PHYS 221

Electromagnetism (1)
2^{nd} semester 1446

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Lecture 2

Electric Field – Introduction

- The electric force is a field force.
- Field forces can act through space.
    - The effect is produced even with no physical contact between objects.
- Faraday developed the concept of a field in terms of electric fields.

Electric Field – Definition

- An **electric field** is said to exist in the region of space around a charged object.
  - This charged object is the **source charge.**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

Electric Field – Definition, cont

- The electric field is defined as the electric force on the test charge per unit charge.

$$\vec{\mathbf{E}}$$

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

- The electric field vector, , at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

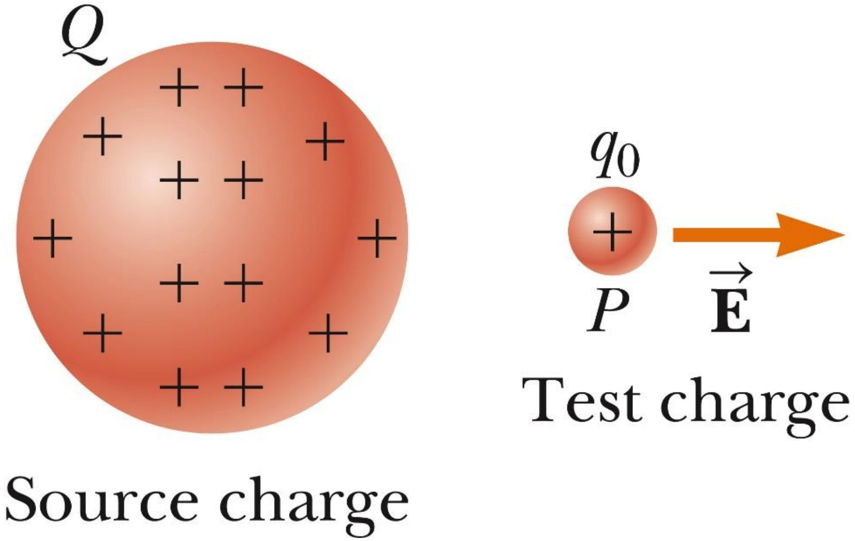
Electric Field, Notes

$\vec{E}$

- is the field produced by some charge or charge distribution, separate from the test charge.
- The existence of an electric field is a property of the source charge.
  - The presence of the test charge is not necessary for the field to exist.
- The test charge serves as a detector of the field.

Electric Field Notes, Final

- The direction of $\vec{E}$ is that of the force on a positive test charge.
- The SI units of $\vec{E}$ are N/C.
- We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.



Relationship Between F and E

- $$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$$
  - This is valid for a point charge only.
  - One of zero size
  - For larger objects, the field may vary over the size of the object.

•If q is positive, the force and the field are in the same direction.

•If q is negative, the force and the field are in opposite directions.

Electric Field, Vector Form

- Remember Coulomb's law, between the source and test charges, can be expressed as

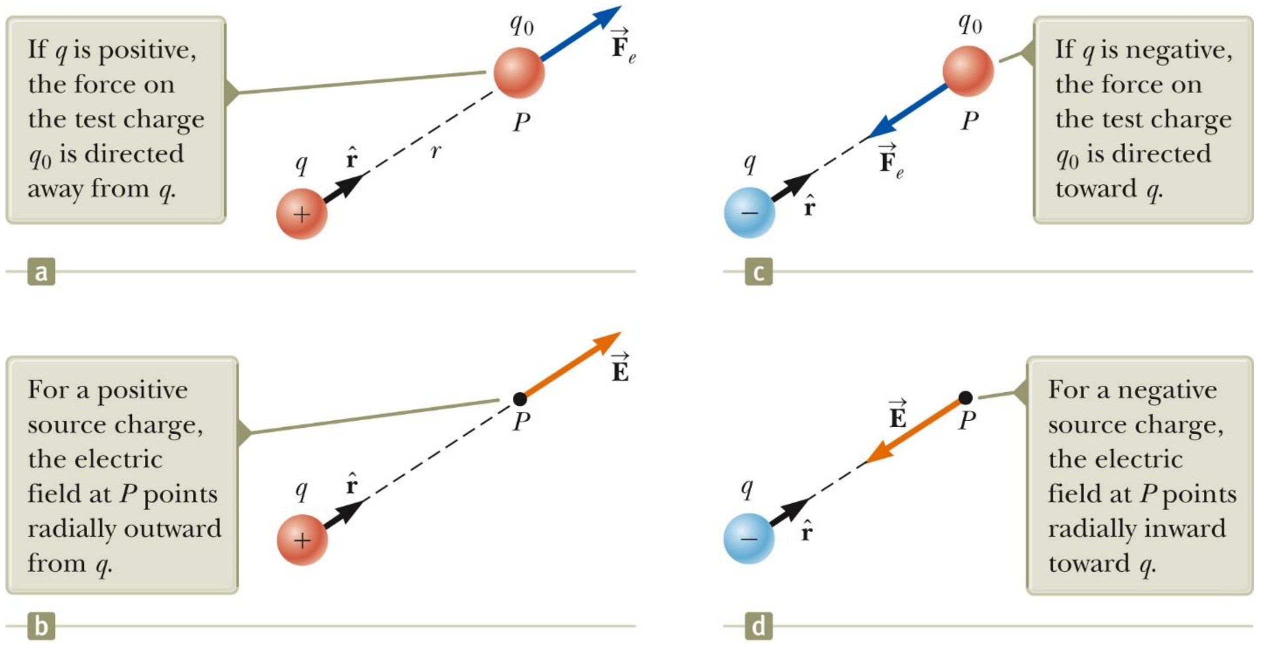
$$\vec{\mathbf{F}}_e = k_e \frac{qq_o}{r^2}\hat{\mathbf{r}}$$

- Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2}\hat{\mathbf{r}}$$

More About Electric Field Direction

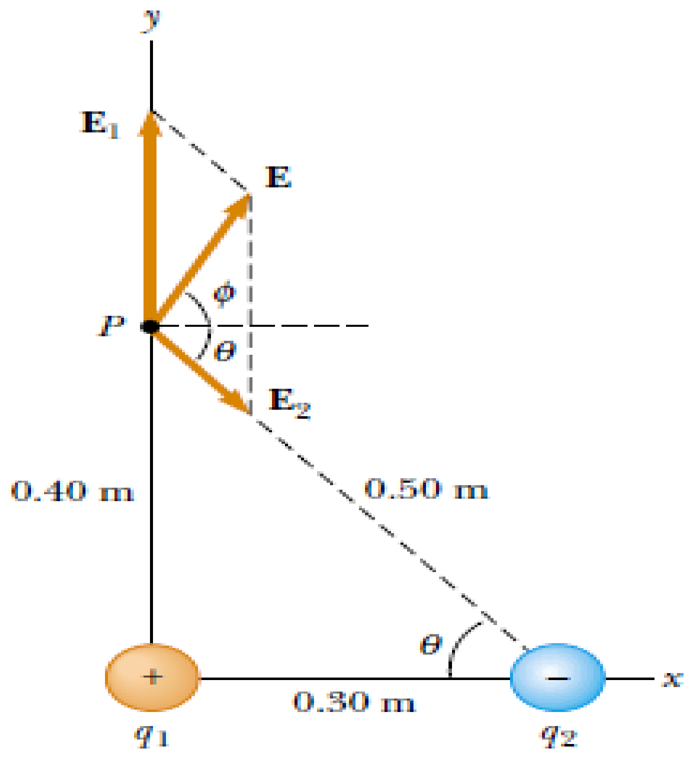
- a) q is positive, the force is directed away from q.
- b) The direction of the field is also away from the positive source charge.
- c) q is negative, the force is directed toward q.
- d) The field is also toward the negative source charge.



Section 23.4

Ex 4

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point P, which has coordinates (0, 0.40) m.



$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(7.0 \times 10^{-6}\text{ C})}{(0.40\text{ m})^2}$$

$$= 3.9 \times 10^5 \text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(5.0 \times 10^{-6}\text{ C})}{(0.50\text{ m})^2}$$

$$= 1.8 \times 10^5 \text{ N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

From this result, we find that $\mathbf{E}$ makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C.

Electric Fields from Multiple Charges

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

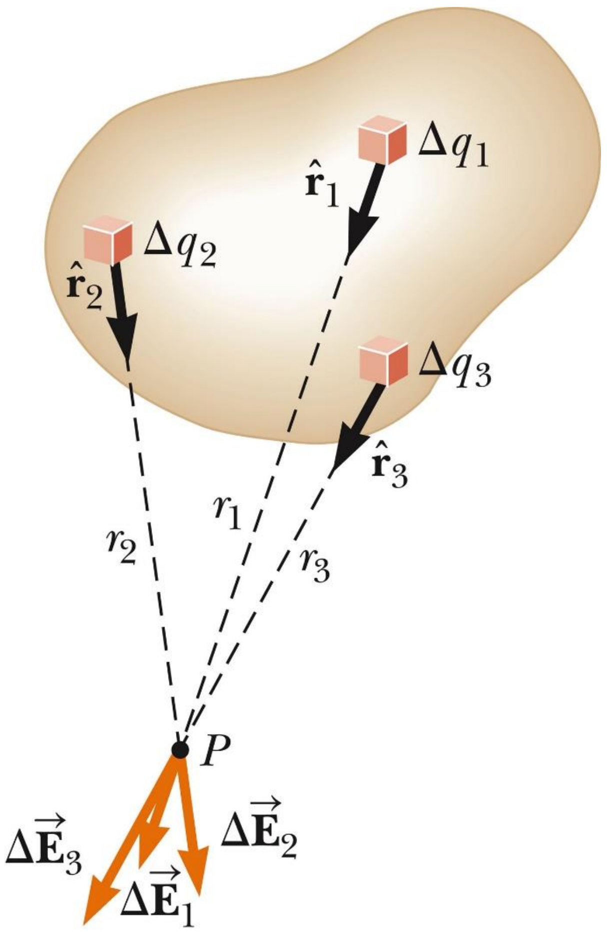
$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

Electric Field – Continuous Charge Distribution

- The distances between charges in a group of charges may be much smaller than the distance between the group and a point of interest.
- In this situation, the system of charges can be modeled as continuous.
- The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

Electric Field – Continuous Charge Distribution, cont

- Procedure:
  - Divide the charge distribution into small elements, each of which contains Δq.
  - Calculate the electric field due to one of these elements at point *P*.
  - Evaluate the total field by summing the contributions of all the charge elements.



Electric Field – Continuous Charge Distribution, equations

- For the individual charge elements

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Because the charge distribution is continuous

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge Densities

- **Volume charge density**: when a charge is distributed evenly throughout a volume
  - $\rho \equiv Q / V$ with units C/m^3
- **Surface charge density**: when a charge is distributed evenly over a surface area
  - $\sigma \equiv Q / A$ with units C/m^2
- **Linear charge density**: when a charge is distributed along a line
  - $\lambda \equiv Q / \ell$ with units C/m

Amount of Charge in a Small Volume

- If the charge is nonuniformly distributed over a volume, surface, or line, the amount of charge, *dq*, is given by
  - For the volume: $dq = \rho\, dV$
  - For the surface: $dq = \sigma\, dA$
  - For the length element: $dq = \lambda\, d\ell$

Problem-Solving Strategy

- *Conceptualize*
  - Establish a mental representation of the problem.
  - Image the electric field produced by the charges or charge distribution.
- *Categorize*
  - Individual charge?
  - Group of individual charges?
  - Continuous distribution of charges?

Problem-Solving Strategy, cont

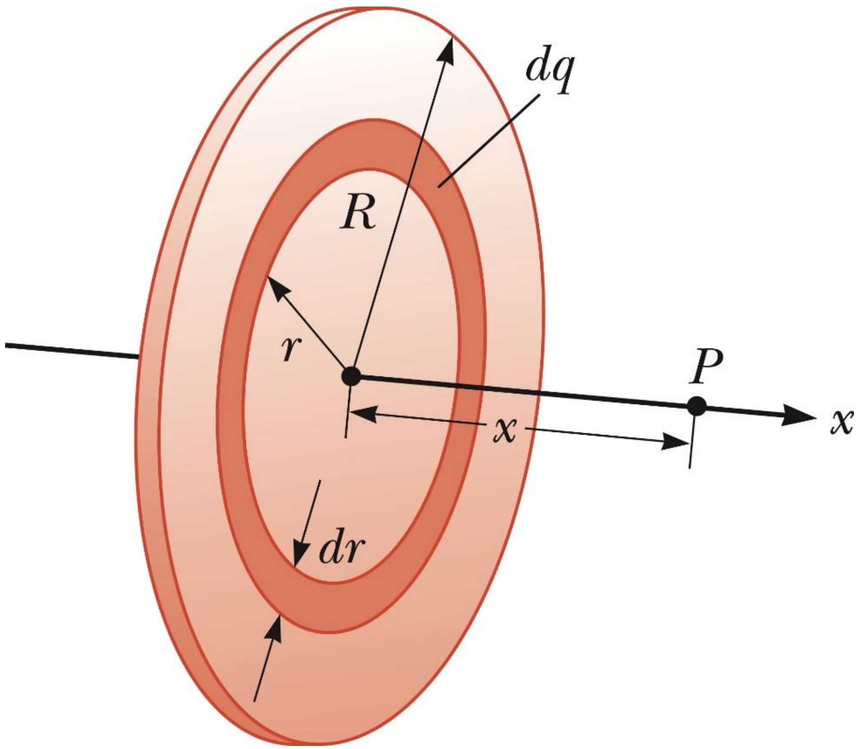
- *Analyze*
  - **Analyzing a group of individual charges**:
    - Use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field.
    - Be careful with the manipulation of vector quantities.
  - **Analyzing a continuous charge distribution:**
    - The vector sums for evaluating the total electric field at some point must be replaced with vector integrals.
    - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution.
  - **Symmetry:**
    - Take advantage of any symmetry to simplify calculations.

Problem Solving Hints, final

- *Finalize*
  - Check to see if the electric field expression is consistent with your mental representation.
  - Check to see if the solution reflects any symmetry present.
  - Image varying parameters to see if the mathematical result changes in a reasonable way.

Example – Charged Disk

- The disk has a radius *R* and a uniform charge density σ.
- Choose *dq* as a ring of radius *r*.
- The ring has a surface area $2\pi r\, dr$.
- Integrate to find the total field.



Ex.8 A ring of radius *a* carries a uniformly distributed positive total charge *Q*. Calculate the electric field due to the ring at a point *P* lying a distance *x* from its center along the central axis perpendicular to the plane of the ring (Fig. 23.18a).

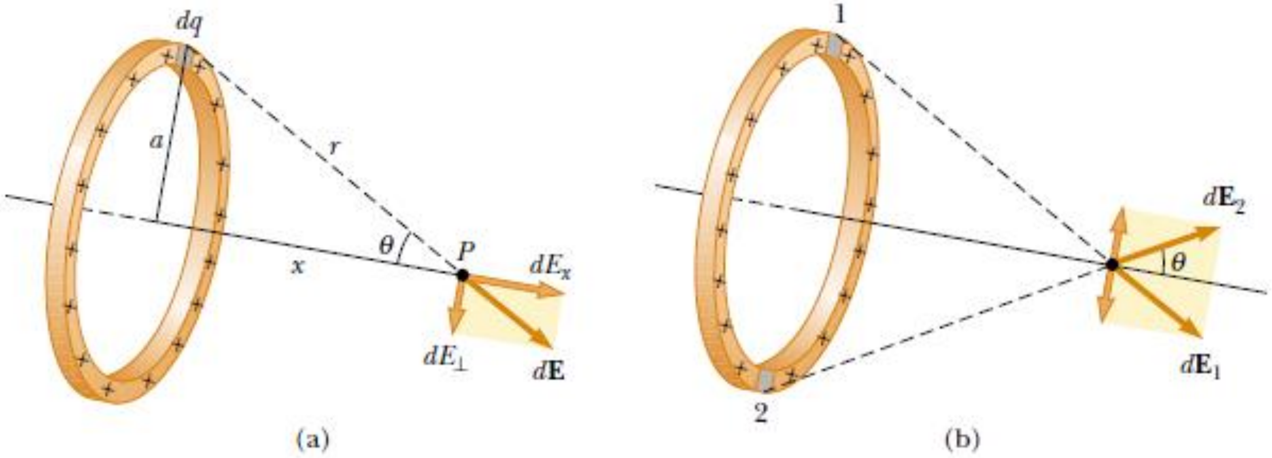


Figure 23.18 (Example 23.8) A uniformly charged ring of radius *a*. (a) The field at *P* on the *x* axis due to an element of charge *dq*. (b) The total electric field at *P* is along the *x* axis. The perpendicular component of the field at *P* due to segment 1 is canceled by the perpendicular component due to segment 2.

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the x axis and a component $dE_\perp$ perpendicular to the x axis. As we see in Figure 23.18b, however, the resultant field at P must lie along the x axis because the perpendicular components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$

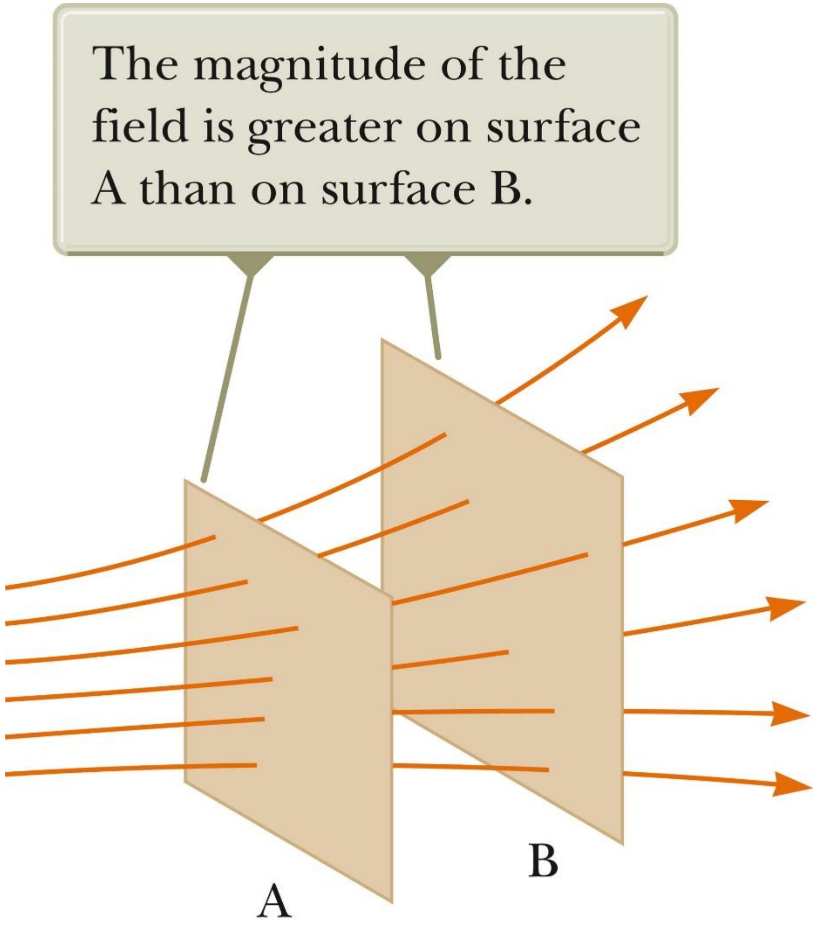
$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q$$

Electric Field Lines

- Field lines give us a means of representing the electric field pictorially.
- The electric field vector is tangent to the electric field line at each point.
  - The line has a direction that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

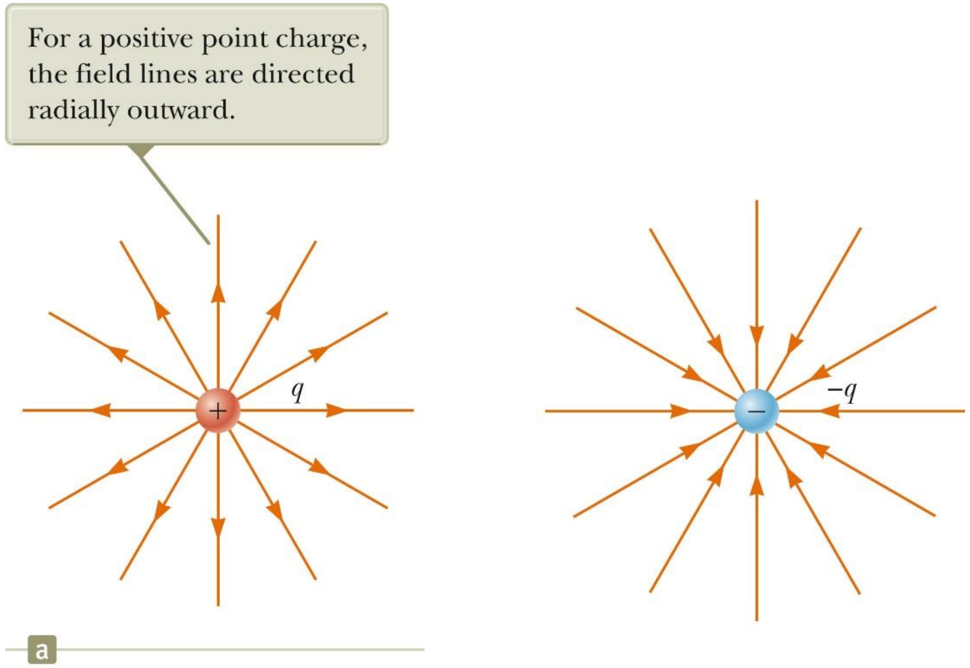
Electric Field Lines, General

- The density of lines through surface A is greater than through surface B.
- The magnitude of the electric field is greater on surface A than B.
- The lines at different locations point in different directions.
  - This indicates the field is nonuniform.



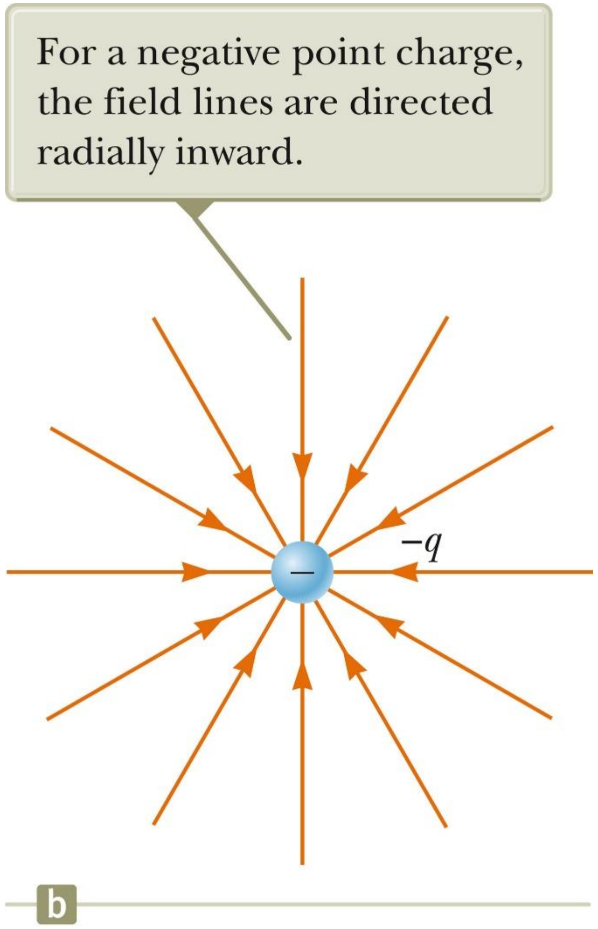
Electric Field Lines, Positive Point Charge

- The field lines radiate outward in all directions.
  - In three dimensions, the distribution is spherical.
- The lines are directed away from the source charge.
  - A positive test charge would be repelled away from the positive source charge.



Electric Field Lines, Negative Point Charge

- The field lines radiate inward in all directions.
- The lines are directed toward the source charge.
  - A positive test charge would be attracted toward the negative source charge.

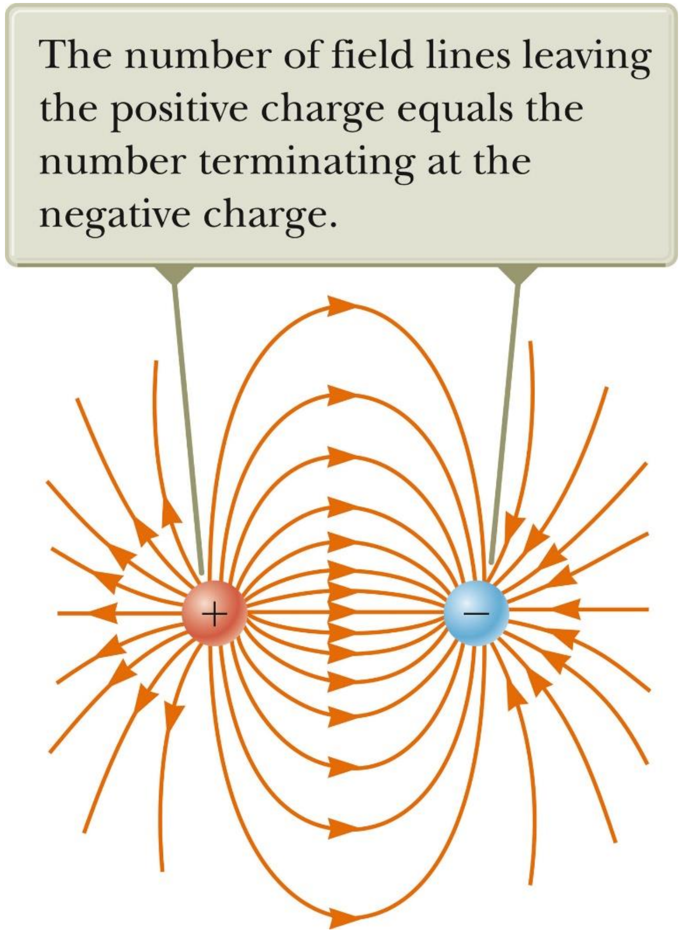


Electric Field Lines – Rules for Drawing

- The lines must begin on a positive charge and terminate on a negative charge.
  - In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.
- Remember field lines are **not** material objects, they are a pictorial representation used to qualitatively describe the electric field.

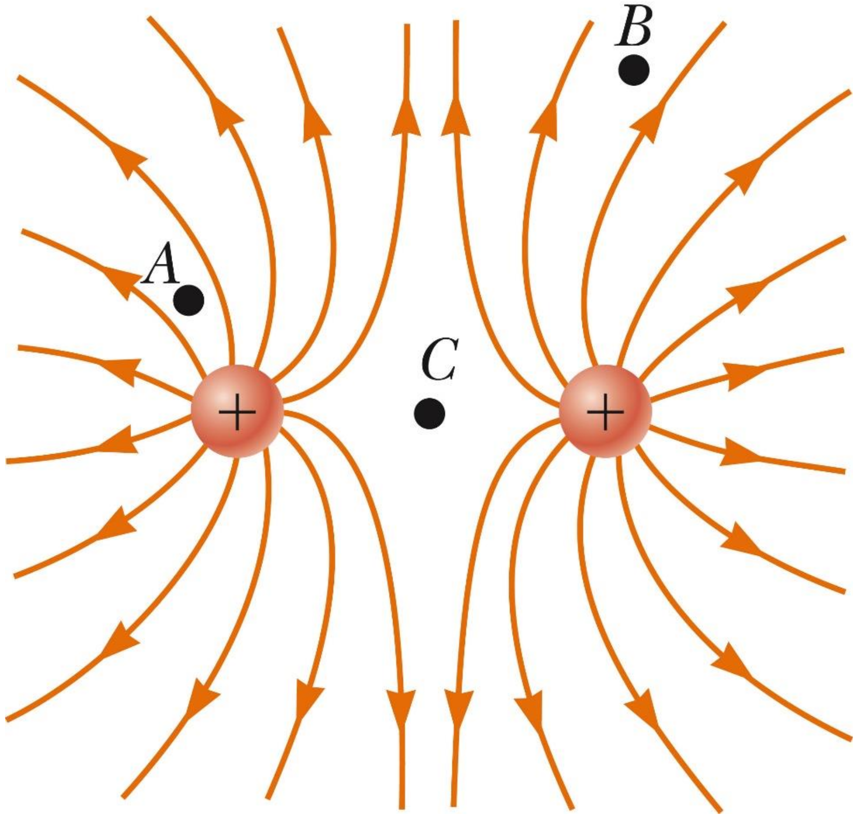
Electric Field Lines – Dipole

- The charges are equal and opposite.
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



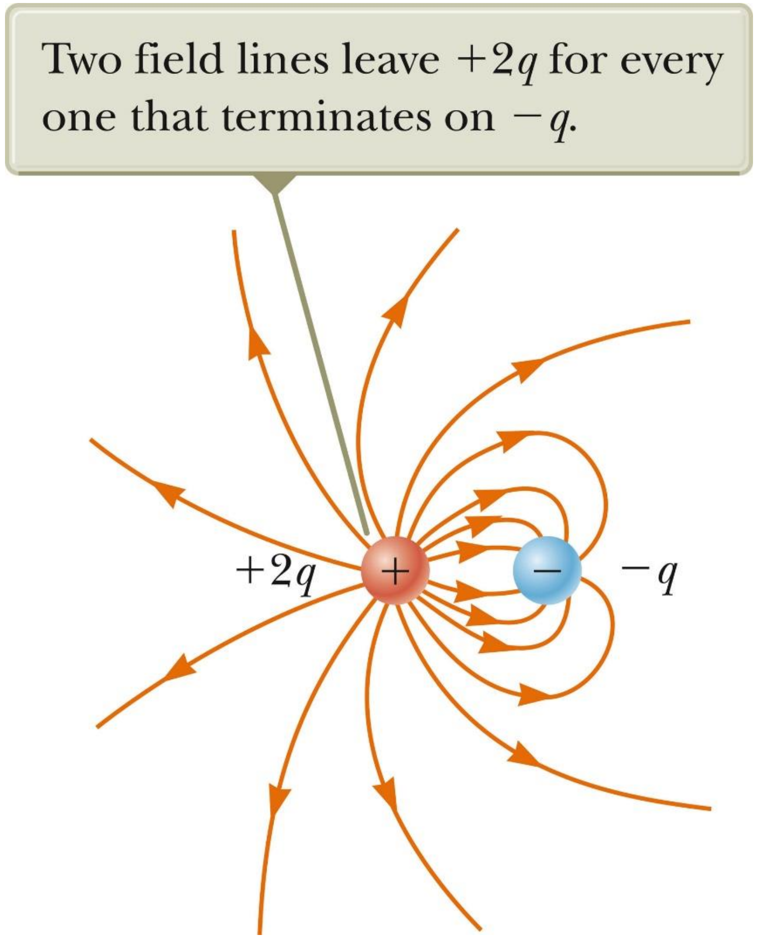
Electric Field Lines – Like Charges

- The charges are equal and positive.
- The same number of lines leave each charge since they are equal in magnitude.
- At a great distance, the field is approximately equal to that of a single charge of 2*q*.
- Since there are no negative charges available, the field lines end infinitely far away.



Electric Field Lines, Unequal Charges

- The positive charge is twice the magnitude of the negative charge.
- Two lines leave the positive charge for each line that terminates on the negative charge.
- At a great distance, the field would be approximately the same as that due to a single charge of +*q*.



Section 23.6

Motion of Charged Particles

- When a charged particle is placed in an electric field, it experiences an electrical force.
- If this is the only force on the particle, it must be the net force.
- The net force will cause the particle to accelerate according to Newton's second law.

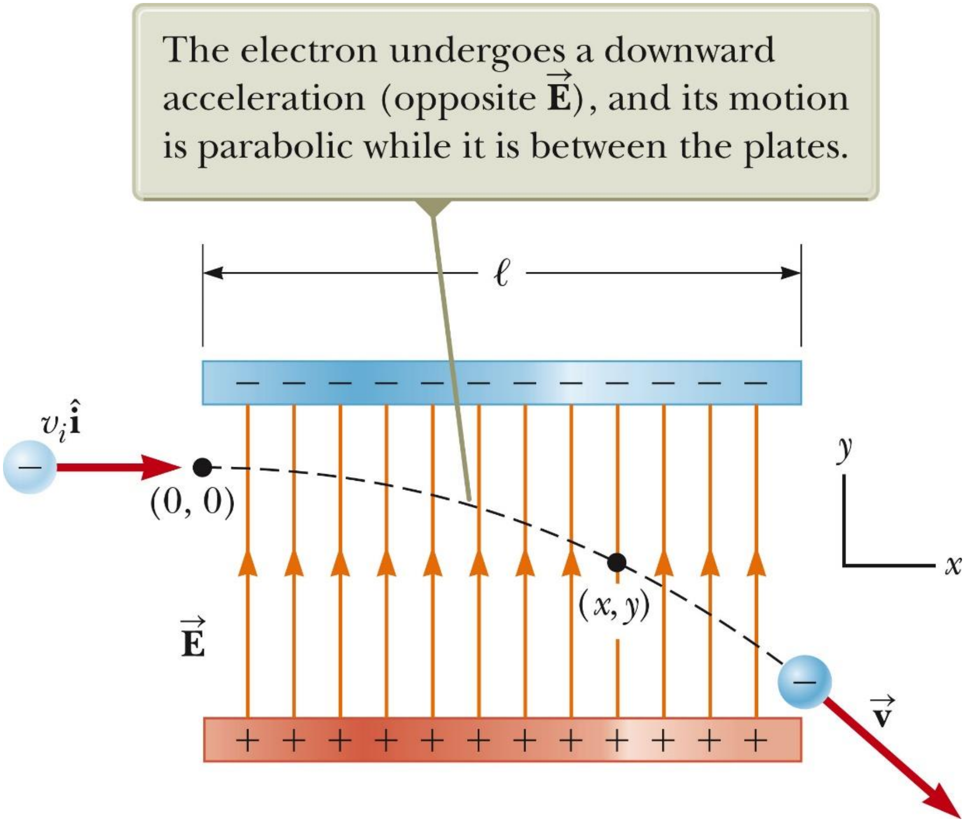
Motion of Particles, cont

- $$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

- If the field is uniform, then the acceleration is constant.
- The particle under constant acceleration model can be applied to the motion of the particle.
  - The electric force causes a particle to move according to the models of forces and motion.
- If the particle has a positive charge, its acceleration is in the direction of the field.
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Electron in a Uniform Field, Example

- The electron is projected horizontally into a uniform electric field.
- The electron undergoes a downward acceleration.
  - It is negative, so the acceleration is opposite the direction of the field.
- Its motion is parabolic while between the plates.



Example 23.10 An Accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field $\mathbf{E}$ directed along the x axis, as shown in Figure 23.25. Describe its motion.

Solution The acceleration is constant and is given by $q\mathbf{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension (see Chapter 2):

$$x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_f = \tfrac{1}{2}at^2 = \frac{qE}{2m}t^2$$

The speed of the particle is given by

$$v_f = at = \frac{qE}{m}t$$

The third kinematic equation gives us

$$v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$:

$$K = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e\Delta x = qE\Delta x$ and $W = \Delta K$.

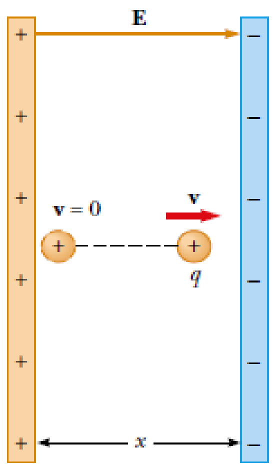


Figure 23.25 (Example 23.10) A positive point charge q in a uniform electric field $\mathbf{E}$ undergoes constant acceleration in the direction of the field.

Example 23.11 An Accelerated Electron

Interactive

An electron enters the region of a uniform electric field as shown in Figure 23.26, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m_e = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.13 gives

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\hat{\mathbf{j}}$$

$$= -3.51 \times 10^{13}\hat{\mathbf{j}}\ \text{m/s}^2$$

(B) If the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Solution The horizontal distance across the field is $\ell = 0.100$ m. Using Equation 23.16 with $x_f = \ell$, we find that the time at which the electron exits the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100\ \text{m}}{3.00 \times 10^6\ \text{m/s}} = 3.33 \times 10^{-8}\ \text{s}$$

(C) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

Solution Using Equation 23.17 and the results from parts (A) and (B), we find that

$$y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(3.51 \times 10^{13}\ \text{m/s}^2)(3.33 \times 10^{-8}\ \text{s})^2$$

$$= -0.019\,5\ \text{m} = -1.95\ \text{cm}$$

If the electron enters just below the negative plate in Figure 23.26 and the separation between the plates is less than the value we have just calculated, the electron will strike the positive plate.