PHYS 221

Electromagnetism (1)
2nd semester 1446

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Lecture 4

Electric potential

25-1 Potential Difference and Electric Potential.
25-2 Potential Difference and Electric Field.
25-3 Electric Potential and Potential Energy due to Point Charges.

Electric Potential

- Electromagnetism has been connected to the study of forces in previous chapters.
- In this chapter, electromagnetism will be linked to energy.
- By using an energy approach, problems could be solved that were insoluble using forces.
- The concept of potential energy is of great value in the study of electricity.
- Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.
- This will enable the definition of electric potential.

Electrical Potential Energy

When a test charge is placed in an electric field, it experiences a force.

The force is conservative.

$$\vec{\mathbf{F}}_e = q_o \vec{\mathbf{E}}$$

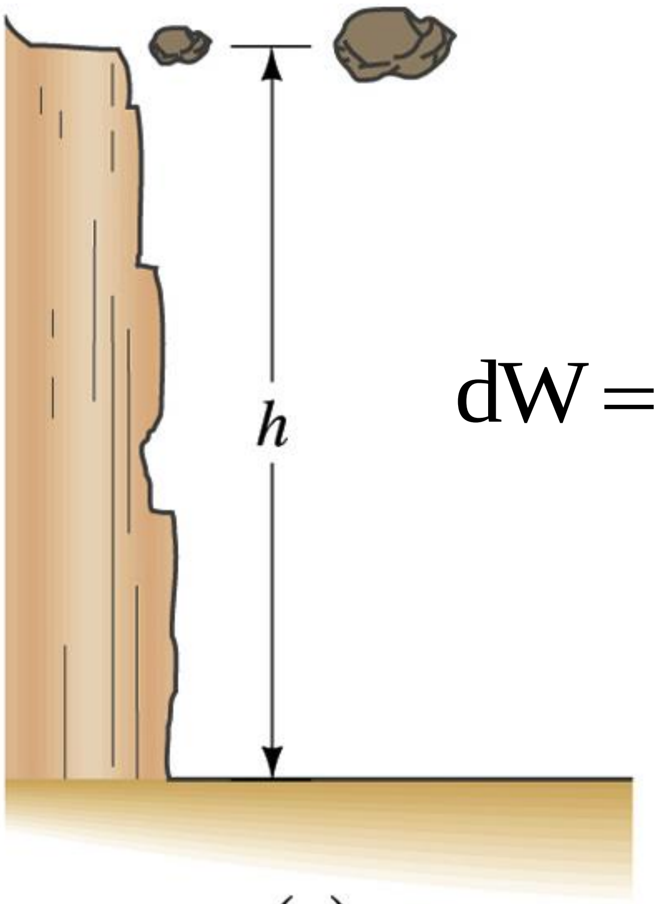
If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent.

is an infinitesimal displacement vector that is oriented tangent to a path through space.

$$d\vec{\mathbf{s}}$$

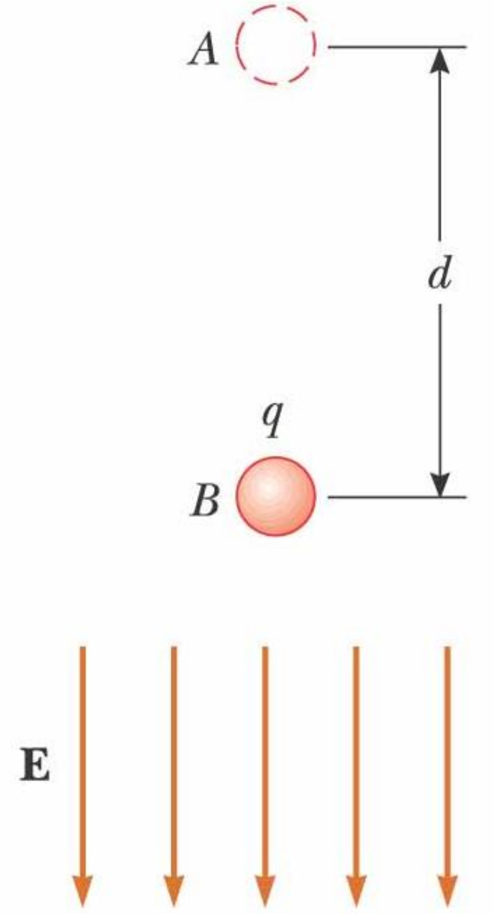
The path may be straight or curved and the integral performed along this path is called either a path integral or a line integral.

25-1 Potential Difference and Electric Potential



(a)

$$dW = -dU$$



(a)

Work and Potential Energy

$$dW = \vec{F} \cdot d\vec{s}$$

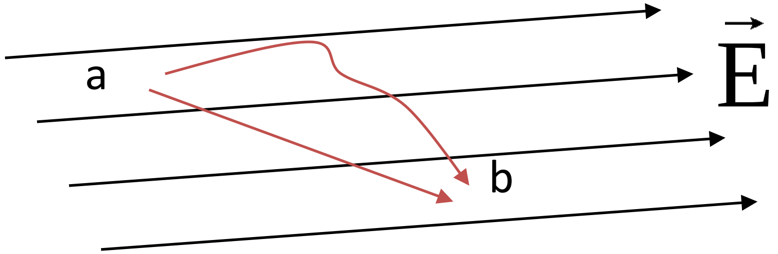
Electric Field Definition:

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}$$

$$dW = q\vec{E} \cdot d\vec{s}$$

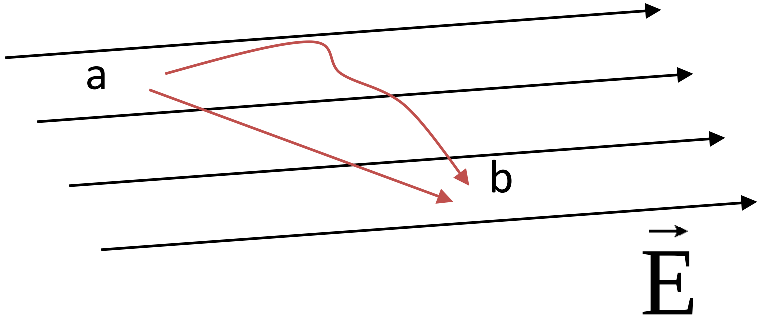
Work Energy Theorem

$$dW = -dU = q\vec{E} \cdot d\vec{s}$$



$$\int_a^b dU = -q\int_a^b \vec{E} \cdot d\vec{s}$$

Electric Potential Difference



$$\int_a^b dU = -q\int_a^b \vec{E}\cdot d\vec{s}$$

$$U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s}$$

$$\frac{U_b - U_a}{q} = -\int_a^b \vec{E}\cdot d\vec{s}$$

Definition: $$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E}\cdot d\vec{s}$$

Conventions for the potential "zero point"

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

"Potential"

Choice 1: $V_a=0$

$$V_b - V_a \equiv \frac{U_b - U_a}{q}$$

(with $V_a \to 0$ and $U_a \to 0$)

$$V_b = \frac{U_b}{q}$$

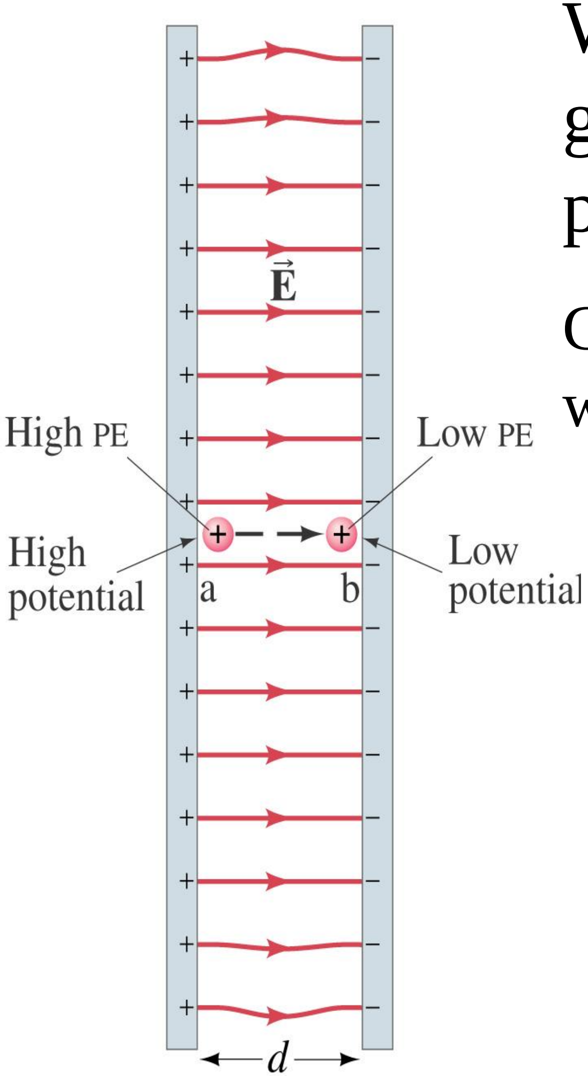
Choice 2: $V_\infty = 0$

$$V_{b\infty} = V_b - V_\infty \equiv \frac{U_b - U_\infty}{q} = -\int_\infty^b \vec{E} \cdot d\vec{s}$$

(with $V_\infty \to 0$ and $U_\infty \to 0$)

$$V_b = \frac{U_b}{q} = -\int_\infty^b \vec{E} \cdot d\vec{s}$$

25-2 Potential Difference and electric field



When a force is “conservative” ie gravitational and the electrostatic force a potential energy can be defined

Change in electric potential energy is negative of work done by electric force:

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$PE_b - PE_a = -qEd$$

$$\Delta V = -\int E\, ds = -Ed$$

•The change in potential energy is directly related to the change in voltage.

DU = q DV

DV = DU/q

- DU: change in electrical potential energy (J)
- q: charge moved (C)
- DV: potential difference (V)

•All charges will spontaneously go to lower potential energies if they are allowed to move.

Units of Potential Difference

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

$$\left[\frac{\text{Joules}}{\text{Coulomb}}\right] = \left[\frac{J}{C}\right] = \text{Volt} = V$$

Because of this, potential difference is often referred to as “voltage”

In addition, 1 N/C = 1 V/m - we can interpret the electric field as a measure of the rate of change with position of the electric potential.

So what is an electron Volt (eV)?

Electron-Volts

- Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt
- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt

  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

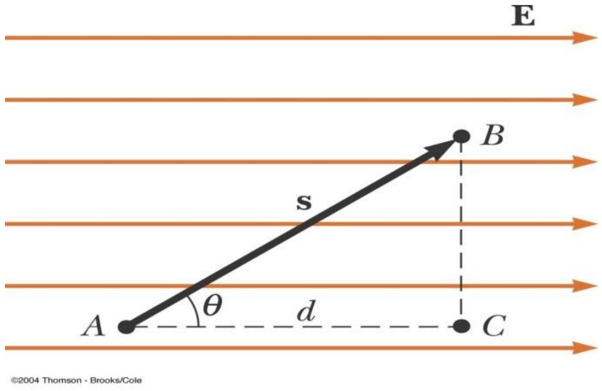
- Since all charges try to **decrease UE, and** $DU_E = qDV$, this means that *spontaneous* movement of charges result in **negative** DU.
- **DV = DU / q**
- Positive charges like to DECREASE their potential $(DV < 0)$
- Negative charges like to INCREASE their potential. $(DV > 0)$

$$\Delta V = V_B - V_A = -\int_A^B \overrightarrow{E}\,.\overrightarrow{ds} = -\overrightarrow{E}\,.\int_A^B \overrightarrow{ds} = -\overrightarrow{E}\,.\overrightarrow{d} = E\,s\cos\theta$$

$V_B - V_A = V_C - V_A$

$V_B = V_C$

A uniform electric field directed along the positive *x* axis. Point *B* is at a lower electric potential than point A. Points *B* and *C* are at the *same* electric potential.



Example

If a 9 V battery has a charge of 46 C how much chemical energy does the battery have?

$$E = V \times Q = 9\ V \times 46C = 414\ Joules$$

Example

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m$$

$$\Delta V = 600V$$

$$E = ?$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = 113{,}207.55 N / C$$

$$q_{e^-} = 1.6x10^{-19} C$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19} C}$$

$$F_e = 1.81 \times 10^{-14}\ \text{N}$$

Example

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_{p^+} = 1.6x10^{-19} C$$

$$m_{p^+} = 1.67x10^{-27} kg$$

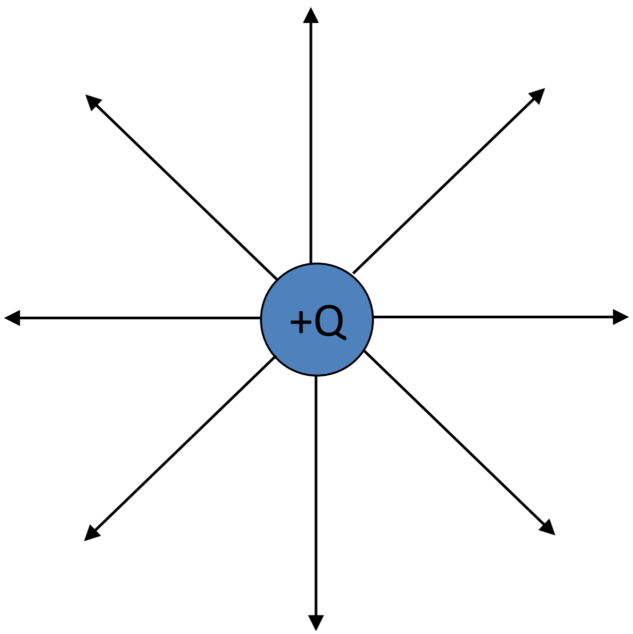
$$V = 120V$$

$$v = ?$$

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{{}^{1}\!/\!{}_{2} m v^2}{q}$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = 1.52 \times 10^{5} \text{ m/s}$$

25-3 Electric Potential and Potential energy due to point charges

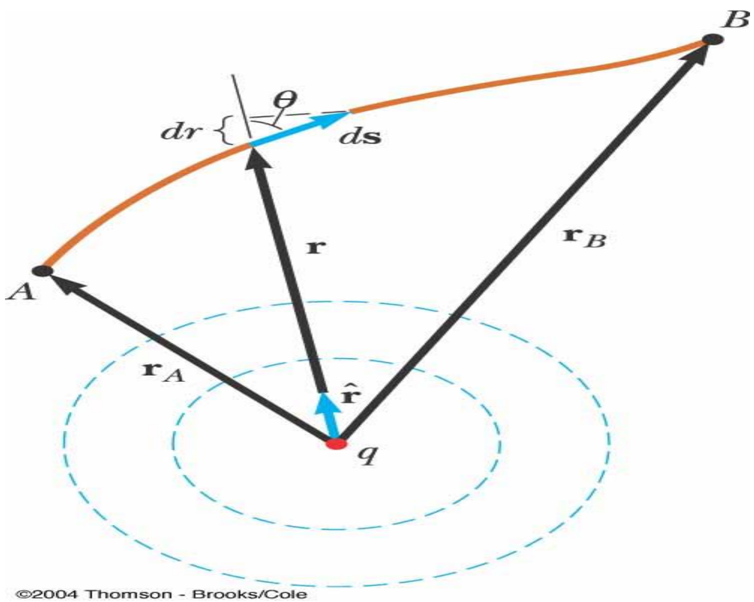


$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$V_{ba} = V_b - V_a = -\frac{kq}{r^2}\int_a^b \hat{r} \cdot d\vec{s}$$

d*s* for a point charge



Recall the convention for the potential "zero point"

$$V_{ba} = V_b - V_a = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

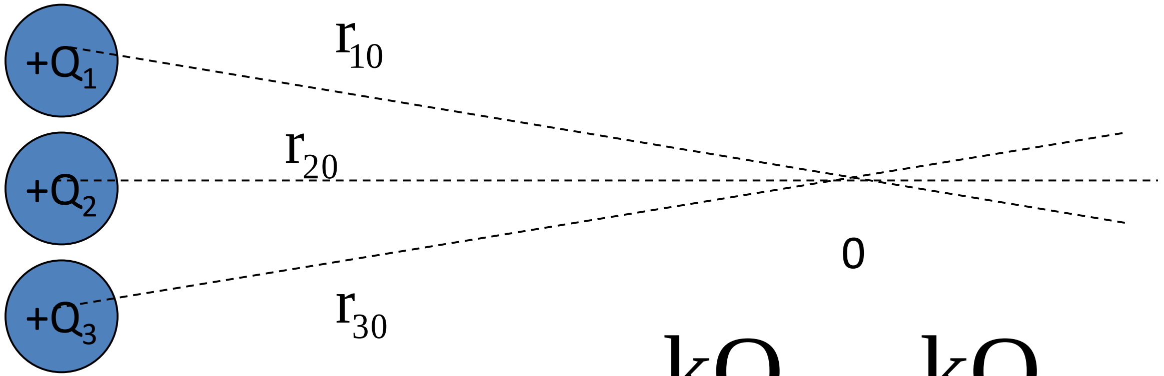
$$V_\infty = 0 \qquad V_{b\infty} = V_b - V_\infty = kq\left(\frac{1}{r_b} - \frac{1}{\infty}\right)$$

$$V(r) = \frac{kq}{r}$$

Equipotential surfaces are concentric spheres

Superposition of potentials

$$V_0 = V_1 + V_2 + V_3 + \ldots$$

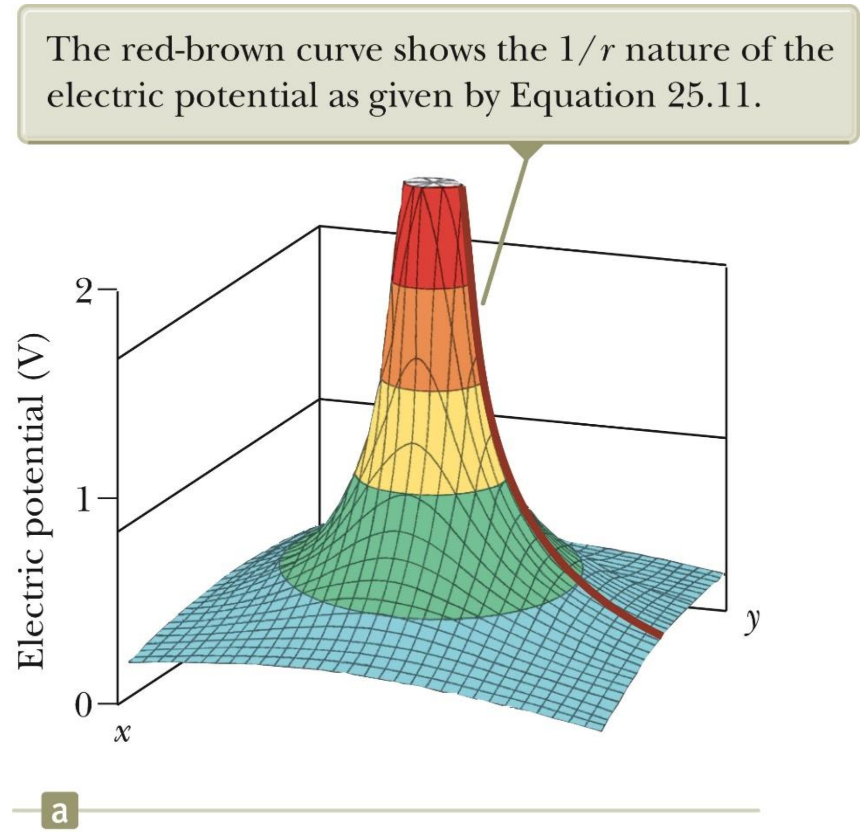


$$V_0 = \frac{kQ_1}{r_{10}} + \frac{kQ_2}{r_{20}} + \frac{kQ_3}{r_{30}} + \ldots$$

$$V_0 = \sum_{i=1}^{N} \frac{kQ_i}{r_{i0}}$$

Electric Potential of a Point Charge

- The electric potential in the plane around a single point charge is shown.
- The red line shows the 1/*r* nature of the potential.



Electric Potential with Multiple Charges

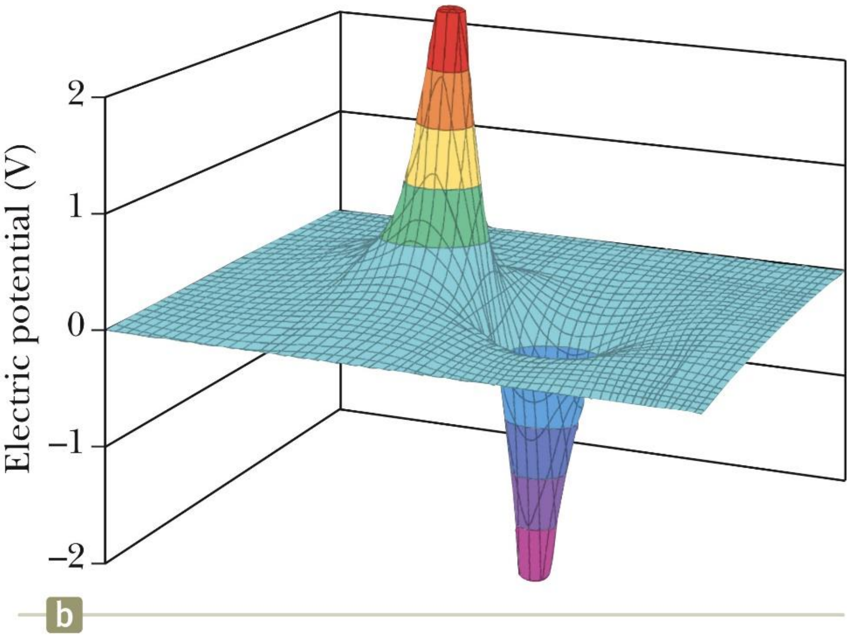
- The electric potential due to several point charges is the sum of the potentials due to each individual charge.
  - This is another example of the superposition principle.
  - The sum is the algebraic sum

$$V = k_e \sum_i \frac{q_i}{r_i}$$

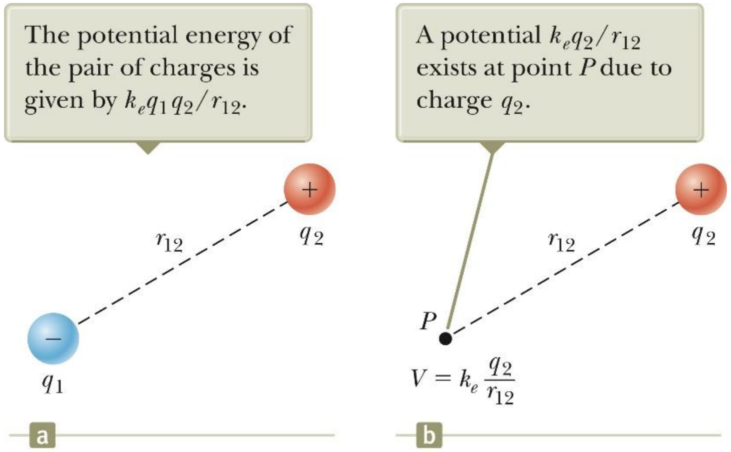
- $V = 0$ at $r = \infty$

Electric Potential of a Dipole

- The graph shows the potential (y-axis) of an electric dipole.
- The steep slope between the charges represents the strong electric field in this region.



Potential Energy of Multiple Charges



- The potential energy of the system is $U = k_e \frac{q_1 q_2}{r_{12}}$

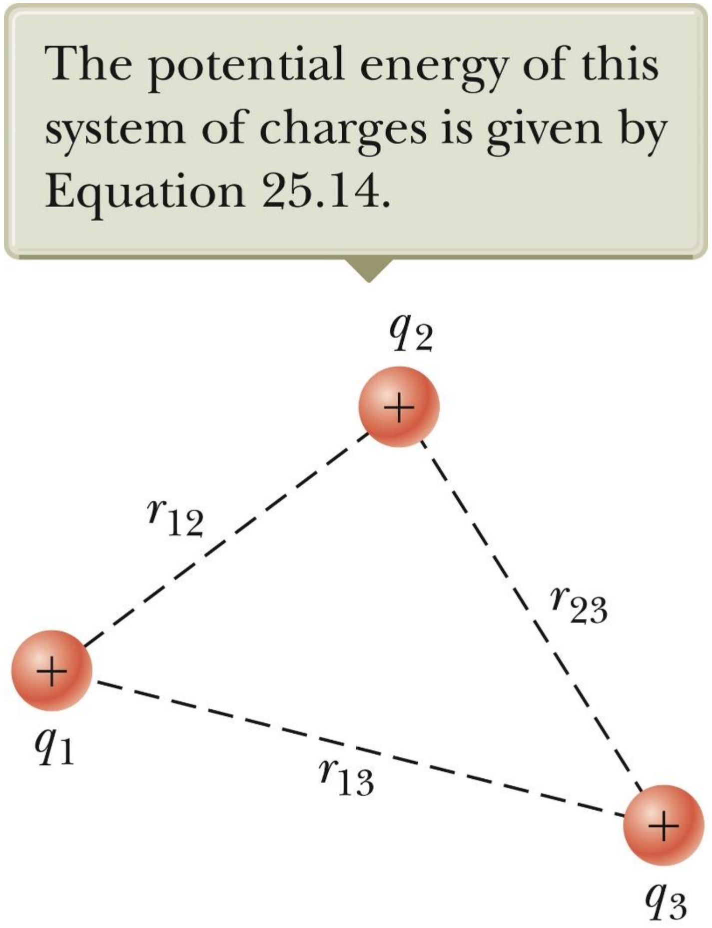
- If the two charges are the same sign, *U* is positive and work must be done to bring the charges together.
- If the two charges have opposite signs, *U* is negative and work is done to keep the charges apart.

U with Multiple Charges, final

- If there are more than two charges, then find *U* for each pair of charges and add them.
- For three charges:

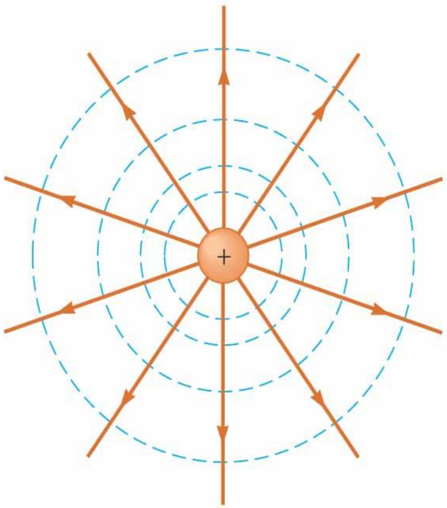
$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

  - The result is independent of the order of the charges.



E and *V* for a Point Charge

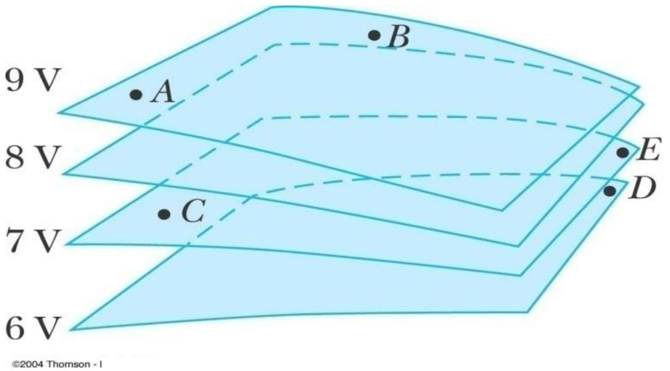
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines

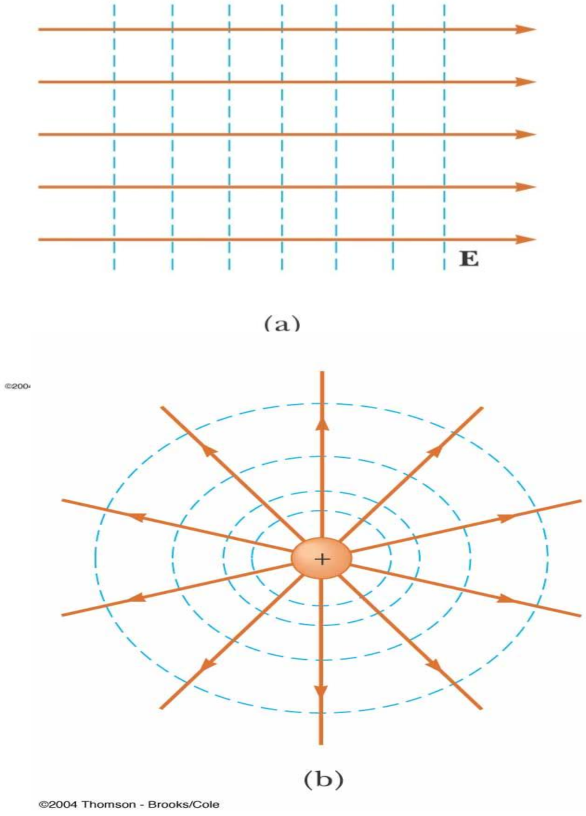


(b)

An ***equipotential*** ***surface*** is a surface on which the electric potential is the same everywhere.

Figure 25.4 (Quick Quiz 25.3) Four equipotential surfaces

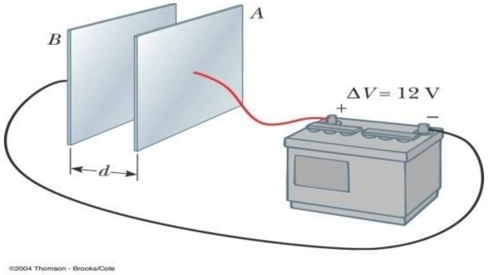




Equipotential surfaces (the dashed blue lines are intersections of these surfaces with the page) and electric field lines (red- rown lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point

Example (25.1)

A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference *V* divided by the plate separation d =0.3 cm



$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = 4.0 \times 10^3\text{ V/m}$$

Example (25.2)

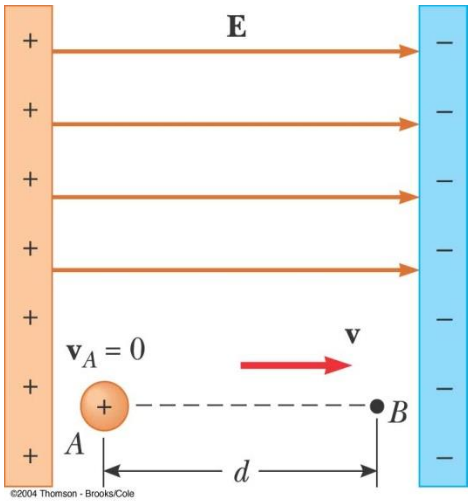
A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of **E**.

(A) Find the change in electric potential between points *A* and *B*.

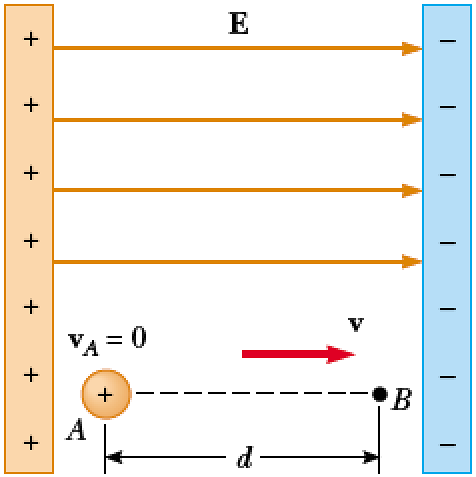
$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m}) = -4.0 \times 10^4\text{ V}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= -6.4 \times 10^{-15}\text{ J}$$



The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.



(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-(2e\,\Delta V)}{m}}$$

$$= \sqrt{\frac{-2(1.6 \times 10^{-19}\ \mathrm{C})(-4.0 \times 10^4\ \mathrm{V})}{1.67 \times 10^{-27}\ \mathrm{kg}}}$$

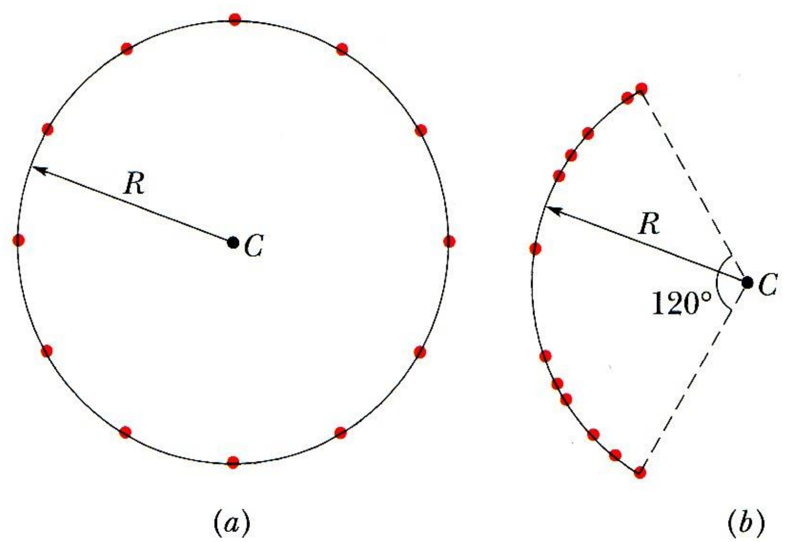
$$= 2.8 \times 10^6\ \mathrm{m/s}$$

Example: (a) In figure a, 12 electrons are equally spaced and fixed around a circle of radius *R*. Relative to *V*=0 at infinity, what are the electric potential and electric field at the center *C* of the circle due to these electrons? (b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° are (figure b), what then is the potential at *C*?

Solution:

$$(a): \quad V = -K\frac{12e}{R} \qquad \mathbf{E} = 0$$

$$(b): \quad V = -K\frac{12e}{R}$$

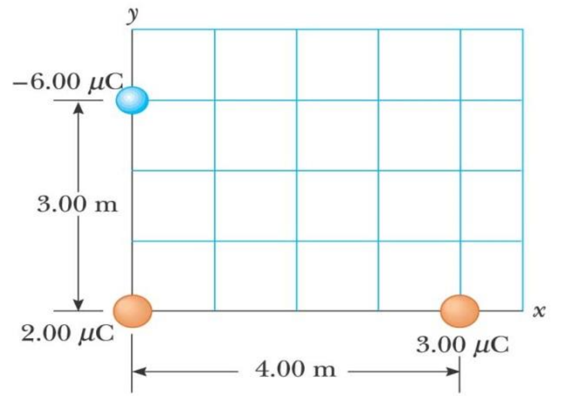


Potential due to a group of point charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i}^{n}\frac{q_i}{r_i}$$

Example (25.3)

(a) The electric potential at *P* due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge q_3 = 3.00 C is brought from infinity to a position near the other charges.



(b)

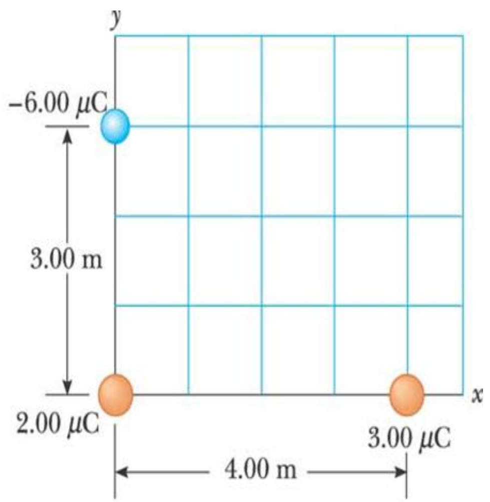
Solution For two charges, the sum

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)$$

$$\times\left(\frac{2.00 \times 10^{-6}\text{ C}}{4.00\text{ m}} - \frac{6.00 \times 10^{-6}\text{ C}}{5.00\text{ m}}\right)$$

$$= -6.29 \times 10^3 \text{ V}$$



(b)

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P

Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ C)(-6.29 \times 10^3\ V)$$

$$= -1.89 \times 10^{-2}\ J$$

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

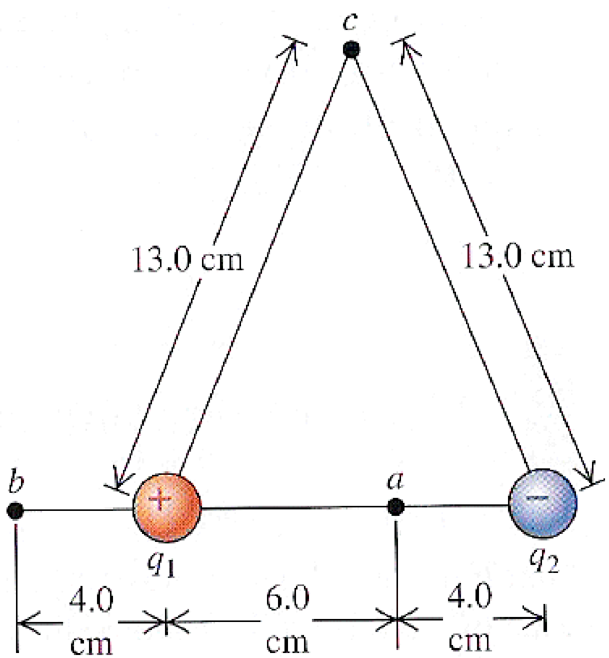
$$= (8.99 \times 10^9\ N \cdot m^2/C^2)$$

$$\times \left(\frac{(2.00 \times 10^{-6}\ C)(-6.00 \times 10^{-6}\ C)}{3.00\ m} + \frac{(2.00 \times 10^{-6}\ C)(3.00 \times 10^{-6}\ C)}{4.00\ m} + \frac{(3.00 \times 10^{-6}\ C)(-6.00 \times 10^{-6}\ C)}{5.00\ m} \right)$$

$$= -5.48 \times 10^{-2}\ J$$

Example

An electric dipole consists of two charges q_1 = +12nC and q_2 = -12nC, placed 10 cm apart as shown in the figure. Compute the potential at points a,b, and c.



$$V_a = k\sum\left(\frac{q_1}{r_a} + \frac{q_2}{r_a}\right)$$

$$V_a = 8.99x10^9\left(\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04}\right)$$

$$V_a = -899\ \mathrm{V}$$

$$V_b = k\sum\left(\frac{q_1}{r_b} + \frac{q_2}{r_b}\right)$$

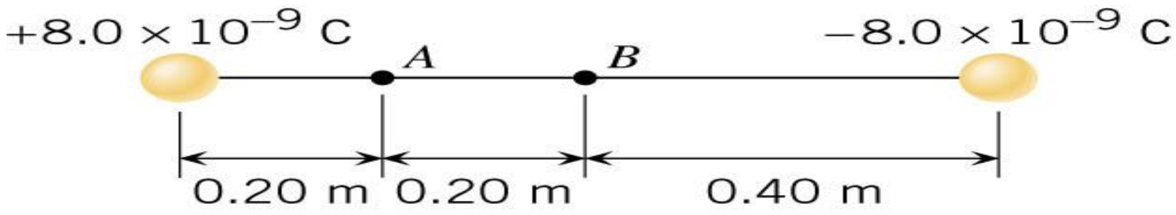
$$V_b = 8.99x10^9\left(\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14}\right)$$

$$V_b = 1926.4\ V$$

$$V_c = 0\ \mathrm{V}$$

Example **The Total Electric Potential**

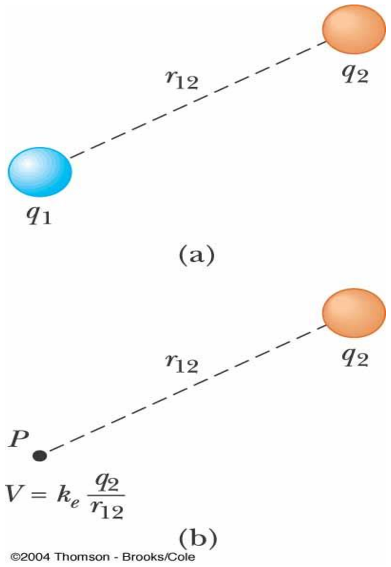
At locations A and B, find the total electric potential.



$$V_A = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.20\,\mathrm{m}} + \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.60\,\mathrm{m}} = +240\ \mathrm{V}$$

$$V_B = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\,\mathrm{m}} + \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\,\mathrm{m}} = 0\ \mathrm{V}$$

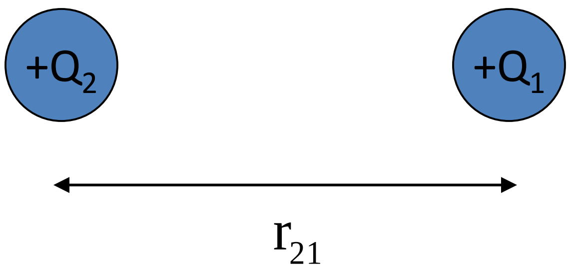
(a) If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$. (b) If charge q_1 is removed, a potential $k_e q_2 / r_{12}$ exists at point *P* due to charge q_2 .



$$U = V q_1$$

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

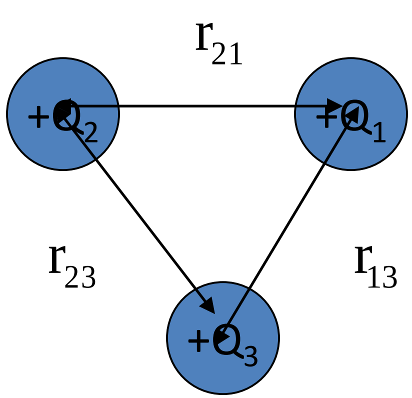
Potential energy due to multiple point charges



$$V(r) = \frac{kq}{r}$$

$$V = \frac{kq_1}{r_{12}}$$

$$U = q_2 V = \frac{kq_1 q_2}{r_{12}}$$

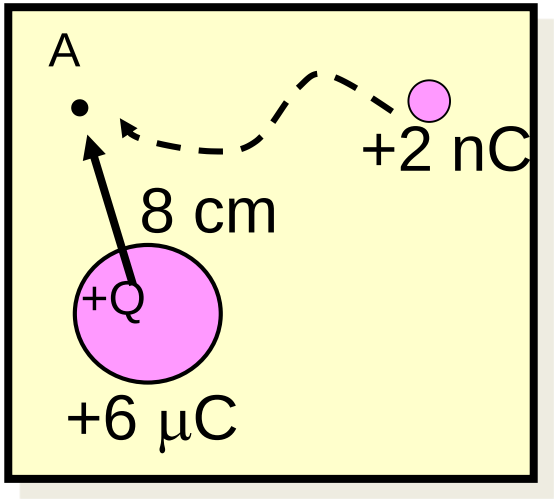


$$V = \frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}$$

$$U = \frac{kq_1 q_2}{r_{12}} + \frac{kq_1 q_3}{r_{13}} + \frac{kq_2 q_3}{r_{23}}$$

Example 1. What is the potential energy if a +2 nC charge moves from ∞ to point A, 8 cm away from a +6 μC charge?

The P.E. will be positive at point A, because the field can do + work if q is released.



Potential Energy:

$$U = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(+6 \text{ x } 10^{-6}\text{C})(+2 \text{ x } 10^{-9}\text{C})}{(0.08 \text{ m})}$$

U = 1.35 mJ

Positive potential energy

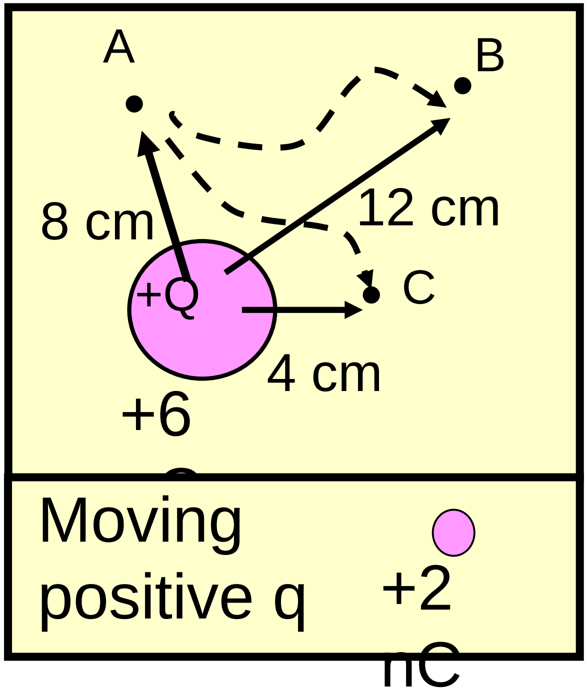
Signs for Potential Energy

Consider Points A, B, and C.

For +2 nC at A: U = +1.35 mJ

Questions:

If +2 nC moves from A to B, does field E do + or – work? Does P.E. increase or decrease?



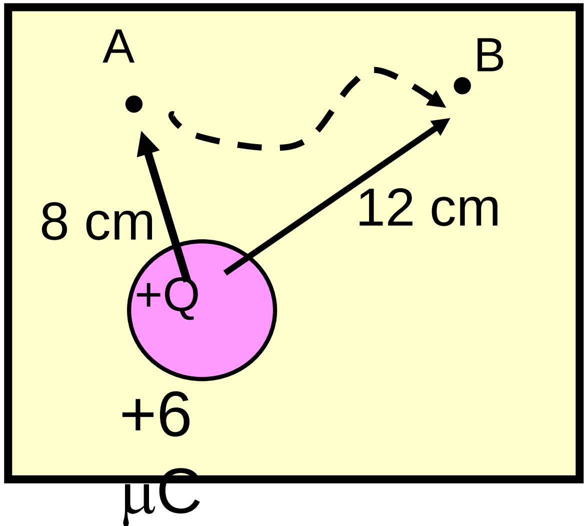
The field E does positive work, the P.E. decreases.

If +2 nC moves from A to C (closer to +Q), the field E does negative work and P.E. increases.

Example. What is the change in potential energy if a +2 nC charge moves from A to B?

Potential Energy:

$$U = \frac{kQq}{r}$$



From Ex-1: U_A = + 1.35 mJ

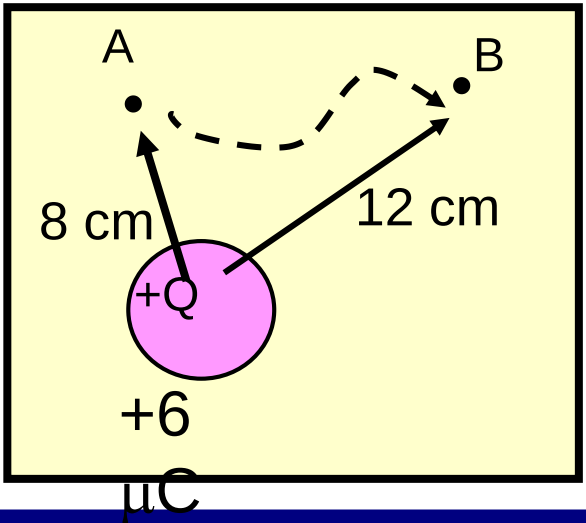
$$U_B = \frac{(9 \text{ x } 19^9 \frac{\text{Nm}^2}{\text{C}^2})(+6 \text{ x } 10^{-6}\text{C})(+2 \text{ x } 10^{-9}\text{C})}{(0.12 \text{ m})} = 0.900 \text{ mJ}$$

$\Delta U = U_B - U_A$ = 0.9 mJ – 1.35 mJ

ΔU = -0.450 mJ

Note that P.E. has decreased as work is done by E.

Example What is the change in potential energy if a -2 nC charge moves from A to B?



Potential Energy:

From Ex-1: U_A = -1.35 mJ

(Negative due to – charge)

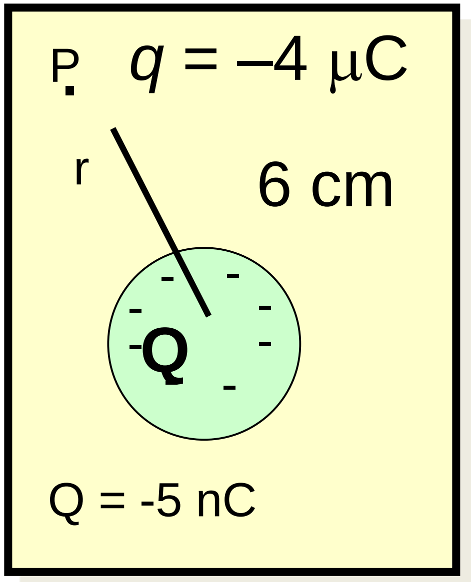
$$U_B = \frac{(9 \text{ x } 19^9 \ \frac{\text{Nm}^2}{\text{C}^2})(6 \text{ x } 10^{-6}\text{C})(-2 \text{ x } 10^{-9}\text{C})}{(0.12 \text{ m})} = -0.900 \text{ mJ}$$

$U_B - U_A$ = -0.9 mJ – (-1.35 mJ)

ΔU = +0.450 mJ

A – charge moved away from a + charge gains P.E.

Example : Find the potential at a distance of 6 cm from a –5 nC charge.



$$V = \frac{kQ}{r} = \frac{\left(9 \times 10^{9}\ \mathrm{Nm^2/C^2}\right)(-5 \times 10^{-9}\,\mathrm{C})}{(0.06\ \mathrm{m})}$$

Negative *V* at Point *P* :

$V_P = -750$ V

What would be the P.E. of a –4 μC charge placed at this point P?

***U* = *qV* = (-4 x 10⁻⁶ μC)(-750 V);**

U = 3.00 mJ

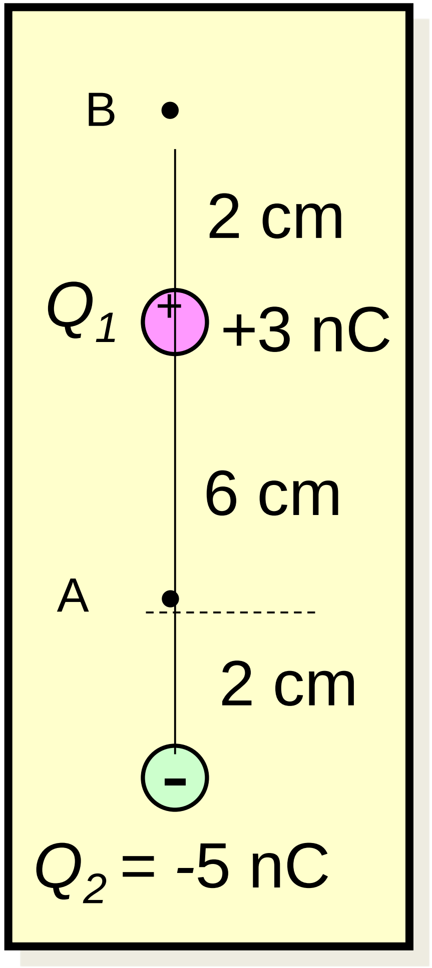
Since P.E. is positive, E will do + work if *q* is released.

Example : Two charges Q_1= +3 nC and Q_2 = -5 nC are separated by 8 cm. Calculate the electric potential at point A.

$$V_A = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$\frac{kQ_1}{r_1} = \frac{\left(9 \text{ x } 10^9 \ \mathrm{Nm^2/C^2}\right)(+3 \text{ x } 10^{-9}\mathrm{C})}{(0.06 \text{ m})} = +450 \text{ V}$$

$$\frac{kQ_2}{r_2} = \frac{\left(9 \text{ x } 10^9 \ \mathrm{Nm^2/C^2}\right)(-5 \text{ x } 10^{-9}\mathrm{C})}{(0.02 \text{ m})} = -2250 \text{ V}$$



V_A = 450 V – 2250 V;

V_A = -1800 V

Example Calculate the electric potential at point B for same charges.

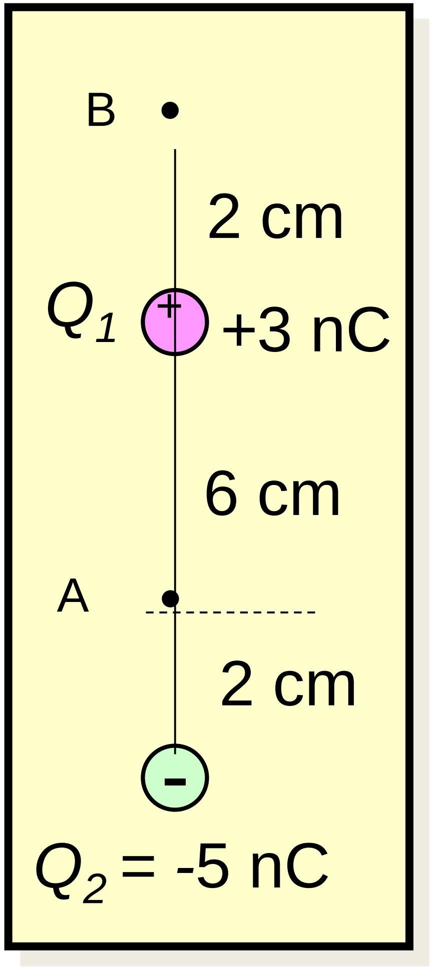
$$V_B = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$\frac{kQ_1}{r_1} = \frac{\left(9 \text{ x } 10^9 \text{ Nm}^2/\text{C}^2\right)(+3 \text{ x } 10^{-9}\text{C})}{(0.02 \text{ m})} = +1350 \text{ V}$$

$$\frac{kQ_2}{r_2} = \frac{\left(9 \text{ x } 10^9 \text{ Nm}^2/\text{C}^2\right)(-5 \text{ x } 10^{-9}\text{C})}{(0.10 \text{ m})} = -450 \text{ V}$$

V_B = 1350 V – 450 V;

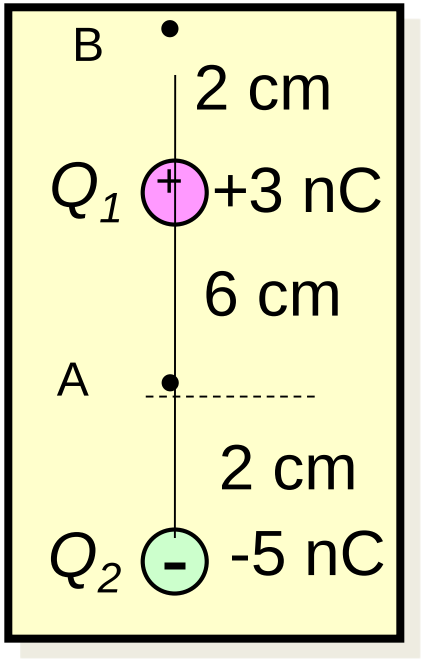
V_B = +900 V



Example : What is the potential difference between points A and B. What work is done by the E-field if a +2 μC charge is moved from A to B?

V_A = -1800 V

V_B = +900 V



$V_{AB} = V_A – V_B$ = -1800 V – 900 V

V_{AB} = -2700 V

Note point B is at higher potential.

$Work_{AB} = q(V_A – V_B) = (2 \times 10^{-6}\text{ C })(-2700\text{ V})$

Work = -5.40 mJ

E-field does negative work.

Thus, **an external force** was required to move the charge.

<u>Example 6 (Cont.):</u> Now suppose the +2 μC charge is moved from back from B to A?

V_A = -1800 V

V_B = +900 V

$V_{BA} = V_B - V_A = 900\text{ V} - (-1800\text{ V})$

V_{BA} = +2700 V

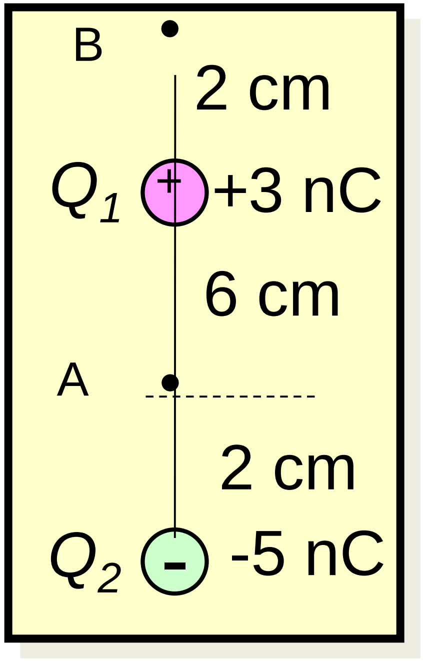
This path is from high to low potential.

$Work_{BA} = q(V_B - V_A) = (2 \times 10^{-6}\text{ C})(+2700\text{ V})$

Work = +5.40 mJ

E-field does positive work.

The work is done BY the E-field this time !



Example

An electron is accelerated in a TV tube through a potential difference of 5000 V.

a) What is the change in PE of the electron?

$V = \Delta PE/q$

$\Delta PE = qV = (-1.60 \times 10^{-19}\ C)(+5000\ V)$ **= -8.0 X 10^-16 J** $\mathbf{= -8.0 \times 10^{-16}\ J}$

What is the final speed of the electron ($m = 9.1 \times 10^{-31}$ kg)

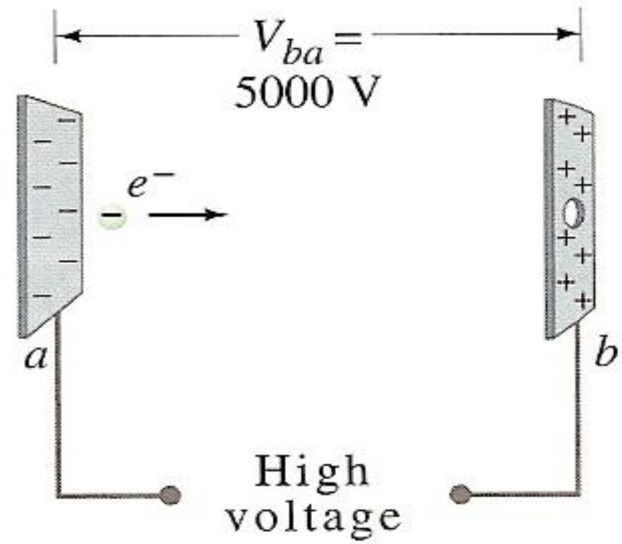
$\Delta PE + \Delta KE = 0$ (Law of conservation of energy)

$\Delta PE = -\Delta KE$

$\Delta PE = -\frac{1}{2} mv^2$

$$v^2 = \frac{(-2)(\Delta PE)}{m} = \frac{(-2)(-8.0 \times 10^{-16}\ J)}{9.1 \times 10^{-31}\ kg}$$

$\mathbf{v = 4.2 \times 10^7\ m/s}$



Summary

- Electric potential energy: $$\mathrm{PE_b} - \mathrm{PE_a} = -qEd$$

- Electric potential difference: work done to move charge from one point to another

- Relationship between potential difference and field:

$$E = -\frac{V_{\mathrm{ba}}}{d}$$

- Equipotential: line or surface along which potential is the same

- Electric potential of a point charge:

$$V = k\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$