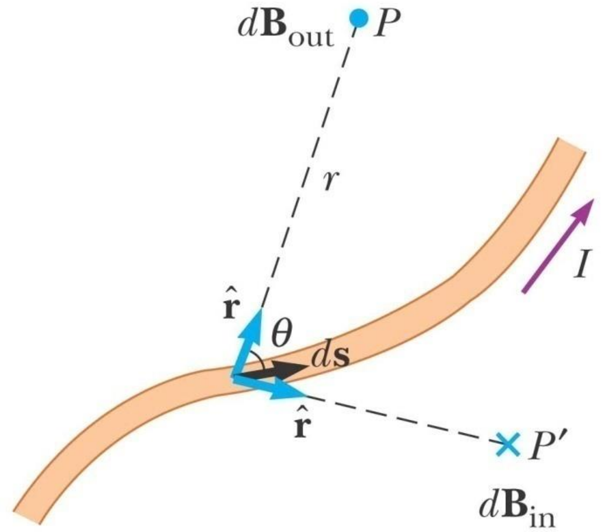
30.4 The Magnetic Field of a Solenoid

30.5 Magnetic Flux

30.6 Gauss's Law in Magnetism

30.1 The Biot–Savart Law

Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet.



The magnetic field dB at a point due to the current I through a length element ds is given by the Biot–Savart law. The direction of the field is out of the page at P and into the page at P'

Properties of the magnetic field created by an electric current

- The vector dB is perpendicular both to ds (which points in the direction of the current) and to the unit vector directed from ds to P.
- The magnitude of dB is inversely proportional to r^2, where r is the distance from ds to P.
- The magnitude of dB is proportional to the current and to the magnitude ds of the length element ds.
- The magnitude of dB is proportional to sin θ , where θ is the angle between the vectors ds and ˆr.

These observations are summarized in the mathematical formula known today as the **Biot–Savart law:**

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{30.1}$$

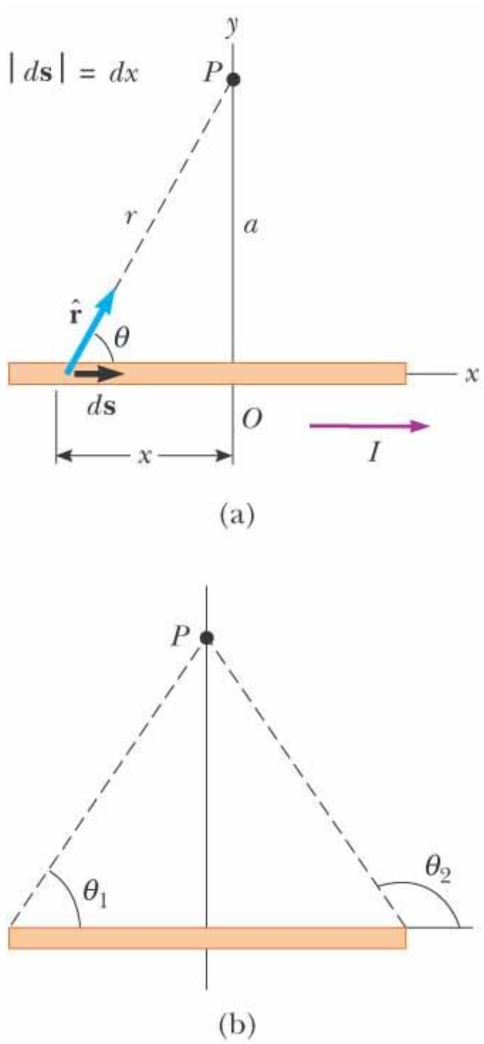
where μ_0 is a constant called the **permeability of free space:**

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \tag{30.2}$$

To find the total magnetic field B created at some point by a current of finite size, we must sum up contributions from all current elements *Ids that make up the current. That is, we* must evaluate B by integrating Equation 30.1

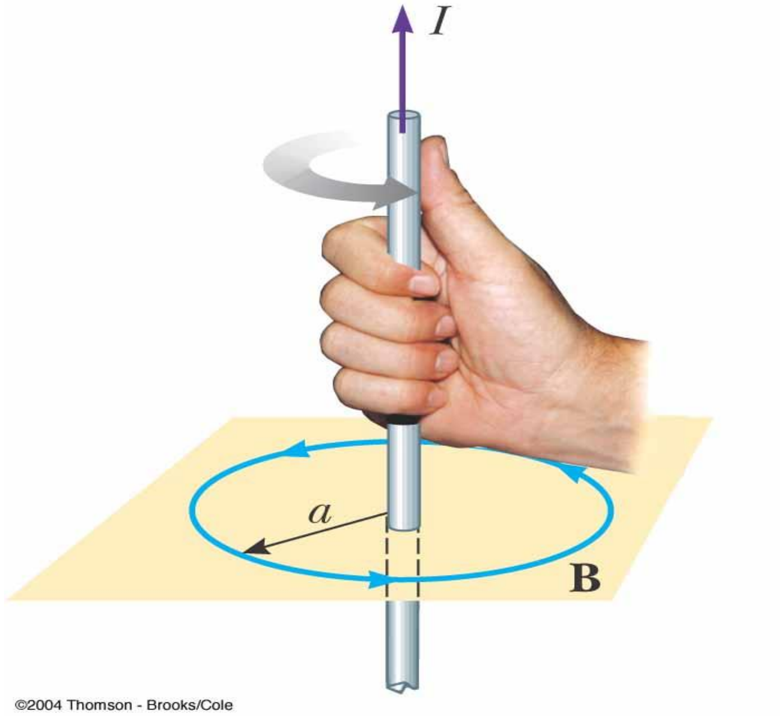
$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \int \frac{Id\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

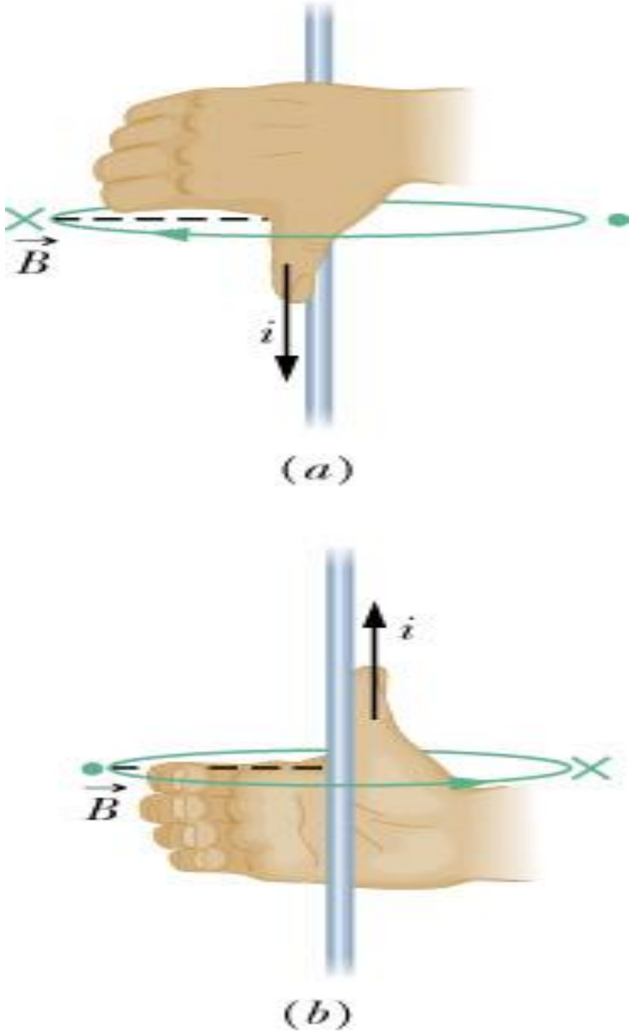
EXAMPLE 30.1 Magnetic Field Surrounding a Thin, Straight Conductor



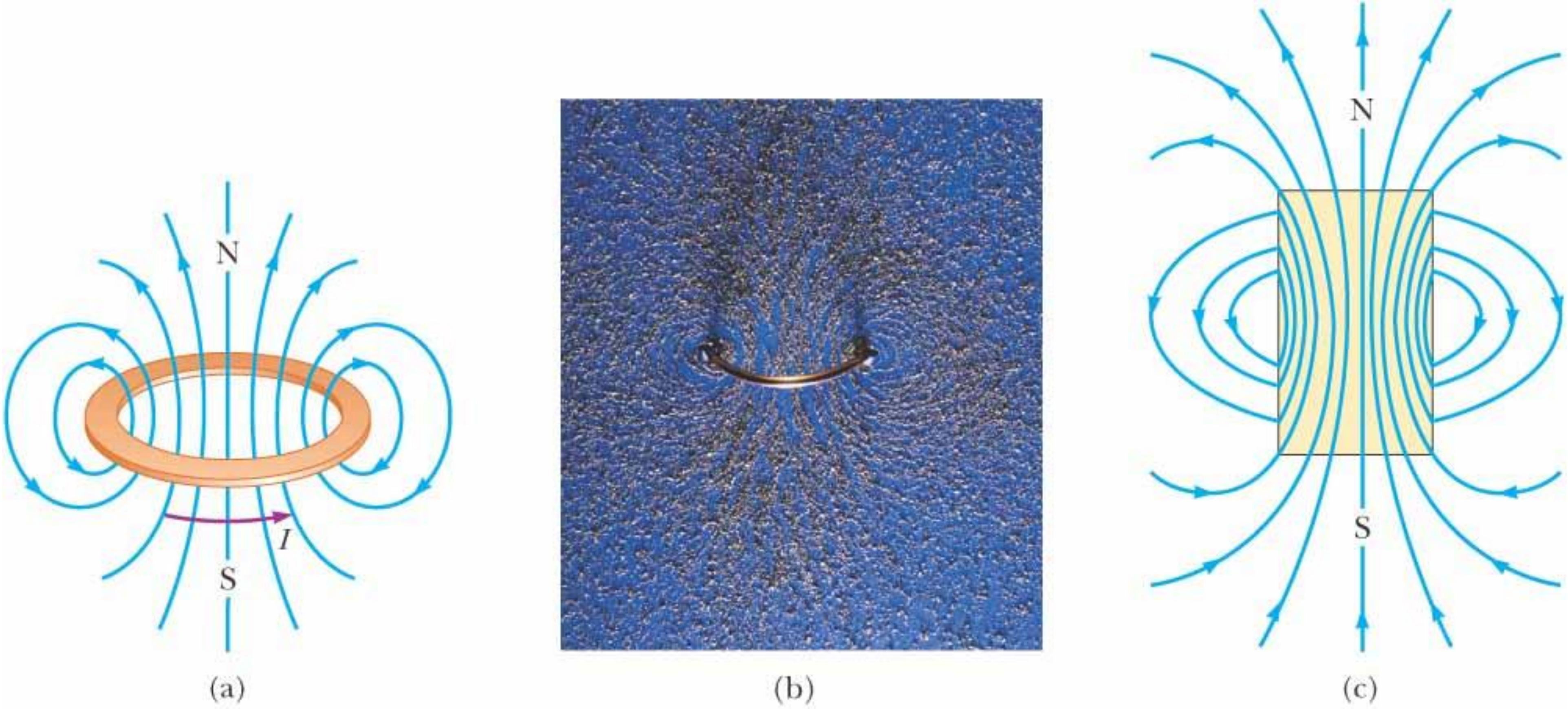
Consider a thin, straight wire carrying a constant current *I*
and placed along the *x axis as shown in Figure ….*
Determine the magnitude and direction of the magnetic field at point *P due to this current*

$$B = \frac{\mu_o I}{2\pi a}$$



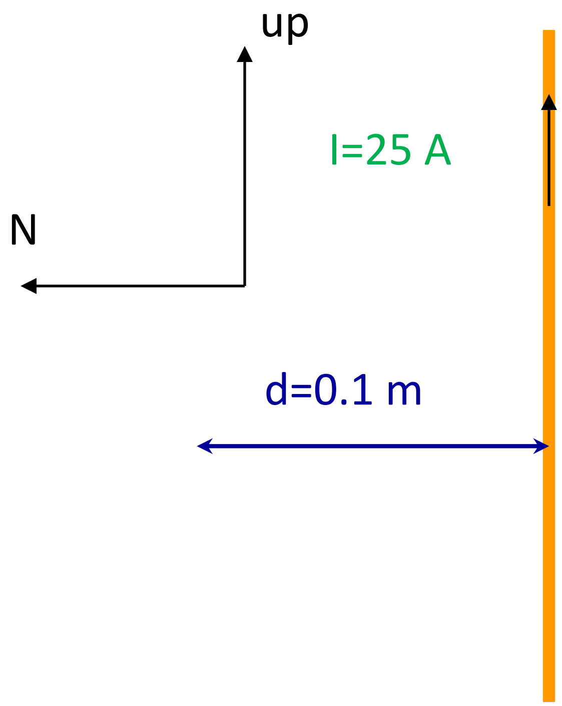


The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Note that the magnetic field lines form circles around the wire.



(a) Magnetic field lines surrounding a current loop. (b) Magnetic field lines surrounding a current loop, displayed with iron filings. (c) Magnetic field lines surrounding a bar magnet. Note the similarity between this line pattern and that of a current loop

Example A vertical electric wire in the wall of a building carries a current of 25 A upward. What is the magnetic field at a point 10 cm due north of this wire?



Let's make north be to the left in this picture, and up be up.

According to the right hand rule, the magnetic field is to the west, coming out of the plane of the "paper."

To calculate the magnitude, B:

$$B = \frac{\mu_0 I}{2\pi\, r}$$

$$B = \frac{4\pi \times 10^{-7}\,\frac{T.m}{A}}{2\pi}\frac{(25A)}{(0.1m)} = 5\times10^{-5}T$$

example

Two infinitely long wires form the *x* and *y* axis of a 2D coordinate system. They each carry current as shown. What is the magnetic field at a point P (2.0 m, 2.0 m)?

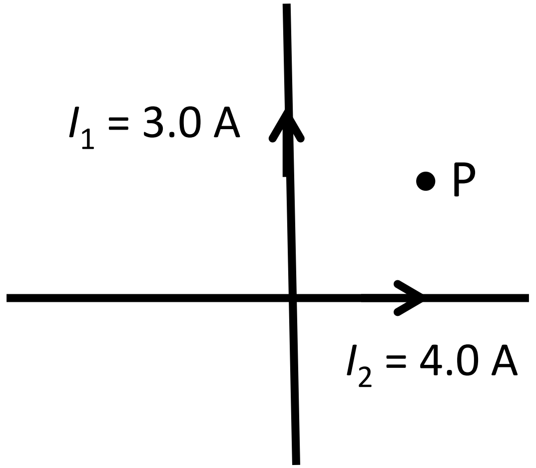
A. 7×10^{-7} T into the page

B. 7×10^{-7} T out of the page

C. 1×10^{-7} T into the page

D. 1×10^{-7} T out of the page

E. 0



$$B_1 = \frac{\mu_0 (3\ A)}{2\pi (2\ m)} \text{ into the page}$$

$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2$$

$$B_2 = \frac{\mu_0 (4\ A)}{2\pi (2\ m)} \text{ out of the page}$$

$$B_{total} = \frac{\mu_0 (1\ A)}{2\pi (2\ m)} = 10^{-7}\ T \text{ out of the page}$$

The Magnetic Field around a Wire

What is the magnetic field strength and direction 1.0×10^{-2} m from the centre of a wire with 1.0 A of current coming out of the page?

Given: $R = 1.0 \times 10^{-2}\,\text{m}$, $I = 1.0$ A, $B = ?$

$$B = \frac{\mu_0 I}{2\pi R}$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(1.0\,\text{A})}{2\pi\left(1.0 \times 10^{-2}\,\text{m}\right)}$$

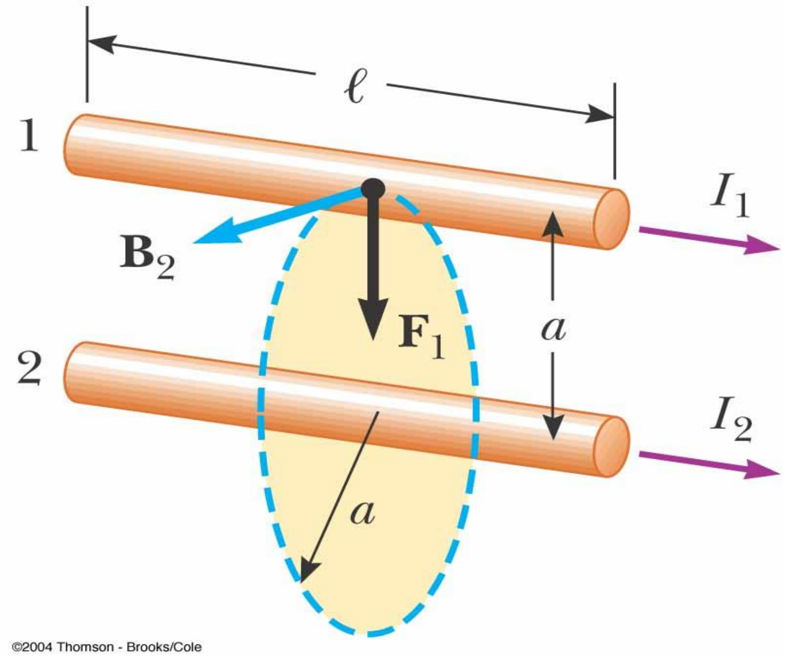
$$B = 2.0 \times 10^{-5}\,\text{T}$$

The strength of the magnetic field is $2.0 \times 10^{-5}\,\text{T}$. Using RHR #1, the direction of the magnetic field is counterclockwise.

30.2 The Magnetic Force Between Two Parallel Conductors

Consider two long, straight, parallel wires separated by a distance *a and carrying* currents I_1 *and* I_2 *in the same direction*

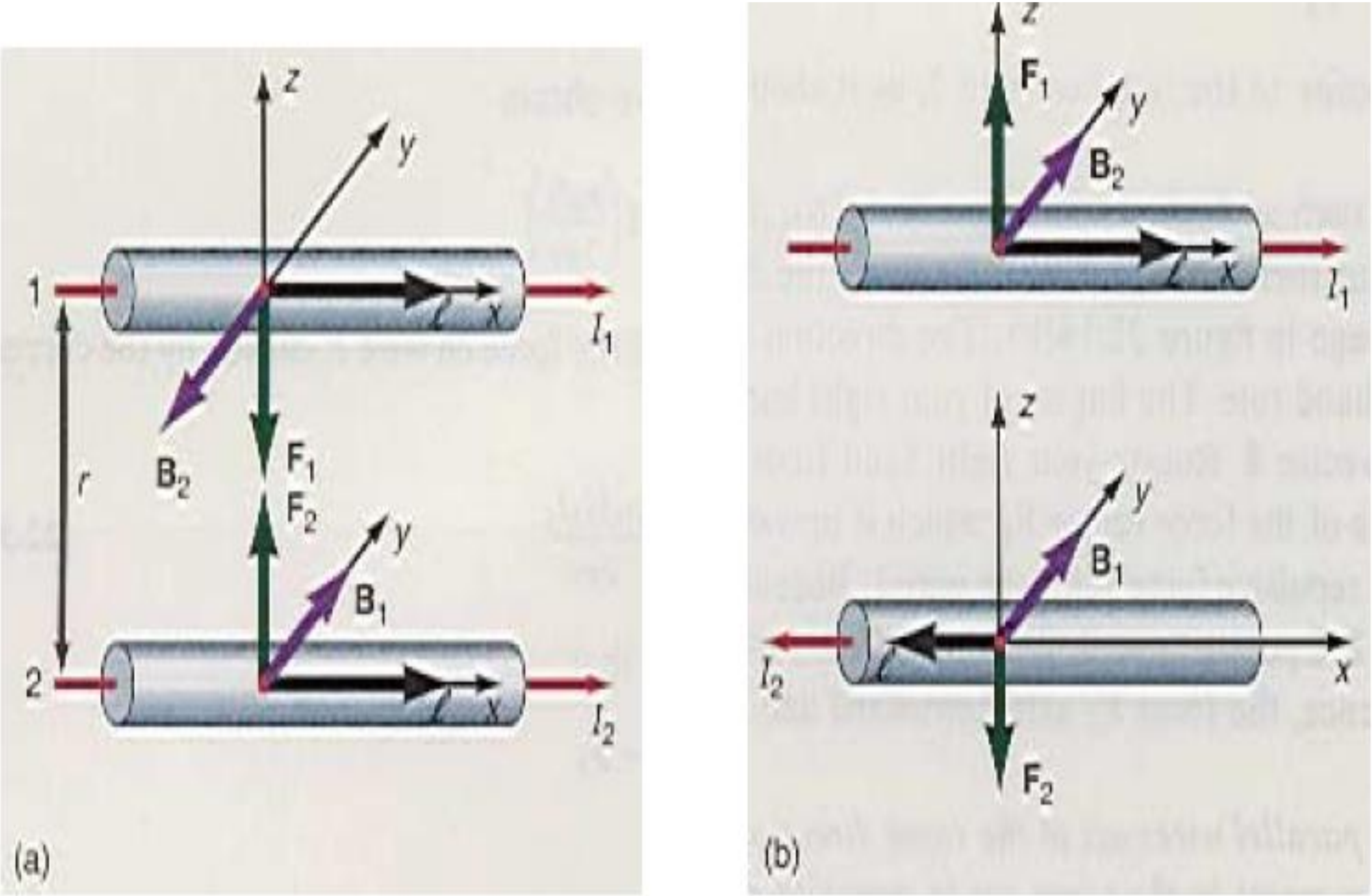
We can determine the force exerted on one wire due to the magnetic field set up by the other wire.



$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$

RHR-
Fingers along B
Thumb along v
Palm shows F

parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.


z
y
1
I_1
r
B_2
F_1
F_2
B_1
2
I_2
x
(a)
z
F_1
y
B_2
I_1
z
y
B_1
I_2
x
F_2
(b)

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B .

Force per unit length: $$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Definition of the Ampere

- When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is 2 x 10^{-7} N/m, the current in each wire is defined to be 1 A

Definition of the Coulomb

- The SI unit of charge, the **coulomb**, is defined in terms of the ampere
- When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C

example

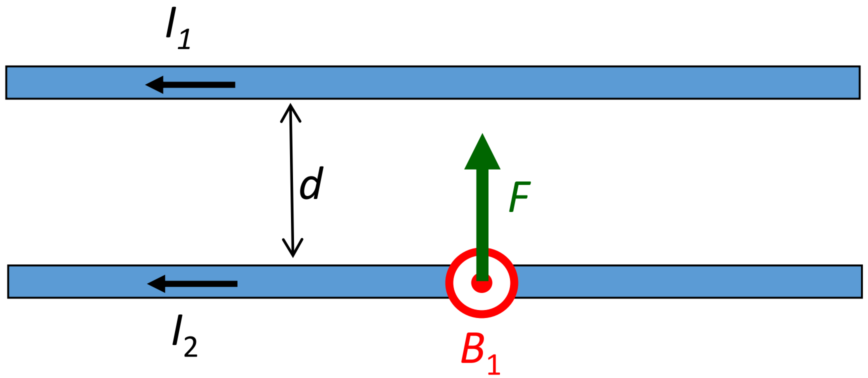
Two very long straight, parallel wires have currents I running through them as shown in the figure. The force on wire 2 due to wire 1 is:

A. Down

B. Up

C. Into the page

D. Out of the page



Magnetic field by 1 at location of wire 2 is out of the page.

Force on I_2 by B_1 is up.

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \text{ out of the page}$$

Force for section of lenght L: $F = I_2 L B_1$

or force per unit length: $$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

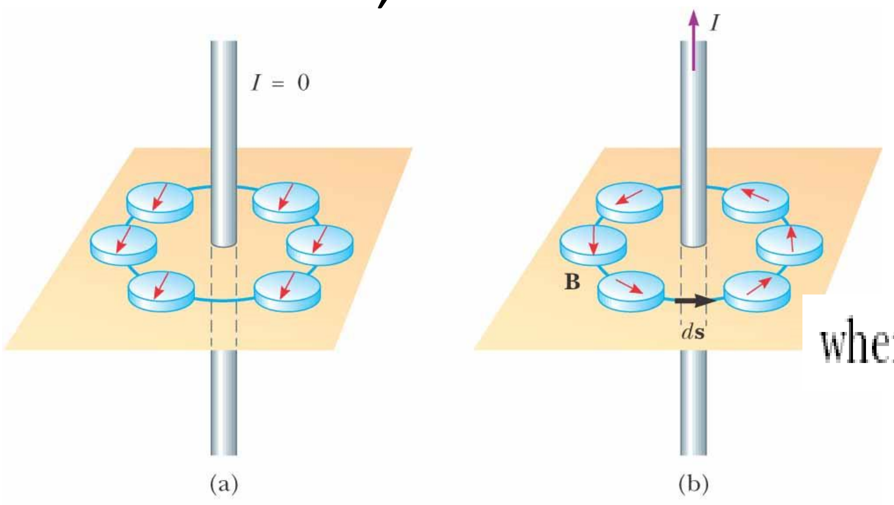
Parallel currents: attraction

Antiparallel currents: repulsion

30.3 AMPERE'S LAW

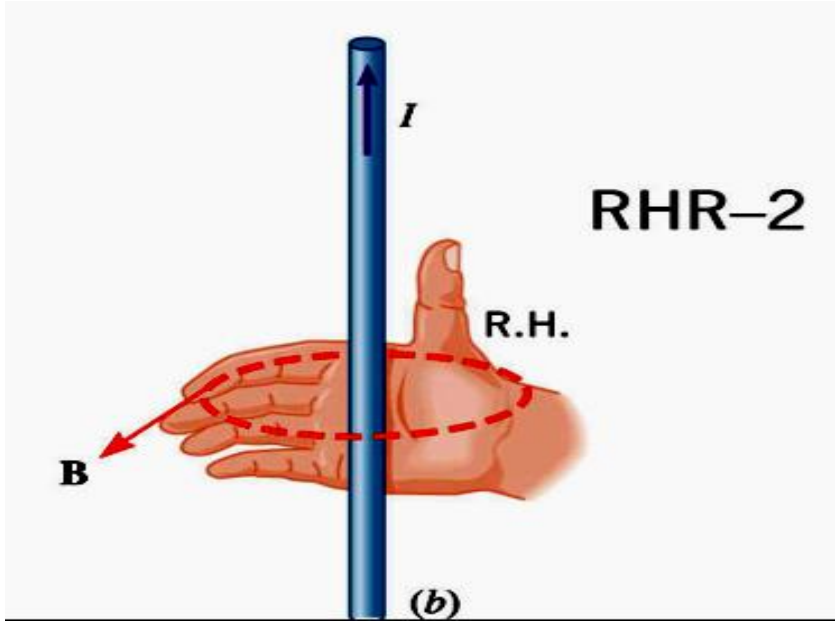
Because the compass needles point in the direction of B, we conclude that the lines of B form circles around the wire,

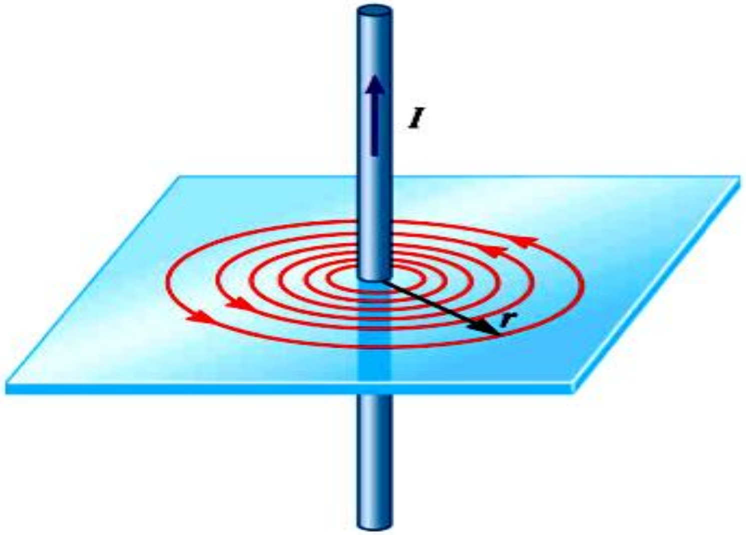
the magnitude of B is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire. By varying the current and distance *a from the wire, we find that B is proportional to the current and inversely* proportional to the distance from the wire,



$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

where $\oint ds = 2\pi r$ is the circumference of the circular path.





$$B \propto \frac{I}{r}$$

Ampère's law

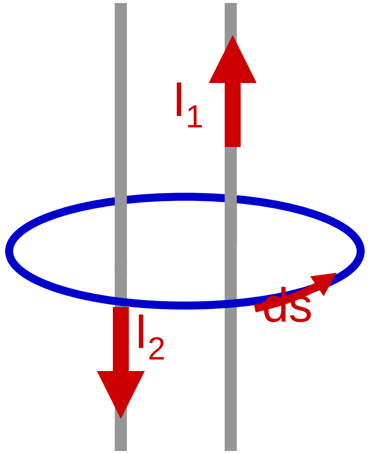
The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path.

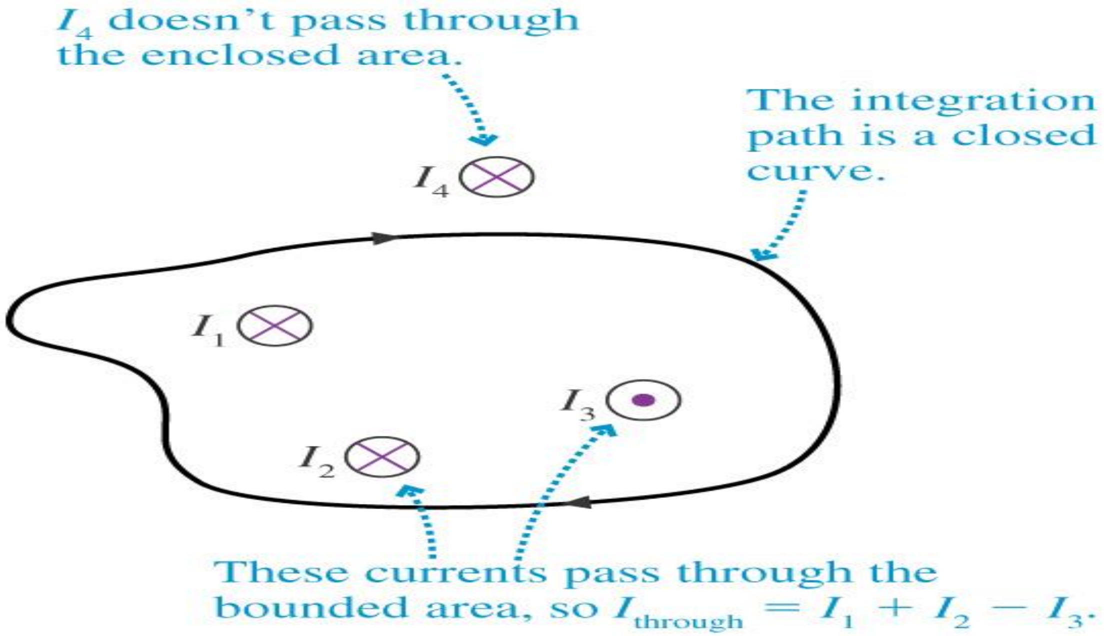
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad (30.13)$$

Ampere's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

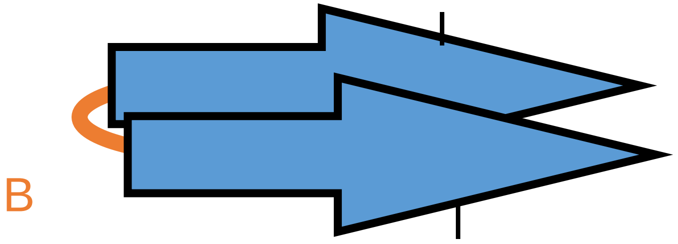
If your path includes more than one source of current, add all the currents (with correct sign).

$$\oint \vec{B}.\vec{ds} = \mu_0 (I_1 - I_2)$$

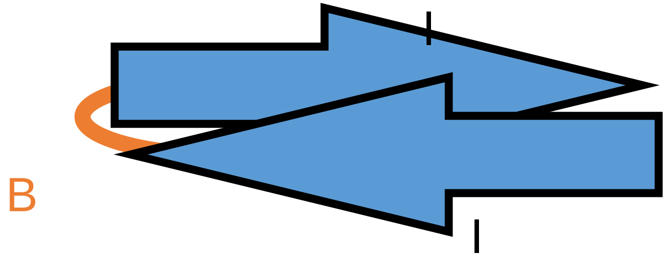




$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = 2\mu_0 I$$

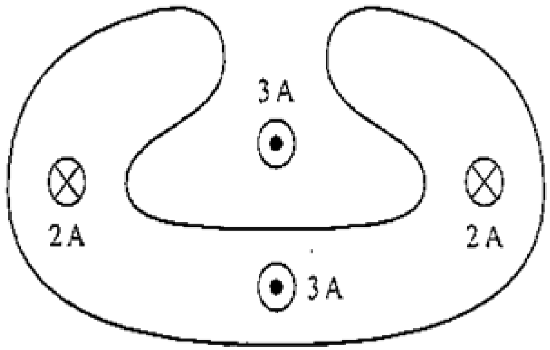


$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = 0$$



Example:

What is the total current through the area bounded by the closed curve?

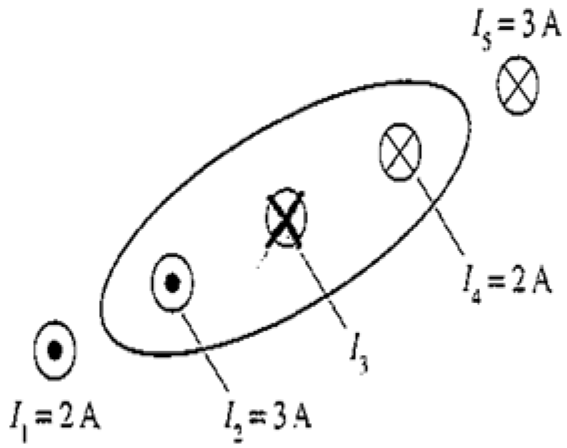


1.0 A

1A into the page

Example:

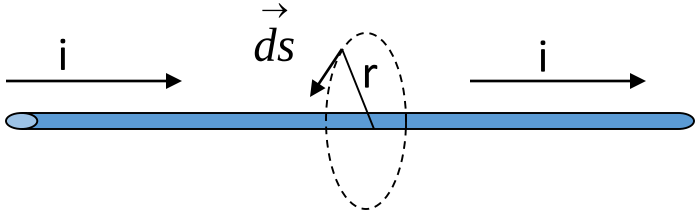
The total current through the area bounded by the closed curve is 2 A. What are the size and direction of I_3?



I_3 = 3A into the page

Example:

Use Ampere's Law to find B near a very long, straight wire. B is independent of position along the wire and only depends on the distance from the wire (symmetry).



By symmetry $\vec{B} \parallel \vec{ds}$

$$\oint \vec{B} \cdot \vec{ds} = \oint Bds = B\int ds = B2\pi r = \mu_0 i$$

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{i}{r}}$$

Suppose i = 10 A

r = 10 cm

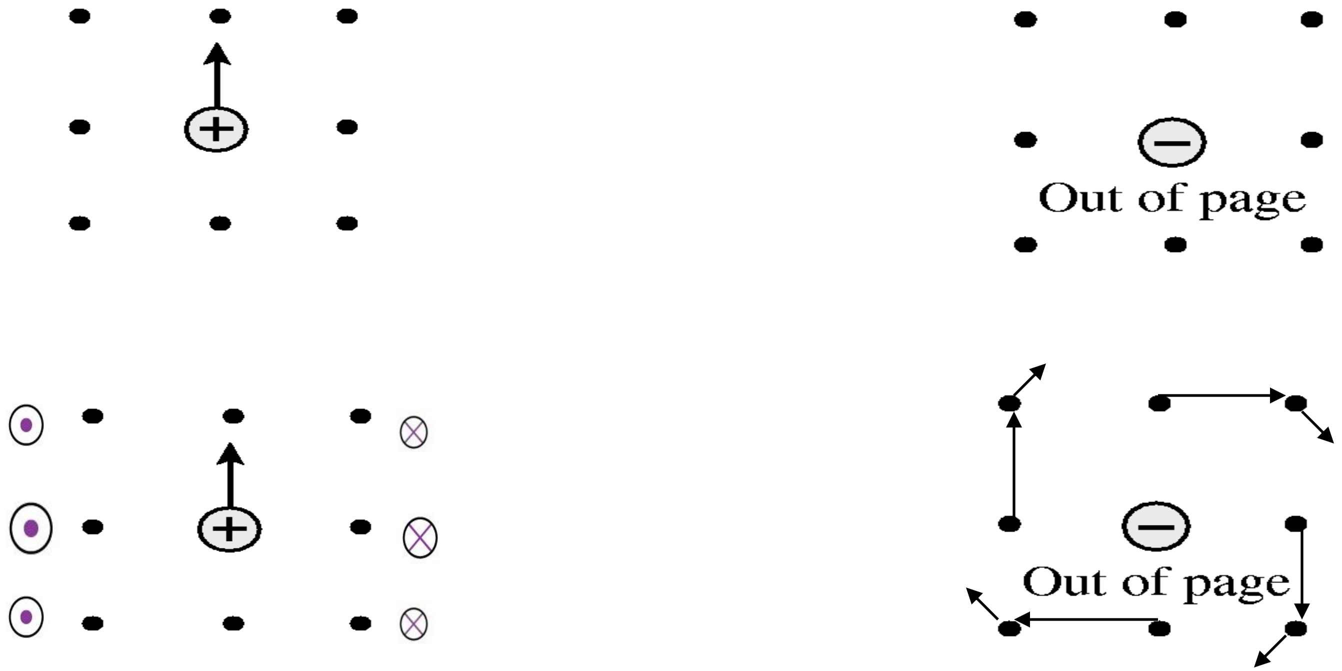
$$B = 2\times 10^{-7} \times 10 \times 10^{-1}$$

$$B = 2\times 10^{-7}\ \mathrm{T}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{N}{A^2}$$

Example:

Draw the magnetic field vector at each dot.
Show relative size with arrow length if possible:

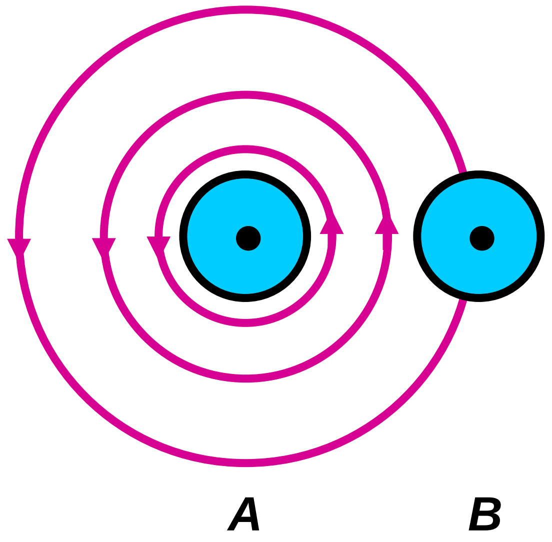


Example

Interaction between current-carrying wires

2 parallel wires *A* & *B*, carry the same size of current.

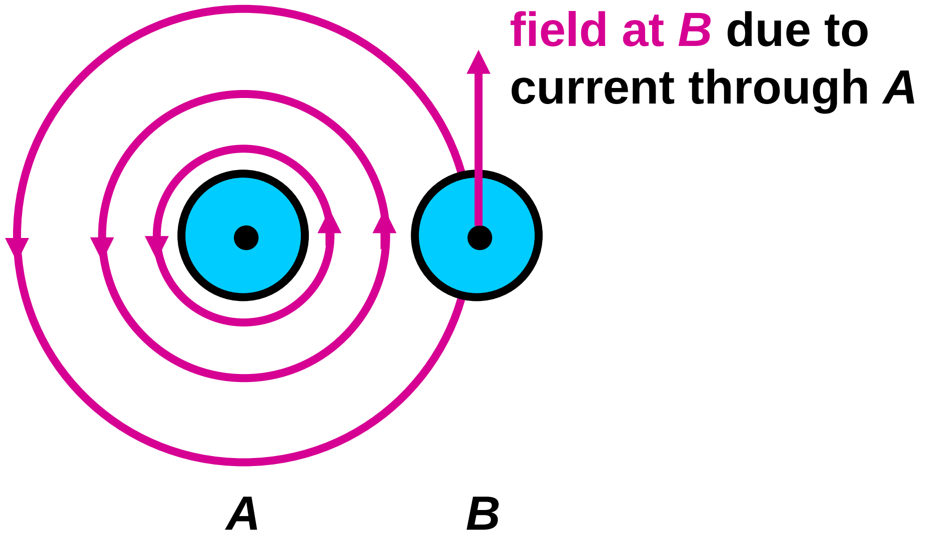
(a) Draw a few magnetic field lines due to the current through wire *A*.



Example

Interaction between current-carrying wires

2 parallel wires *A* & *B*, carry the same size of current.

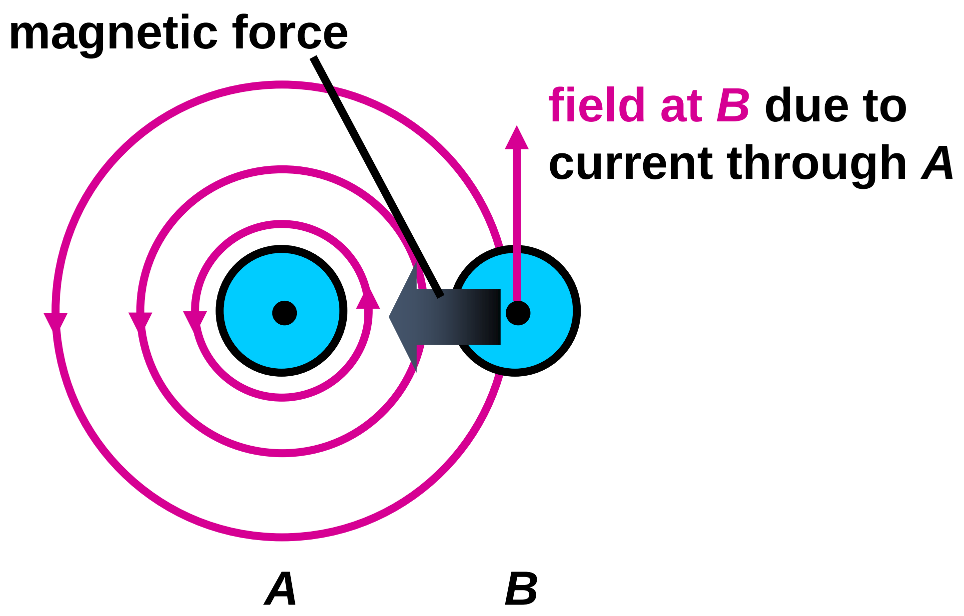


(b) Mark the direction of this magnetic field at wire *B* with an arrow.

Example

Interaction between current-carrying wires

2 parallel wires *A* & *B*, carry the same size of current.

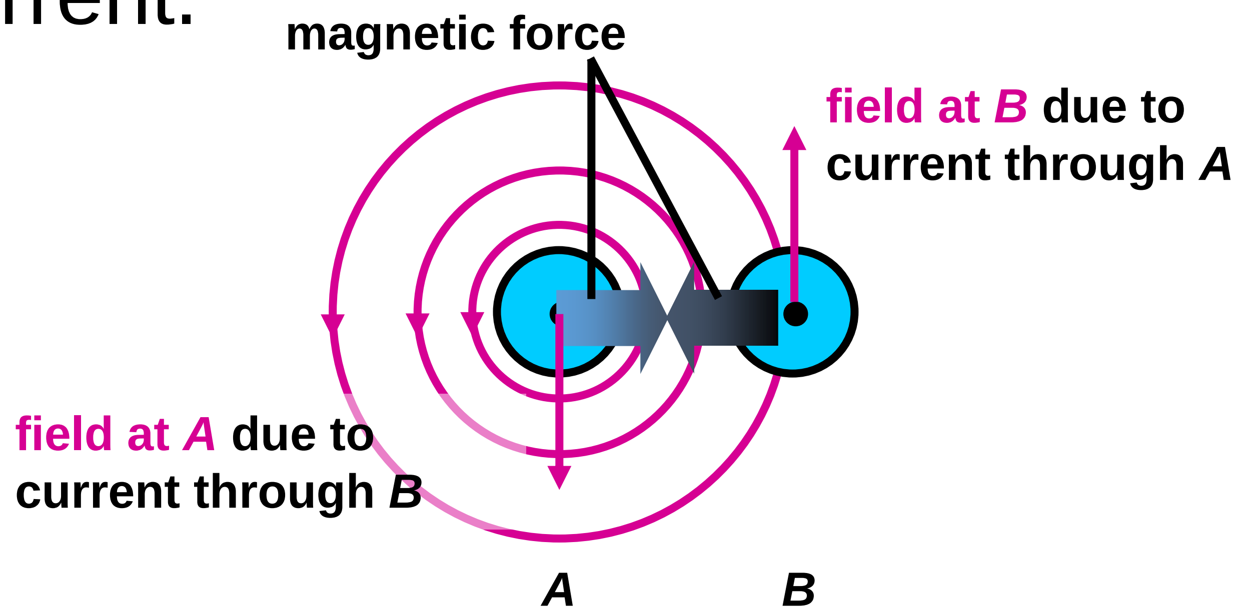


(c) Mark the direction of magnetic force acting on *B* due to magnetic field of the current through *A*.

Example

Interaction between current-carrying wires

2 parallel wires *A* & *B*, carry the same size of current.

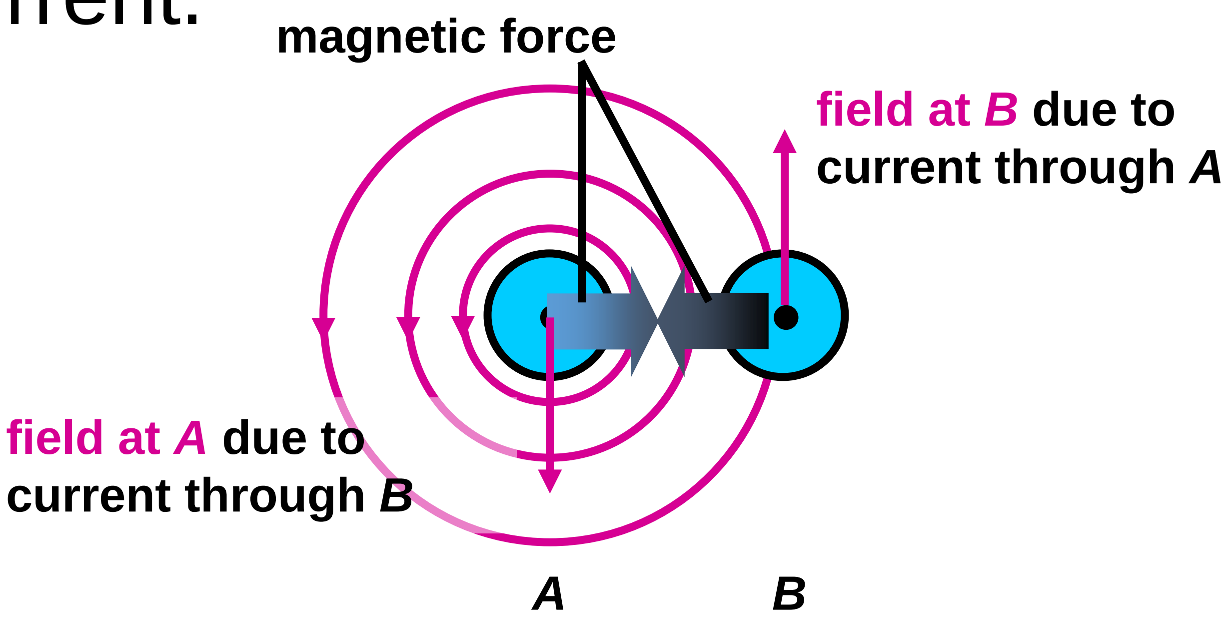


(d) Deduce the direction of magnetic force acting on *A* due to magnetic field of the current through *B*.

Example

Interaction between current-carrying wires

2 parallel wires *A* & *B*, carry same size of current.



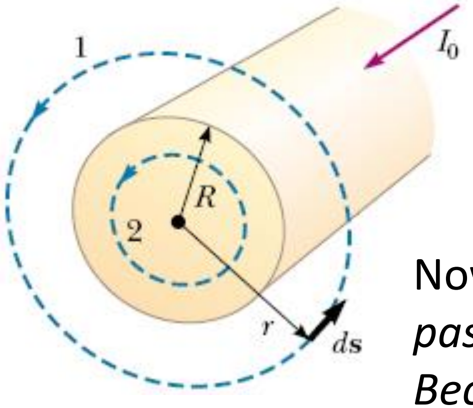
(e) Do *A* & *B* attract or repel each other?

They **attract** each other.

EXAMPLE 30.4 The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire (Fig. 30.11). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Let us choose for our path of integration circle 1 in Figure …From symmetry, B must be constant in magnitude and parallel to *ds at every* point on this circle. Because the total current passing through the plane of the circle is I_0, *Ampère's law gives*



$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$

Now consider the interior of the wire, where $r < R$. *Here* the current I *passing through the plane of circle 2 is less than* the total current I_0. *Because the current is uniform over the* cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area r^2 *enclosed by* circle 2 to the cross-sectional area R^2 *of the wire:*

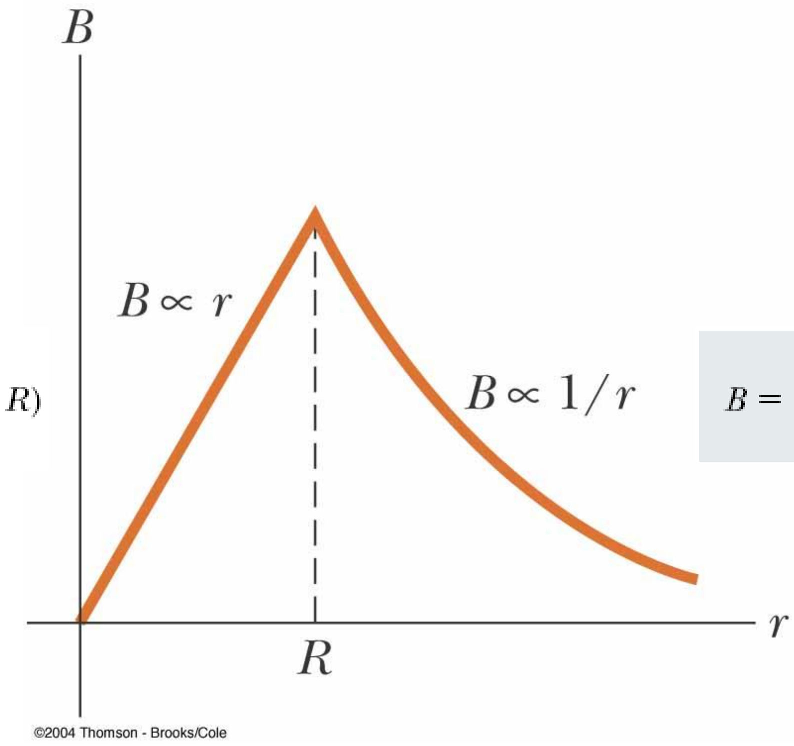
$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$I = \frac{r^2}{R^2} I_0$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0 \left(\frac{r^2}{R^2} I_0\right)$$

$$B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r \qquad \text{(for } r < R\text{)}$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad \text{(for } r \geq R\text{)}$$



$$B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r \qquad \text{(for } r < R\text{)}$$

Fig 30-13, p.936

30.4 The Magnetic Field of a Solenoid

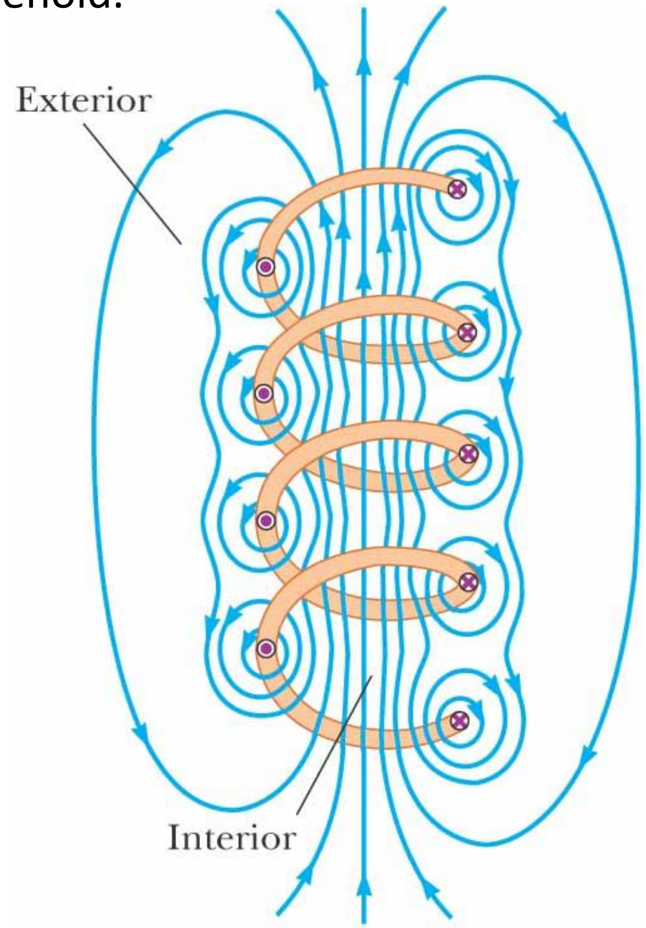
➢ A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the *interior of the solenoid—when the solenoid* carries a current.

* The magnetic field lines for a loosely wound solenoid.

➢ the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is uniform and strong.

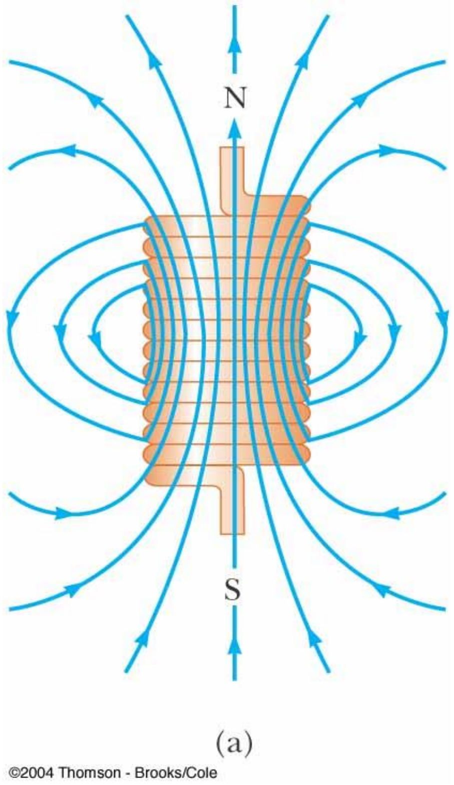
➢ The field lines between current elements on two adjacent turns tend to cancel each other because the field vectors from the two elements are in opposite directions.

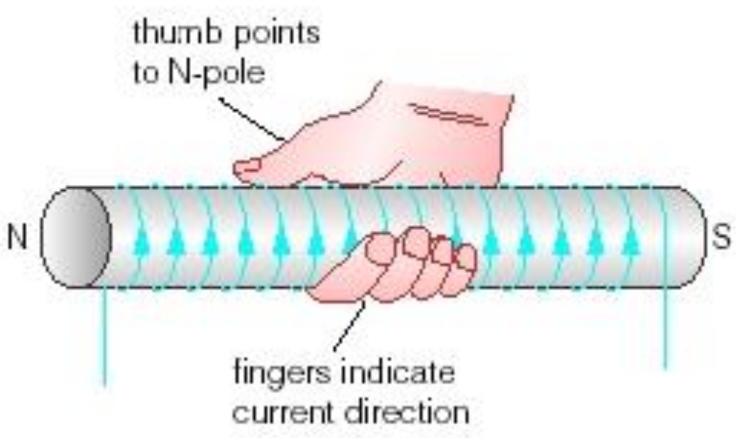
➢ The field at exterior points such as *P is weak because* the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion.

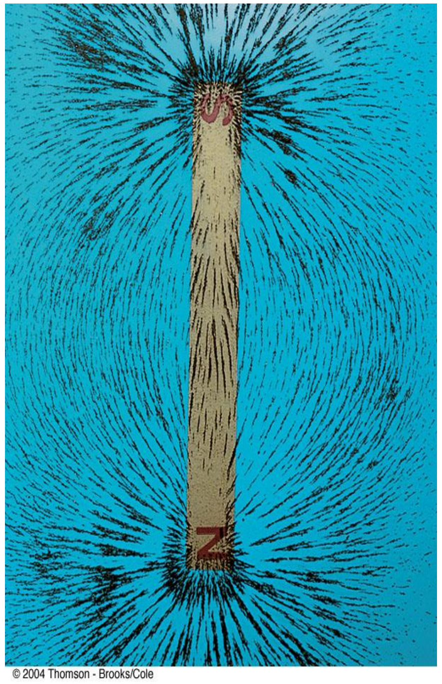


- Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current

  - The field at exterior points such as *P is weak because* the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion.

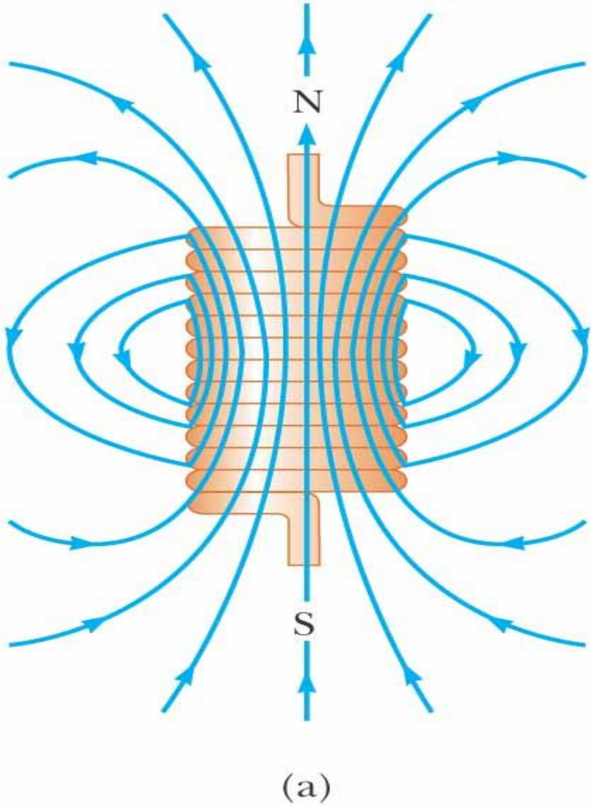


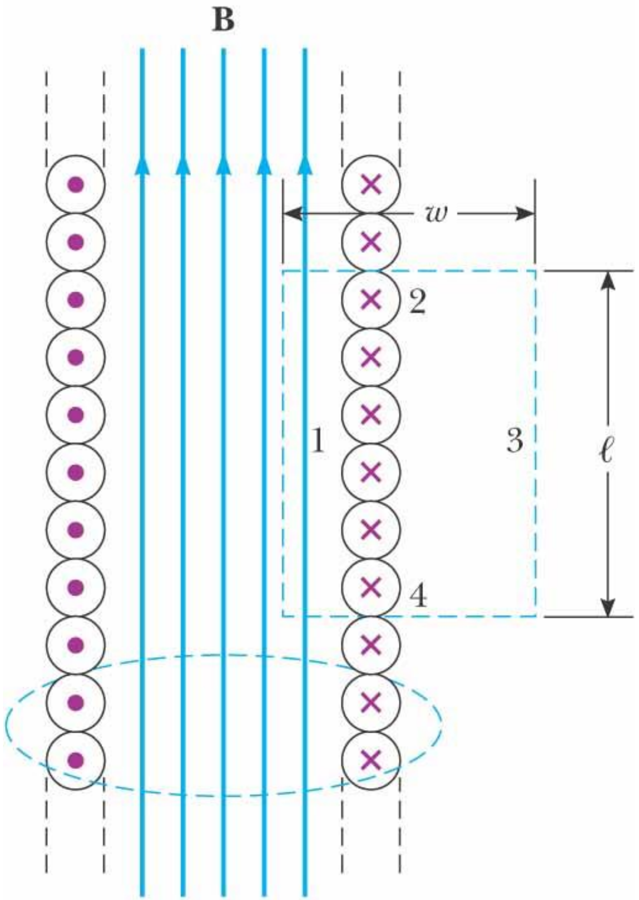




The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.

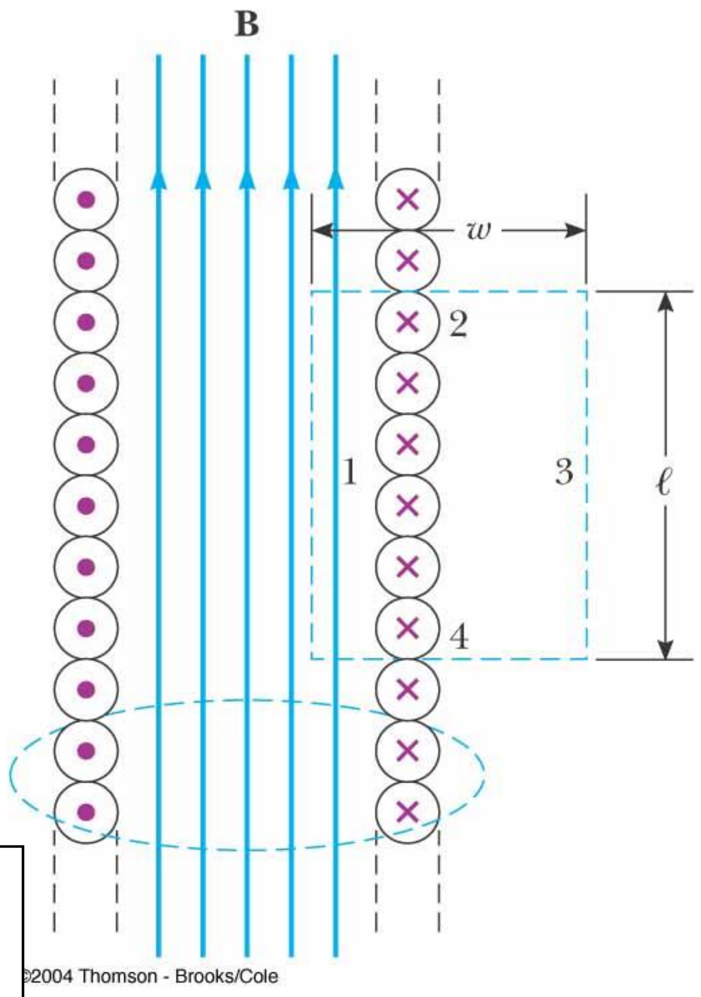
An *ideal solenoid is approached* when the turns are closely spaced and the length is much greater than the radius of the turns. In this case, the external field is zero, and the interior field is uniform over a great volume.





Because the solenoid is ideal, B in the interior space is uniform and parallel to the axis, and B in the exterior space is zero.

Consider the rectangular path of length and width w shown in Figure…. We can apply Ampere's law to this path by evaluating the integral of B.*ds* over each side of the rectangle.



$$\oint \vec{B}.\vec{ds} = \int_1 \vec{B}.\vec{ds} + \int_2 \vec{B}.\vec{ds} + \int_3 \vec{B}.\vec{ds} + \int_4 \vec{B}.\vec{ds}$$

$$\oint \vec{B}.\vec{ds} = Bl + 0 + 0 + 0 = \mu_0 I_{enclosed}$$

$$Bl = \mu_0 NI$$

N is the number of turns enclosed by our surface

$$n = \frac{N}{l}$$

n= N/L is the number of turns per unit length

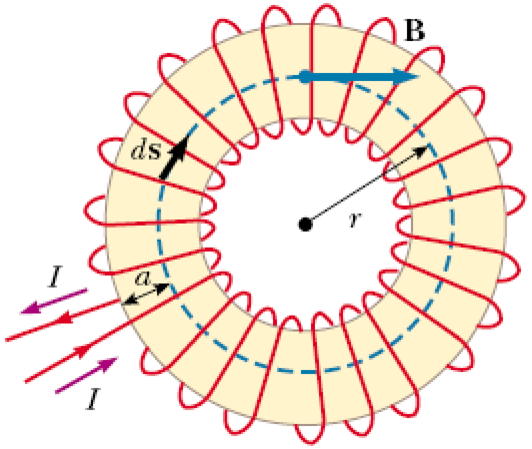
$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

valid only for points near the center (that is, far from the ends) of a very long solenoid.

If the radius *r of the torus in Figure 30.13 containing N turns* is much greater than the toroid's cross-sectional radius *a, a short section of the* toroid approximates a solenoid for which

$$n = N/2\pi r.$$



Example:

a thin 10-cm long solenoid has a total of 400 turns of wire and carries a current of 2 A. Calculate the magnetic field inside near the center.

$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right) \frac{(400)}{(0.1\,m)} (2\,A)$$

$$\boxed{B = 0.01\ T}$$

Example

A 10 cm long solenoid has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the magnetic field inside the solenoid

n = 400 turns/0.10 m = 4000 m^{-1}

$B = m_o nI$

B = (4p X 10^{-7} T m/A)(4000m-1)(2.0 A)

B = 0.01 T

Example:

A solenoid 20 cm long and 4 cm in diameter is wounded with a total of 200 turns of wire. The solenoid is aligned with its axis parallel to the earth's magnetic field which is $3x10^{-5}$ T in magnitude. What should the current in the solenoid be in order for its field to exactly cancel the earth's field inside the solenoid?

Solution: Since l=0.2 m and N=200 turns, $\mathbf{B=N\mu_o I/l}$

$I=Bl/\mu_o N=(3x10^{-5}T)(0.2)/(4\pi x10^{-7}Tm/A)(200)=0.024\ A=24\ mA$

The solenoid diameter has no significance except as a check that the solenoid is long relative to its diameter.

Example

A 0.100 T magnetic field is required. A student makes a solenoid of length 10.0 cm. Calculate how many turns are required if the wire is to carry 10.0 A.

$$B = \mu_0 \frac{N}{L} I$$

$$N = \frac{BL}{\mu_0 I} = \frac{o.1 \times 10 \times 10^{-2}}{4\pi x 10^{-7} \times 10} \qquad \mathrm{T}$$

30.5 Magnetic Flux

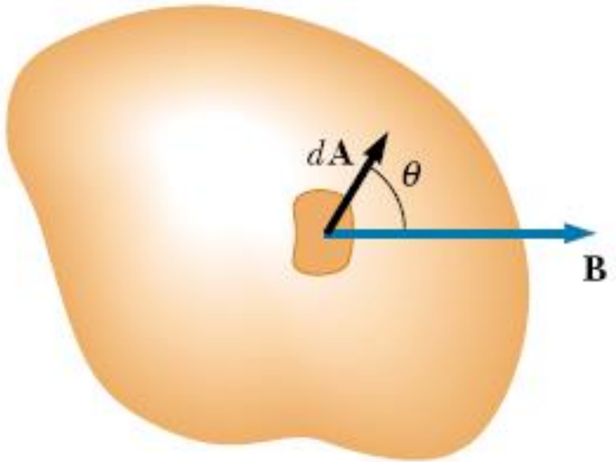
Magnetic Flux, Φ: The number of magnetic (flux) field lines which pass through a given cross-sectional area A

Consider the special case of a plane of area *A in a uniform field B that makes* an angle θ with *dA. The magnetic flux through the plane in this case is*

$$\Phi = \int \vec{B}.\vec{ds}$$

$$for \text{ constant B and A}$$

$$\Phi = \text{BA } \cos\theta$$



Units:

Φ webers B Tesla

A area m^2

θ angle formed between B and the normal to the loop (area vector A)

The area vector A is perpendicular to the surface A and has a magnitude equal to the area A.

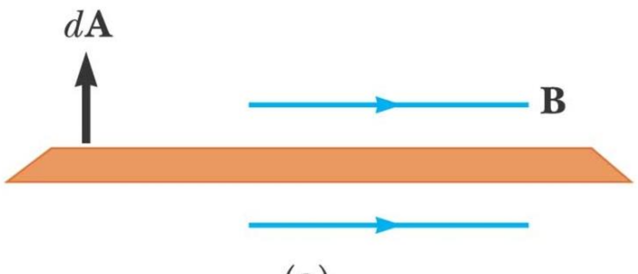
Hint: θ is the angle formed between B and the normal to the loop.

When B is along the plane of the loop?

$\Phi = B\,A \cos\theta$

$\Theta = 90^0$

$\Phi = 0$



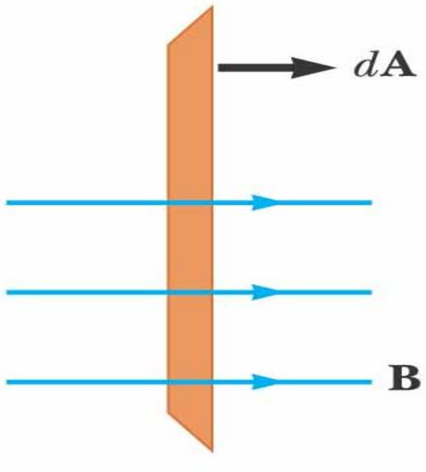
(a)

When B is perpendicular to the loop?

$\Phi = B\,A \cos\theta$

$\Theta = 0^0$

$\Phi = B\,A$



(b)

Example:

Which has the largest magnetic flux?

Answer: A

EXAMPLE 30.8 Magnetic Flux Through a Rectangular Loop

A rectangular loop of width *a and length b is located near a* long wire carrying a current *I (Fig. 30.21). The distance between* the wire and the closest side of the loop is *c . The wire* is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

$$B = \frac{\mu_0 I}{2\pi r}$$

Because B is parallel to *dA at any point within the loop,* the magnetic flux through an area element *dA is*

$$\Phi_B = \int B\, dA = \int \frac{\mu_0 I}{2\pi r}\, dA$$

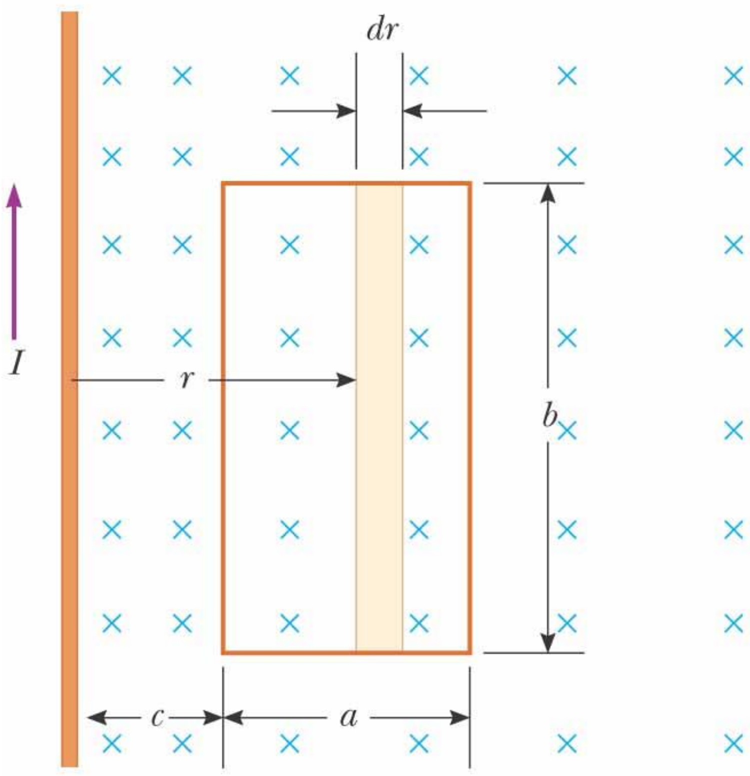


Fig 30-22, p.941

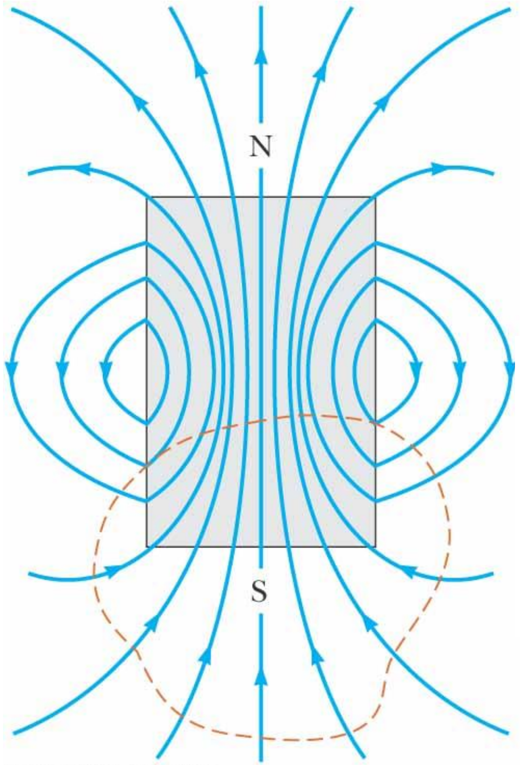
To integrate, we first express the area element (the tan region in Fig. 30.21) as $dA = b\,dr$. Because r is now the only variable in the integral, we have

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \Big]_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$

30.6 GAUSS'S LAW IN MAGNETISM

In Chapter 24 we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges.



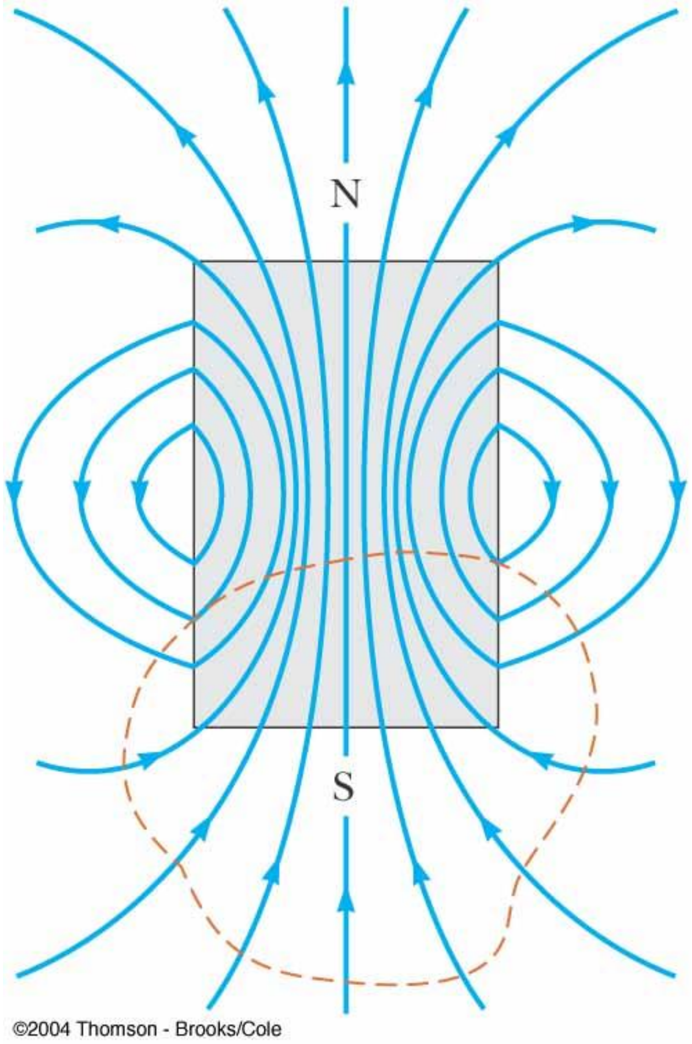
The situation is quite different for magnetic fields, which are continuous and form closed loops

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

Fig 30-23, p.942



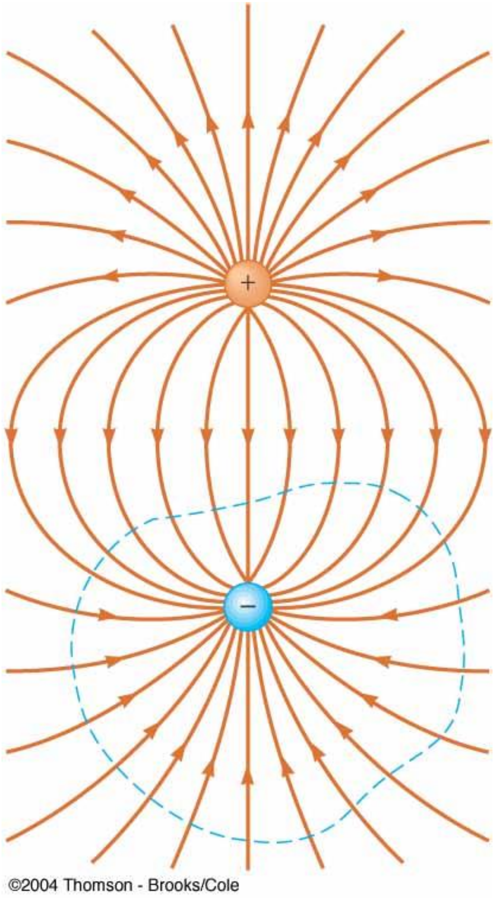


Fig 30-24, p.942

SUMMARY

The **Biot–Savart law** says that the magnetic field $d\mathbf{B}$ at a point P due to a length element $d\mathbf{s}$ that carries a steady current I is

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad \textbf{(30.1)}$$

where $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$ is the **permeability of free space,** r is the distance from the element to the point P, and $\hat{\mathbf{r}}$ is a unit vector pointing from $d\mathbf{s}$ to point P. We find the total field at P by integrating this expression over the entire current distribution.

The magnetic field at a distance a from a long, straight wire carrying an electric current I is

$$B = \frac{\mu_0 I}{2\pi a} \qquad \textbf{(30.5)}$$

The field lines are circles concentric with the wire.

The magnetic force per unit length between two parallel wires separated by a distance a and carrying currents I_1 and I_2 has a magnitude

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \tag{30.12}$$

The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Ampère's law says that the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \tag{30.13}$$

Using Ampère's law, one finds that the fields inside a toroid and solenoid are

$$B = \frac{\mu_0 N I}{2\pi r} \qquad \text{(toroid)} \tag{30.16}$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I \qquad \text{(solenoid)} \tag{30.17}$$

where N is the total number of turns.

The **magnetic flux Φ_B** through a surface is defined by the surface integral

$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} \tag{30.18}$$

Gauss's law of magnetism states that the net magnetic flux through any closed surface is zero.

$$B = \frac{\mu_0 I}{2\pi r}$$

long straight wire

use Ampere's law (or note the lack of N)

$$B = \frac{\mu_0 N I}{2a}$$

center of N loops of radius a

probably not a starting equation

$$B = \mu_0 \frac{N}{\ell} I$$

solenoid, length l, N turns

field inside a solenoid is constant

$$B = \mu_0 n I$$

solenoid, n turns per unit length

field inside a solenoid is constant

$$B = \frac{\mu_0 N I}{2\pi r}$$

toroid, N loops

field inside a toroid depends on position (r)