PHYS 221

Electromagnetism (1)
2^{nd} semester 1446

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Problems 5

Problems 7, 9, 12, 30, 37, 41

7. A proton moving at 4.00 × 10^6 m/s through a magnetic field of 1.70 T experiences a magnetic force of magnitude 8.20 × 10^{-13} N. What is the angle between the proton's velocity and the field?

$F_B = qvB\sin\theta$ so $8.20\times10^{-13}\ \text{N} = \left(1.60\times10^{-19}\ \text{C}\right)\left(4.00\times10^{6}\ \text{m/s}\right)(1.70\ \text{T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9^\circ \text{ or } 131^\circ}$.

9. A proton moves with a velocity of v = (2 **i** – 4 **j** + **k**) m/s in a region in which the magnetic field is B = (**i** + 2 **j** – 3 **k**) T. What is the magnitude of the magnetic force this charge experiences?

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\hat{\mathbf{i}} + (1+6)\hat{\mathbf{j}} + (4+4)\hat{\mathbf{k}} = 10\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

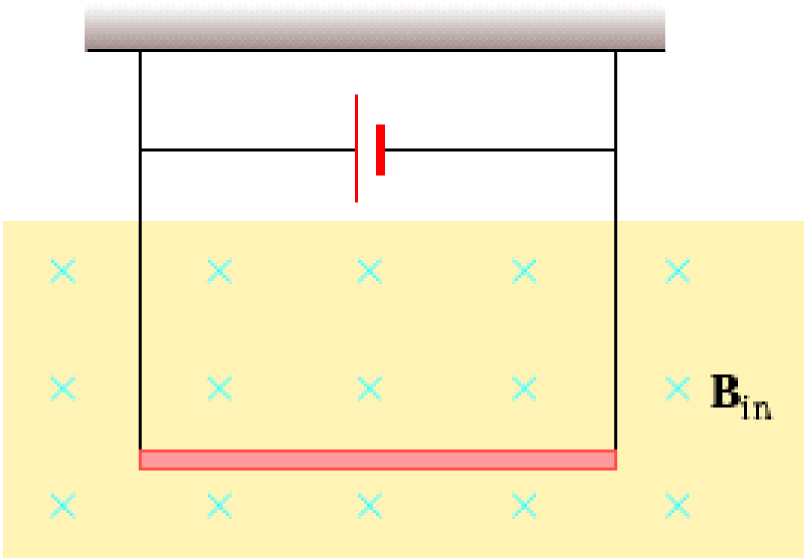
$$|\mathbf{v} \times \mathbf{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6 \text{ T} \cdot \text{m/s}$$

$$|\mathbf{F}_B| = q|\mathbf{v} \times \mathbf{B}| = (1.60 \times 10^{-19} \text{ C})(14.6 \text{ T} \cdot \text{m/s}) = \boxed{2.34 \times 10^{-18} \text{ N}}$$

12. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, B = 1.60 T. If the current is in the +x direction, what is the magnetic force on the section of wire?

$$\mathbf{F}_B = I\boldsymbol{\ell} \times \mathbf{B} = (2.40\ \text{A})(0.750\ \text{m})\hat{\mathbf{i}} \times (1.60\ \text{T})\hat{\mathbf{k}} = \boxed{\left(-2.88\hat{\mathbf{j}}\right)\ \text{N}}$$

14. A conductor suspended by two flexible wires as shown in Figure P29.14 has a mass per unit length of 0.040 0 kg/m. What current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.60 T into the page? What is the required direction for the current?



$$\frac{|\mathbf{F}_B|}{\ell} = \frac{mg}{\ell} = \frac{I|\ell \times \mathbf{B}|}{\ell}$$

$$I = \frac{mg}{B\ell} = \frac{(0.040\,0\ \text{kg/m})(9.80\ \text{m/s}^2)}{3.60\ \text{T}} = \boxed{0.109\ \text{A}}$$

The direction of I in the bar is $\boxed{\text{to the right}}$.

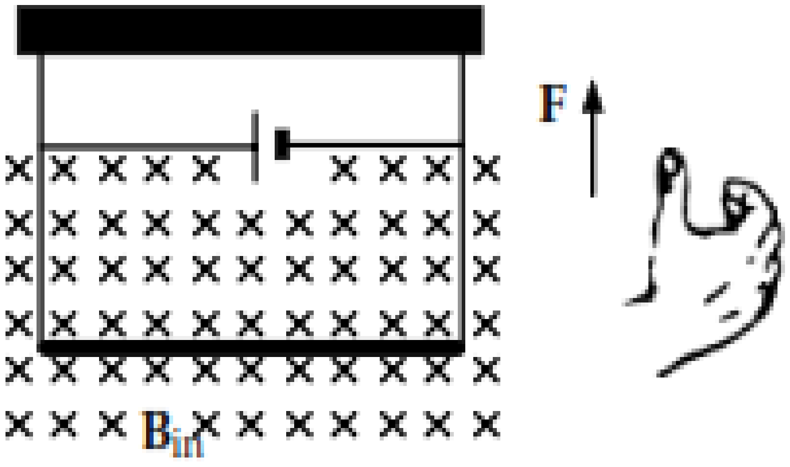


FIG. P29.14

30. A singly charged positive ion has a mass of 3.20 × 10–26 kg. After being accelerated from rest through a potential difference of 833 V, the ion enters a magnetic field of 0.920 T along a direction perpendicular to the direction of the field. Calculate the radius of the path of the ion in the field.

$$\frac{1}{2}mv^2 = q(\Delta V) \qquad \frac{1}{2}\left(3.20\times10^{-26}\text{ kg}\right)v^2 = \left(1.60\times10^{-19}\text{ C}\right)(833\text{ V}) \qquad v = 91.3\text{ km/s}$$

The magnetic force provides the centripetal force: $qvB\sin\theta = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB\sin 90.0^\circ} = \frac{\left(3.20\times10^{-26}\text{ kg}\right)\left(9.13\times10^{4}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.920\text{ N}\cdot\text{s/C}\cdot\text{m}\right)} = \boxed{1.98\text{ cm}}.$$

37. A cosmic-ray proton in interstellar space has an energy of 10.0 MeV and executes a circular orbit having a radius equal to that of Mercury's orbit around the Sun (5.80×10^{10} m). What is the magnetic field in that region of space?

$$E = \frac{1}{2}mv^2 = e\Delta V$$

and $$evB\sin 90° = \frac{mv^2}{R}$$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80 \times 10^{10}\ \text{m}}\sqrt{\frac{2\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(10.0 \times 10^{6}\ \text{V}\right)}{1.60 \times 10^{-19}\ \text{C}}} = \boxed{7.88 \times 10^{-12}\ \text{T}}$$

41. Singly charged uranium-238 ions are accelerated through a potential difference of 2.00 kV and enter a uniform magnetic field of 1.20 T directed perpendicular to their velocities. (a) Determine the radius of their circular path. (b) Repeat for uranium-235 ions. **What If?** How does the ratio of these path radii depend on the accelerating voltage and on the magnitude of the magnetic field?

$$K = \frac{1}{2}mv^2 = q(\Delta V) \qquad \text{so} \qquad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

$$|\mathbf{F}_B| = |q\mathbf{v} \times \mathbf{B}| = \frac{mv^2}{r} \qquad r = \frac{mv}{qB} = \frac{m}{q}\sqrt{\frac{2q(\Delta V)/m}{B}} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $$r_{238} = \sqrt{\frac{2\left(238 \times 1.66 \times 10^{-27}\right)2\,000}{1.60 \times 10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28 \times 10^{-2} \text{ m} = \boxed{8.28 \text{ cm}}$$

(b) $$r_{235} = \boxed{8.23 \text{ cm}}$$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.006\,4$$

The ratios of the orbit radius for different ions are independent of ΔV and B.