PHYS 221

Electromagnetism (1)
2nd semester 1446

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Problems 8

Chapter 32 Problems
6,7,9,16,29,30,31,37

6. An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

From $|\varepsilon| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\varepsilon}{(\Delta I/\Delta t)} = \frac{24.0\times10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40\times10^{-3}\ \text{H}$.

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40\times10^{-3}\ \text{H})(4.00\ \text{A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$.

7. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm^2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 (420)^2 (3.00 \times 10^{-4})}{0.160} = 4.16 \times 10^{-4}\ \text{H}$$

$$\varepsilon = -L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = \frac{-\varepsilon}{L} = \frac{-175 \times 10^{-6}\ \text{V}}{4.16 \times 10^{-4}\ \text{H}} = \boxed{-0.421\ \text{A/s}}$$

9. A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) **What If?** If the current were different, which of these quantities would change?

(a) $$B = \mu_0 nI = \mu_0 \left(\frac{450}{0.120}\right)(0.0400\ \text{A}) = \boxed{188\ \mu\text{T}}$$

(b) $$\Phi_B = BA = \boxed{3.33 \times 10^{-8}\ \text{T}\cdot\text{m}^2}$$

(c) $$L = \frac{N\Phi_B}{I} = \boxed{0.375\ \text{mH}}$$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$

16. Show that $I = I_0\, e^{-t/\tau}$ is a solution of the differential equation

$$IR + L\frac{dI}{dt} = 0$$

Taking $\tau = \frac{L}{R}$, $I = I_0 e^{-t/\tau}$: $\frac{dI}{dt} = I_0 e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$IR + L\frac{dI}{dt} = 0$ will be true if $I_0 R e^{-t/\tau} + L\left(I_0 e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) =$

Because $\tau = \frac{L}{R}$, we have agreement with $0 = 0$.

29. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of 3.70×10^{-4} Wb in each turn.

$$L = \frac{N\Phi_B}{I} = \frac{200\left(3.70 \times 10^{-4}\right)}{1.75} = 42.3 \text{ mH} \quad \text{so} \quad U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423 \text{ H})(1.75 \text{ A})^2 = \boxed{0.0648 \text{ J}}.$$

30. The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

(a) The magnetic energy density is given by

$$\mu = \frac{B^2}{2\mu_0} = \frac{(4.50\ \text{T})^2}{2(1.26\times10^{-6}\ \text{T}\cdot\text{m/A})} = \boxed{8.06\times10^{6}\ \text{J/m}^3}.$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = \left(8.06\times10^{6}\ \text{J/m}^3\right)\left[(0.260\ \text{m})\pi(0.0310\ \text{m})^2\right] = \boxed{6.32\ \text{kJ}}$$

31. An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

$$L = \mu_0 \frac{N^2 A}{\ell} = \mu_0 \frac{(68.0)^2\left[\pi\left(0.600\times 10^{-2}\right)^2\right]}{0.0800} = 8.21\ \mu\text{H}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(8.21\times 10^{-6}\ \text{H}\right)(0.770\ \text{A})^2 = \boxed{2.44\ \mu\text{J}}$$

37. A uniform electric field of magnitude 680 kV/m throughout a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

We have $u = \epsilon_0 \dfrac{E^2}{2}$ and $u = \dfrac{B^2}{2\mu_0}$.

Therefore $\epsilon_0 \dfrac{E^2}{2} = \dfrac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0 \mu_0} = \frac{6.80 \times 10^5 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.27 \times 10^{-3} \text{ T}}.$$