PHYS 221

Electromagnetism (1)
2^{nd} semester 1446

Prof. Omar Abd-Elkader

Problems 7

Chapter 31 Problems 2,5,13,20

2. A flat loop of wire consisting of a single turn of cross-sectional area 8.00 cm² is perpendicular to a magnetic field that increases uniformly in magnitude from 0.500 T to 2.50 T in 1.00 s. What is the resulting induced current if the loop has a resistance of 2.00 Ω?

$$|\varepsilon| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = \frac{(2.50\ \mathrm{T} - 0.500\ \mathrm{T})\left(8.00\times10^{-4}\ \mathrm{m}^2\right)}{1.00\ \mathrm{s}}\left(\frac{1\ \mathrm{N}\cdot\mathrm{s}}{1\ \mathrm{T}\cdot\mathrm{C}\cdot\mathrm{m}}\right)\left(\frac{1\ \mathrm{V}\cdot\mathrm{C}}{1\ \mathrm{N}\cdot\mathrm{m}}\right)$$

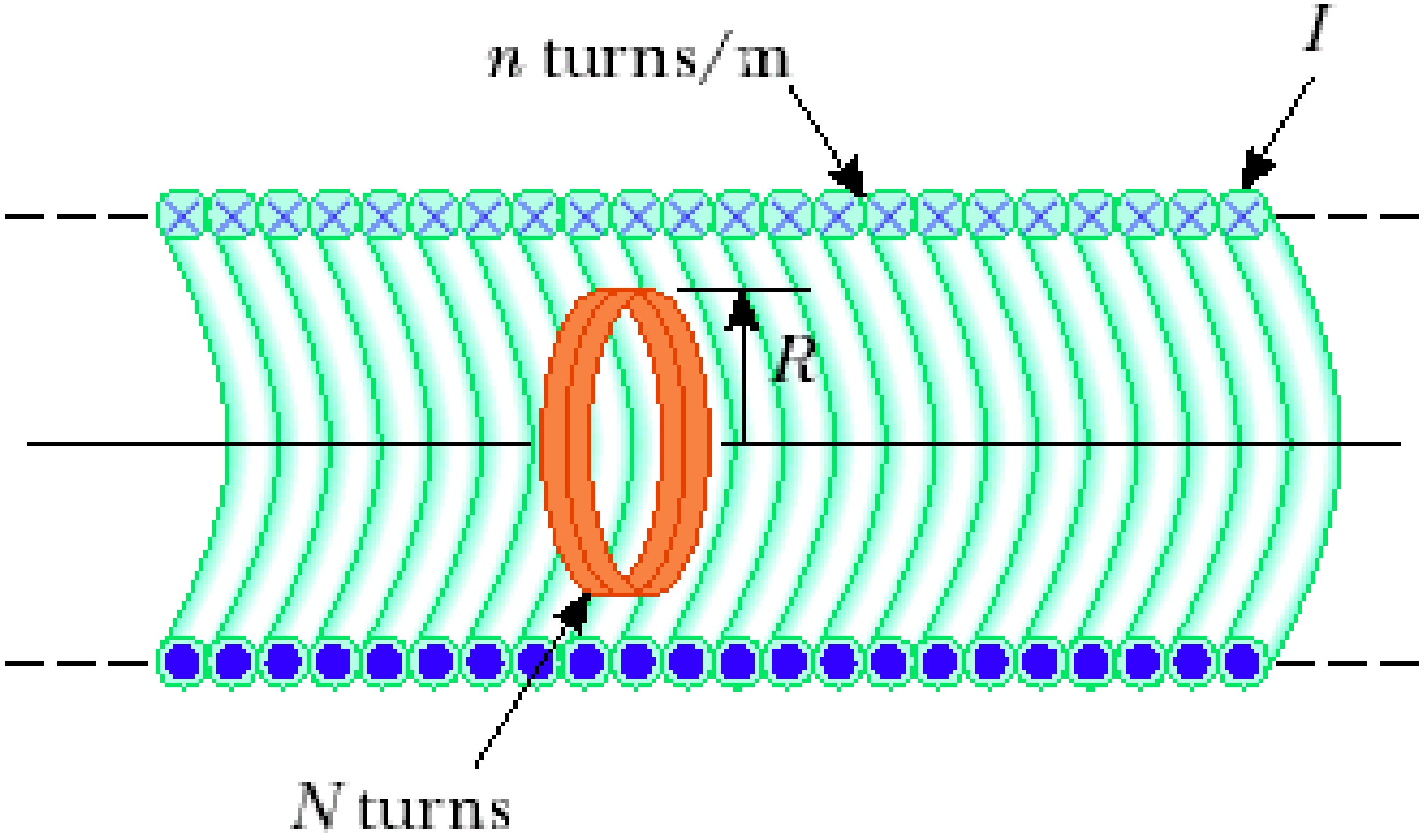
5. A strong electromagnet produces a uniform magnetic field of 1.60 T over a cross-sectional area of 0.200 m². We place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet. We then smoothly reduce the current in the electromagnet until it reaches zero in 20.0 ms. What is the current induced in the coil?

Noting unit conversions from $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ and $U = qV$, the induced voltage is

$$\varepsilon = -N\frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} = -N\left(\frac{0 - B_i A\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})\left(0.200\text{ m}^2\right)\cos 0°}{20.0\times 10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3\,200\text{ V}$$

$$I = \frac{\varepsilon}{R} = \frac{3\,200\text{ V}}{20.0\ \Omega} = \boxed{160\text{ A}}$$

13. A long solenoid has 400 turns per meter and carries a current given by $I = (30.0\ \text{A})(1 - e^{-1.60\,t})$. Inside the solenoid and coaxial with it is a coil that has a radius of 6.00 cm and consists of a total of 250 turns of fine wire (Fig. P31.13). What emf is induced in the coil by the changing current?



$$B = \mu_0 nI = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)$$

$$\Phi_B = \int B\,dA = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\int dA$$

$$\Phi_B = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_0 n(30.0\text{ A})\pi R^2(1.60)e^{-1.60t}$$

$$\varepsilon = -(250)\left(4\pi\times 10^{-7}\text{ N/A}^2\right)\left(400\text{ m}^{-1}\right)(30.0\text{ A})\left[\pi(0.060\,0\text{ m})^2\right]1.60\text{ s}^{-1}e^{-1.60t}$$

$$\varepsilon = \boxed{(68.2\text{ mV})e^{-1.60t}\text{ counterclockwise}}$$

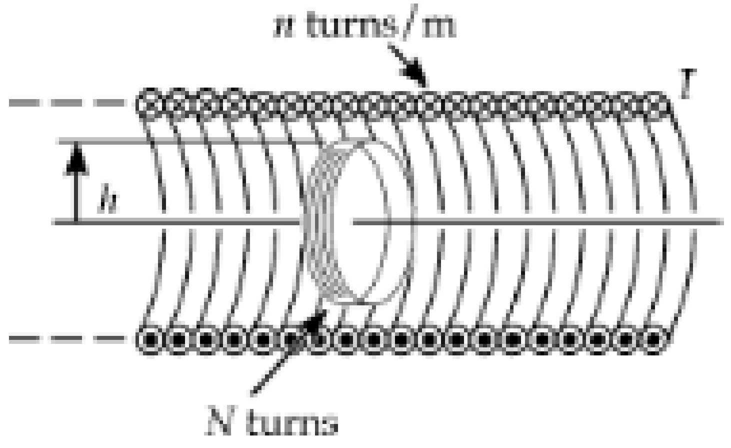


FIG. P31.13

20. Consider the arrangement shown in Figure P31.20. Assume that *R* = 6.00 Ω, *ℓ* = 1.20 m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

$$I = \frac{\varepsilon}{R} = \frac{B\ell v}{R}$$

$$\boxed{v = 1.00\ \text{m/s}}$$