# PHYS 221

# Electromagnetism (1)

## $2^{nd}$ semester 1446

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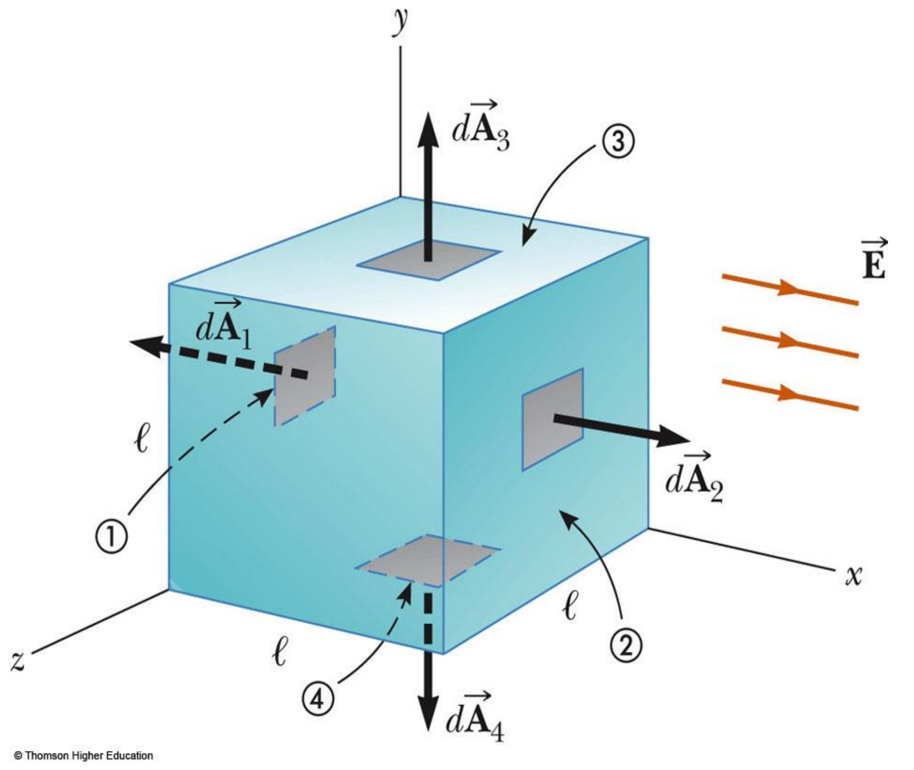
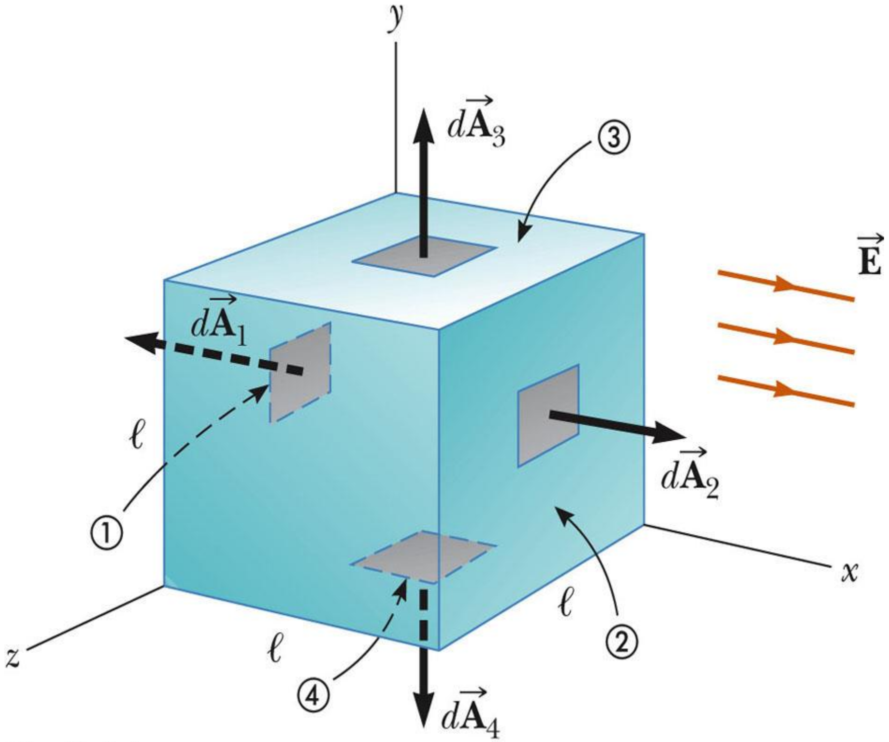
## Problems 2

# Flux through a cube of a uniform electric field

Consider a uniform electric field $\vec{E}$ oriented in the $x$ direction in empty space. A cube of edge length $\ell$ is placed in the field, oriented as shown in Figure 24.5. Find the net electric flux through the surface of the cube.



- **The field lines pass through l h two surfaces perpendicularly and are parallel to the other four surfaces**
- **For side 1, $\Phi_E = -El^2$**
- **For side 2, $\Phi_E = El^2$**
- **For the other sides, $\Phi_E = 0$**
- **Therefore, $\Phi_{total} = 0$**



# Flux Due to a Point Charge

- **A spherical Gaussian surface surrounds a point charge *q*. Describe what happens to the total Flux through the surface if: (A) the charge is tripled. (B) the radius of sphere is doubled. (C) the surface is changed to a cube, and (D) the charge is moved to another location inside the surface.**

***Solution:***

- ***(A) F is tripled because F is proportional to the amount of charge.***
- ***(B) F does not change because All E lines from a charge pass through the sphere regardless of its radius.***
- ***(C) F does not change because All E lines from a charge pass through the surface regardless of its shape .***
- ***(D) F does not change because Gauss's law refers to the total charge enclosed regardless of where the charge is located inside the surface.***

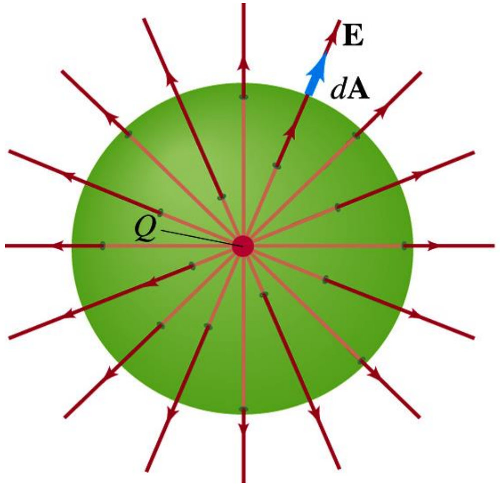
# The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge *q*.

- The field lines are directed radially outwards and are perpendicular to the surface at every point, so

The angle θ between $\vec{E}$ and $d\vec{A}$ is zero at any point on the surface, we can re-write Gauss' Law as

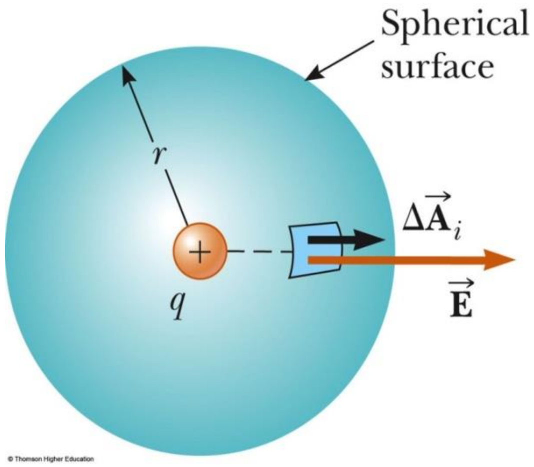
$$\Phi_E \equiv \oint \vec{E} \cdot d\vec{A} = \oint E_n \; dA = \oint E dA = E \oint dA = E \cdot 4\pi r^2$$




E has the has same value at all points on the surface

$$\Phi_E = \oint \vec{E} \,.\, d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$q_{enc} = q$$




A spherical Gaussian surface centered on a point charge q

E is can be moved out $\quad \epsilon_0 E \oint dA = q$

Integral is the sum of surface area $\quad \epsilon_0 E (4\pi r^2) = q$

Coulomb's Law $\quad E = \dfrac{q}{4\pi\epsilon_0 r^2}$

# A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius *a* has a uniform volume charge density $\rho$ and carries total charge *Q*.

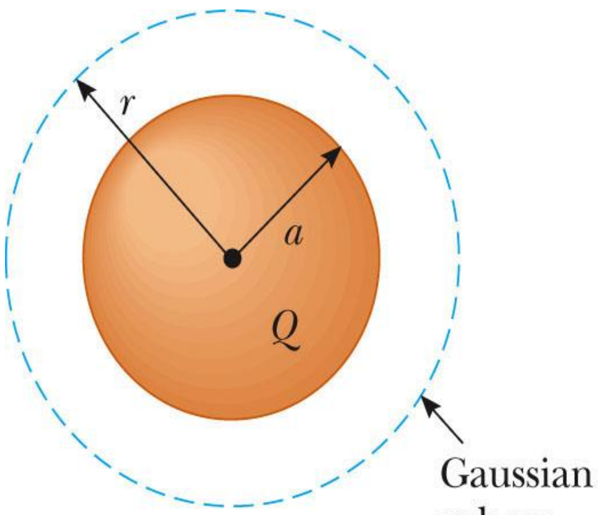
(A)Find the magnitude of the E-field at a point outside the sphere

(B)Find the magnitude of the E-field at a point inside the sphere

- For $r > a$

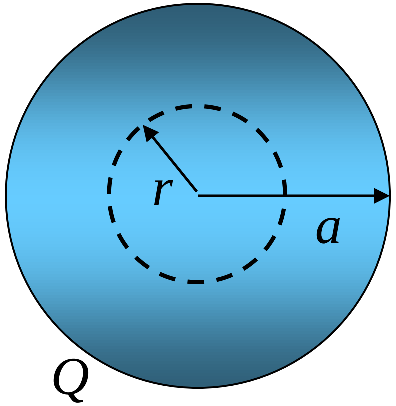
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = k_e \frac{Q}{r^2}$$



(a)

Now we select a spherical Gaussian surface with radius $r < a$. Again the symmetry of the charge distribution allows us to simply evaluate the left side of Gauss's law just as before.




$$\oint E \, dA = E \oint dA = E\,(4\pi r^2)$$

❶ The charge inside the Gaussian sphere is no longer $Q$. If we call the Gaussian sphere volume $V'$ then

Volume charge density“: ρ = charge / unit volume is used to characterize the charge distribution

❷ Right side: $Q_{in} = \rho V' = \rho \frac{4}{3}\pi r^3$

❸ $$E\left(4\pi r^2\right) = \frac{Q_{in}}{\varepsilon_0} = \frac{4\rho\pi r^3}{3\varepsilon_0}$$

❹ $$E = \frac{4\rho\pi r^3}{3\varepsilon_0\left(4\pi r^2\right)} = \frac{\rho}{3\varepsilon_0} r \quad \text{but} \quad \rho = \frac{Q}{\frac{4}{3}\pi a^3} \quad \text{so} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3} r = k_e \frac{Q}{a^3} r$$

❹

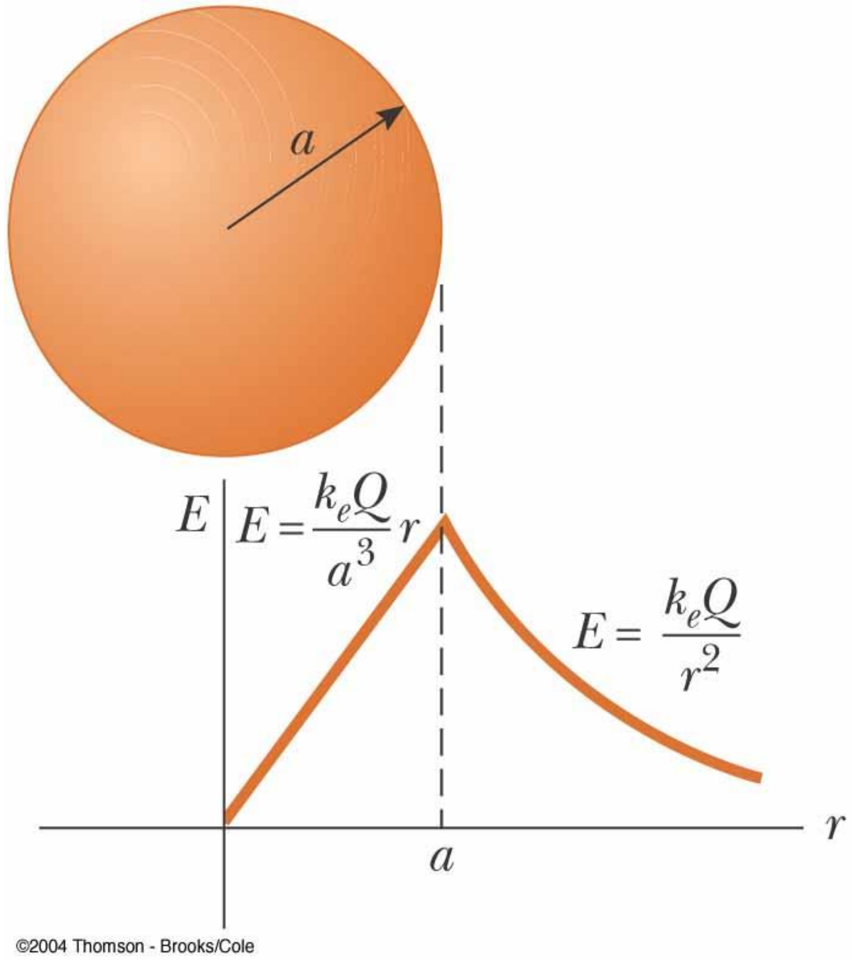
$$E = \frac{4\rho\pi r^3}{3\varepsilon_0\left(4\pi r^2\right)} = \frac{\rho}{3\varepsilon_0} r \quad \text{but} \quad \rho = \frac{Q}{\frac{4}{3}\pi a^3} \quad \text{so} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3} r = k_e \frac{Q}{a^3} r$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}$$

$$E = \lim_{r \to a} (K_e\, Q/r^2) = K_e\, Q/a^2$$

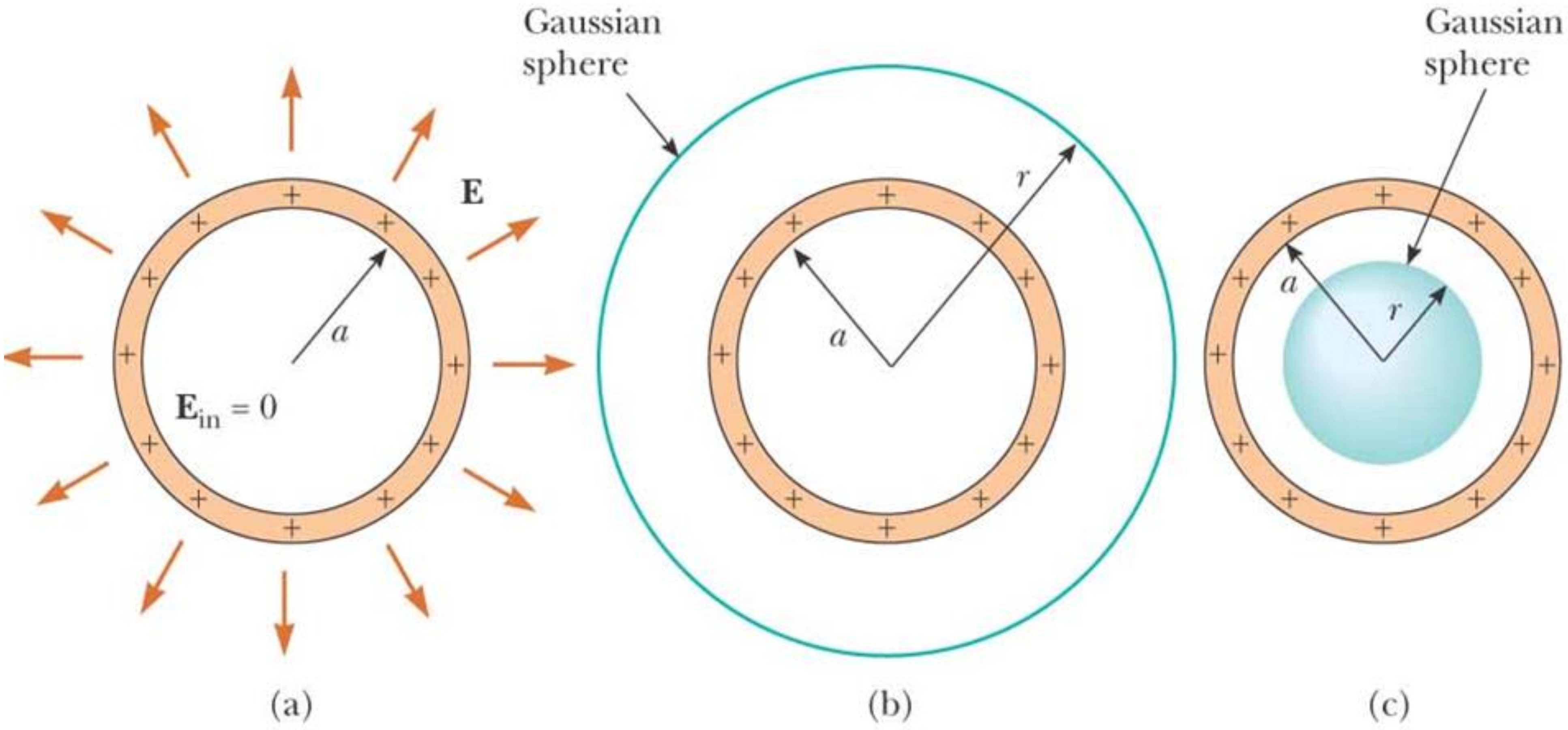
$$E = \lim_{r \to a} (K_e\, Q/a^3) r = K_e\, Q/a^3) a$$

- Inside the sphere, $E$ varies linearly with $r$ ($E \to 0$ as $r \to 0$)
- The value of the field is the same as we approach the surface from both directions.



# The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface Find the electric field at points **(A) outside and (B) inside the shell.**

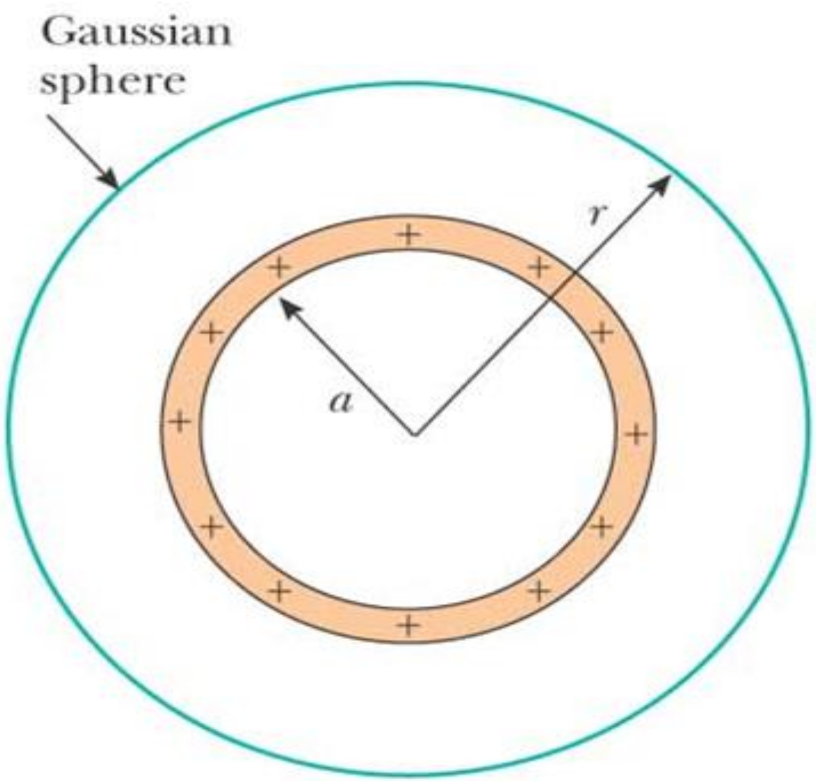


- Let's start with the Gaussian surface outside the sphere of charge, r > a
- We know from symmetry arguments that the electric field will be radial outside the charged sphere
  - If we rotate the sphere, the electric field cannot change
    - Spherical symmetry
- Thus we can apply Gauss' Law and get

$$\text{Flux} = \oint E \cdot dA = E \times (4\pi r^2)$$
$$= q / \varepsilon_0 \qquad \text{(Gauss)}$$

- … so the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$



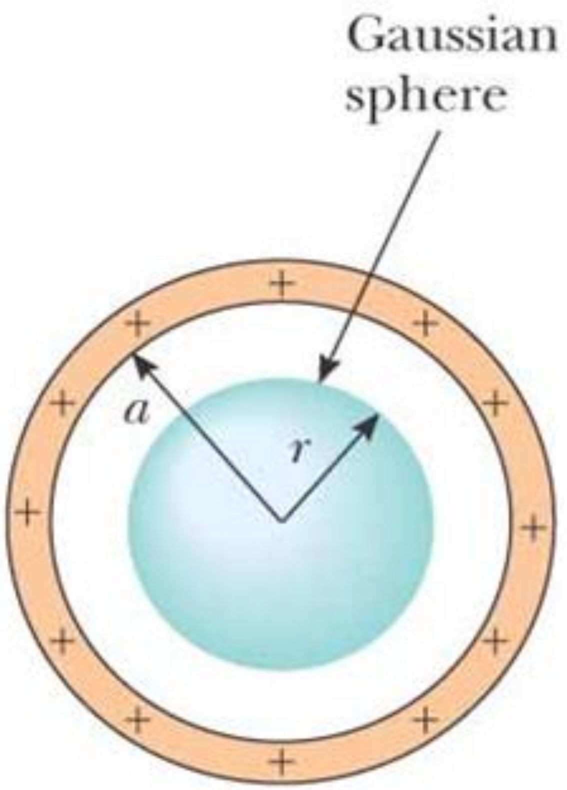
- Let's take the Gaussian surface inside the sphere of charge, $r < a$
- We know that the enclosed charge is zero so

$$\text{Flux} = \Phi_E = EA = 0$$

- We find that the electric field is zero everywhere inside spherical shell of charge

$$E = 0$$

- Thus we obtain two results
  - The electric field outside a spherical shell of charge is the same as that of a point charge.
  - The electric field inside a spherical shell of charge is zero.

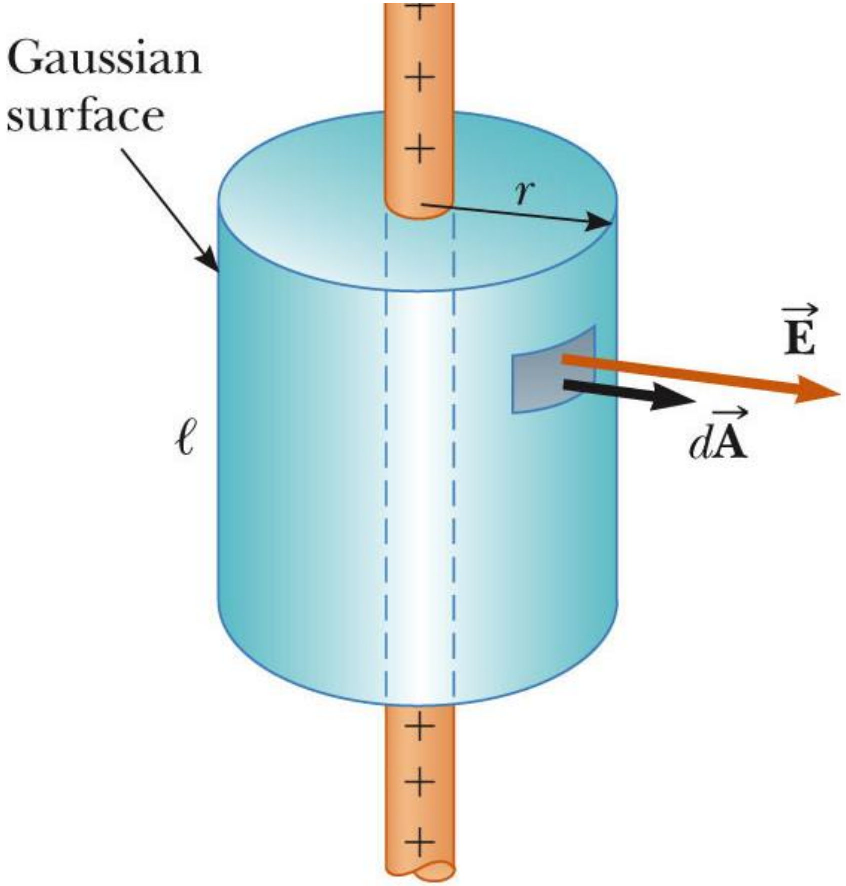


# A Cylindrically Symmetric Charge Distribution

Find the E-field a distance *r* from a line of positive charge of infinite length and constant charge per unit length $\lambda$.

- Symmetry $\Rightarrow$ E field must be $\perp$ to line and directed outward

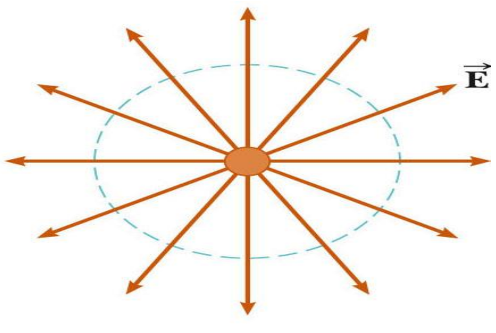
- Select a cylinder as Gaussian surface. The cylinder has a radius of *r* and a length of $\ell$
- $\vec{E}$ is constant in magnitude and parallel to the surface (the direction of a surface is its normal!) at every point on the curved part of the surface (the body of the cylinder.



$$\frac{q}{L} = \lambda$$

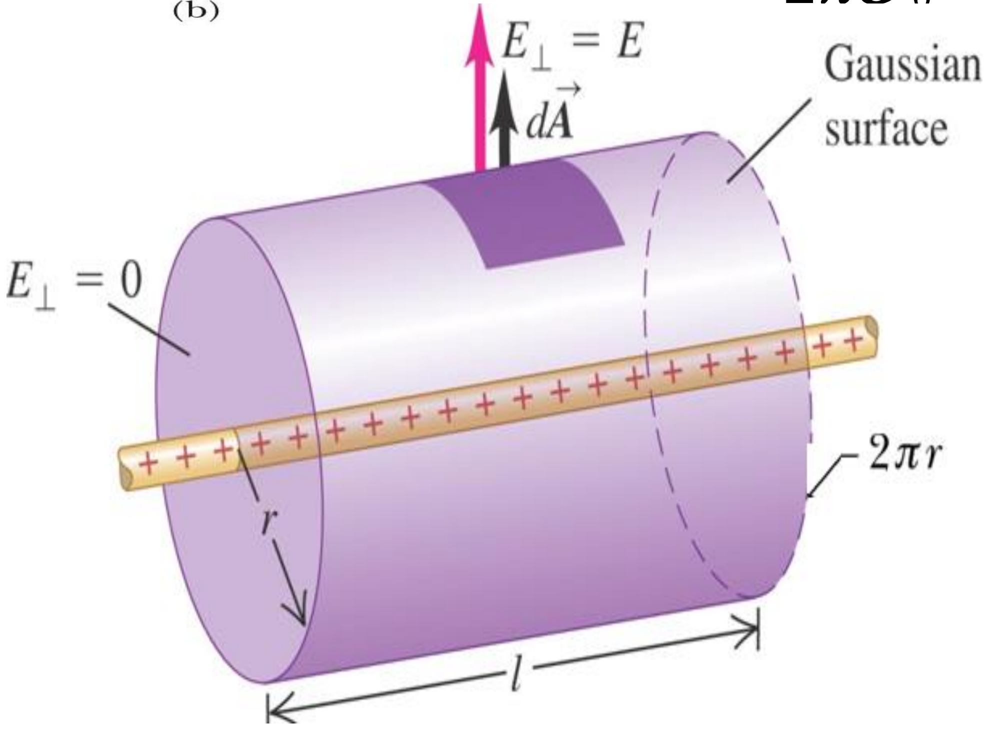
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Gauss s law




$$\oint E \; dA = E\oint dA = 0 + E\,(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$



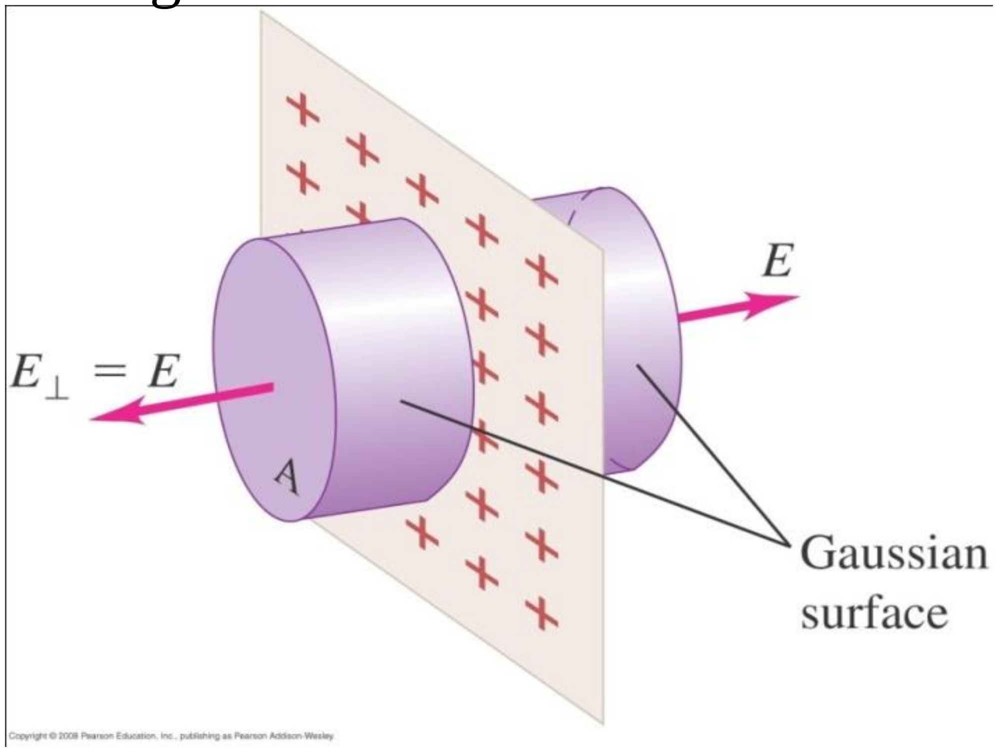

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Line of Charge

$$\vec{E} = \frac{2k\lambda}{r}\hat{r}$$

# A Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.




- Assume that we have a thin, infinite non-conducting sheet of positive charge
- The charge density in this case is the charge per unit area, $\sigma$
- From symmetry, we can see that the electric field will be perpendicular to the surface of the sheet

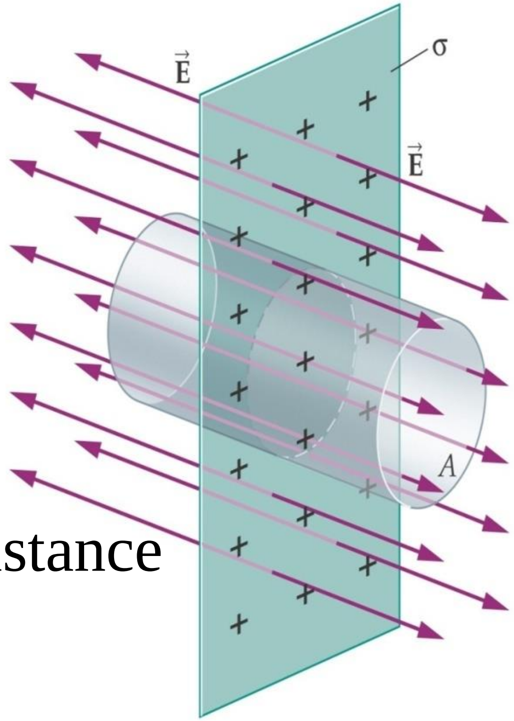
- To calculate the electric field using Gauss' Law, we assume a Gaussian surface in the form of a right cylinder with cross sectional area *A* and height 2*r*, chosen to cut through the plane perpendicularly.
- Because the electric field is perpendicular to the plane everywhere, the electric field will be parallel to the walls of the cylinder and perpendicular to the ends of the cylinder.
- Using Gauss' Law we get

$$\text{Flux} = \Phi_E = \oint E \cdot dA = q / \varepsilon_0 \qquad \text{(Gauss)}$$

$$EA + EA = \sigma A / \varepsilon_0$$

$$2EA = \sigma A / \varepsilon_0$$

$$E = \sigma / 2\varepsilon_0$$




- The distance from each flat end does not appear we conclude that at any distance
- From the plane. The electric field is uniform everywhere

$$E = \sigma / 2\varepsilon_0$$

- **3. A 40.0-cm-diameter loop is rotated in a uniform electric field until the position of electric flux is found. The flux in this position is measured to be $5.20 \times 10^5$ N · $m^2$/C. What is the magnitude of the electric field?**

**3. A 40.0-cm-diameter loop is rotated in a uniform electric field until the position of electric flux is found. The flux in this position is measured to be $5.20 \times 10^5$ N · $m^2$/C. What is the magnitude of the electric field?**

$$\Phi E = EA \cos \theta$$

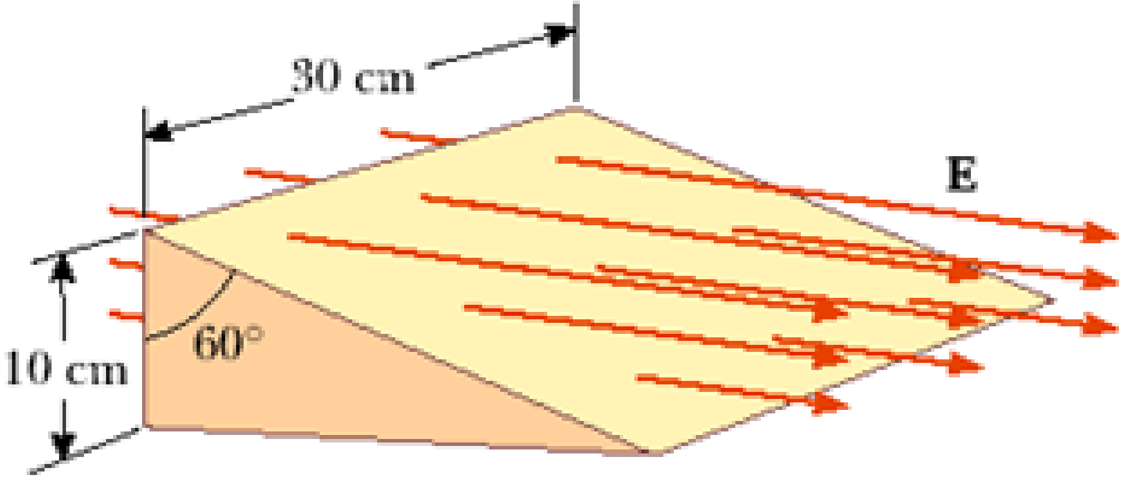
The flux is a maximum when the surface is perpendicular to the field

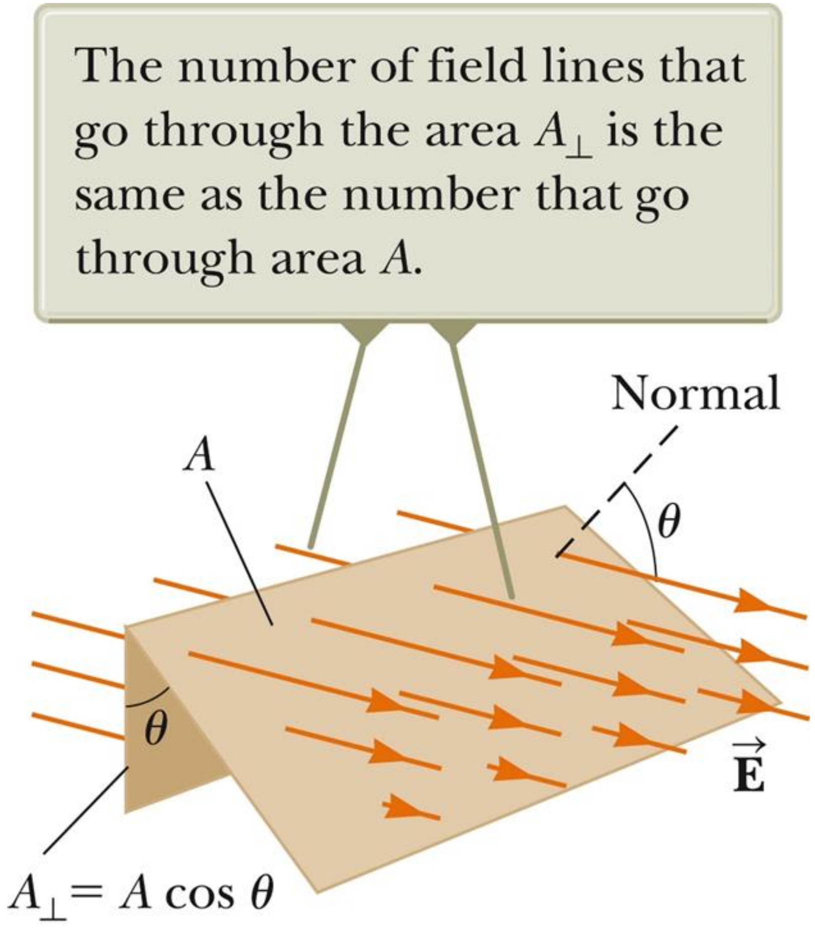
The flux is a minimum (zero) when the surface is (parallel) to the field

$$\Phi_E = EA\cos\theta \qquad A = \pi r^2 = \pi(0.200)^2 = 0.126 \text{ m}^2$$

$$5.20 \times 10^5 = E(0.126)\cos 0° \qquad E = 4.14 \times 10^6 \text{ N/C} = \boxed{4.14 \text{ MN/C}}$$

**4. Consider a closed triangular box resting within a horizontal electric field of magnitude E = 7.80 × 104 N/C as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface. (b) the slanted surface, and (c) the entire surface of the box.**



(a) $A' = (10.0\text{ cm})(30.0\text{ cm})$

$A' = 300\text{ cm}^2 = 0.030\,0\text{ m}^2$

$\Phi_{E,\,A'} = EA'\cos\theta$

$\Phi_{E,\,A'} = \left(7.80\times 10^4\right)(0.030\,0)\cos 180°$

$\Phi_{E,\,A'} = \boxed{-2.34\text{ kN}\cdot\text{m}^2/\text{C}}$

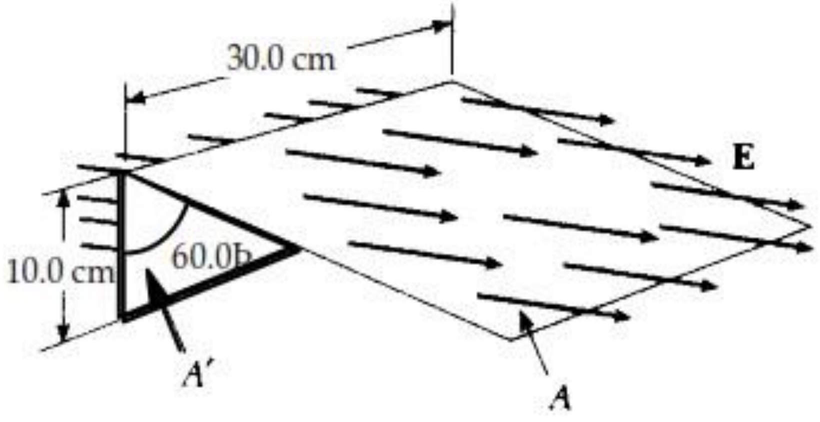



FIG. P24.4

(b) $\Phi_{E,\,A} = EA\cos\theta = \left(7.80\times 10^4\right)(A)\cos 60.0°$

$A = (30.0\text{ cm})(w) = (30.0\text{ cm})\left(\dfrac{10.0\text{ cm}}{\cos 60.0°}\right) = 600\text{ cm}^2 = 0.060\,0\text{ m}^2$

$\Phi_{E,\,A} = \left(7.80\times 10^4\right)(0.060\,0)\cos 60.0° = \boxed{+2.34\text{ kN}\cdot\text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to **E**, so $\Phi_E = 0$ for each of these. Thus,

$\Phi_{E,\,\text{total}} = -2.34\text{ kN}\cdot\text{m}^2/\text{C} + 2.34\text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$.

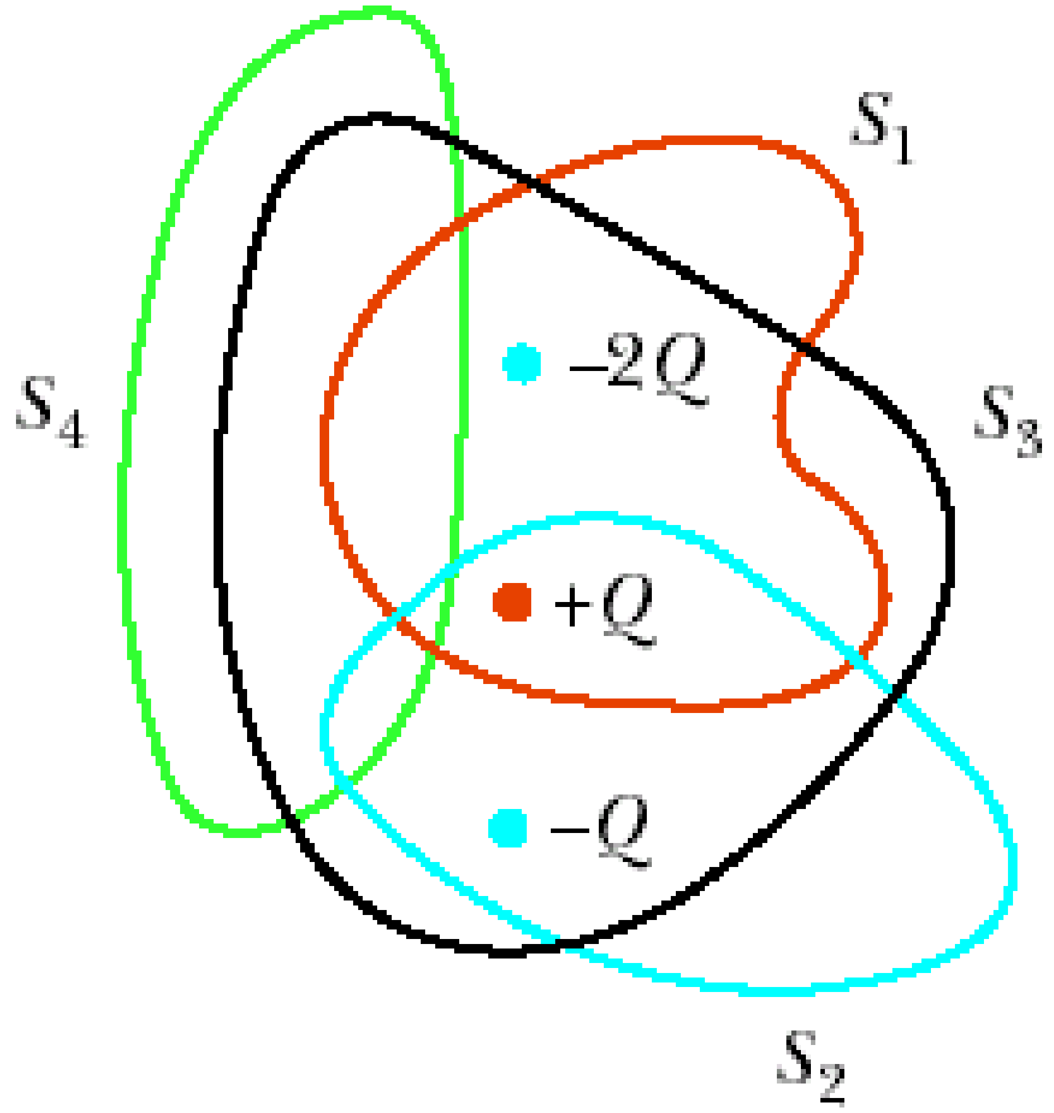
**9. The following charges are located inside a submarine: 5.00 μC, –9.00 μC, 27.0 μC, and –84.0 μC. (a) Calculate the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?**

(a) $$\Phi_E = \frac{q_{in}}{\epsilon_0} = \frac{(+5.00\ \mu C - 9.00\ \mu C + 27.0\ \mu C - 84.0\ \mu C)}{8.85 \times 10^{-12}\ C^2/N \cdot m^2} = -6.89 \times 10^6\ N \cdot m^2/C^2$$

$$\Phi_E = \boxed{-6.89\ MN \cdot m^2/C}$$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

**11. Four closed surfaces, S1 through S4, together with the charges –2Q, Q, and –Q are sketched in Figure P24.11. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.**



$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$$

Through $S_1$ $\quad \Phi_E = \frac{-2Q+Q}{\epsilon_0} = \boxed{-\frac{Q}{\epsilon_0}}$

Through $S_2$ $\quad \Phi_E = \frac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through $S_3$ $\quad \Phi_E = \frac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\frac{2Q}{\epsilon_0}}$

Through $S_4$ $\quad \Phi_E = \boxed{0}$

**21. A charge of 170 μC is at the center of a cube of edge 80.0 cm. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) What If? Would your answers to parts (a) or (b) change if the charge were not at the center? Explain.**

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.92 \times 10^7\ \text{N}\cdot\text{m}^2/\text{C}$$

(a) $$(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1.92 \times 10^7\ \text{N}\cdot\text{m}^2/\text{C}}{6}$$ $$(\Phi_E)_{\text{one face}} = \boxed{3.20\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(b) $$\Phi_E = \boxed{19.2\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

**24.A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.**

(a) $$E = \frac{k_e Q r}{a^3} = \boxed{0}$$

(b) $$E = \frac{k_e Q r}{a^3} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365\ \text{kN/C}}$$

(c) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46\ \text{MN/C}}$$

(d) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649\ \text{kN/C}}$$

The direction for each electric field is $\boxed{\text{radially outward}}$.

**31.Consider a thin spherical shell of radius 14.0 cm with a total charge of 32.0 μC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.**

(a) $\mathbf{E} = \boxed{0}$

(b) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = 7.19 \text{ MN/C}$ $\quad \mathbf{E} = \boxed{7.19 \text{ MN/C radially outward}}$

**35.A uniformly charged, straight filament 7.00 m in length has a total positive charge of 2.00 μC. An uncharged cardboard cylinder 2.00 cm in length and 10.0 cm in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.**

(a) $$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left[\left(2.00\times10^{-6}\ \mathrm{C}\right)/7.00\ \mathrm{m}\right]}{0.100\ \mathrm{m}}$$

$$E = \boxed{51.4\ \mathrm{kN/C},\ \text{radially outward}}$$

(b) $$\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos 0^\circ$$

$$\Phi_E = \left(5.14\times10^{4}\ \mathrm{N/C}\right)2\pi(0.100\ \mathrm{m})(0.0200\ \mathrm{m})(1.00) = \boxed{646\ \mathrm{N\cdot m^2/C}}$$

**37.A large flat horizontal sheet of charge has a charge per unit area of 9.00 μC/m2. Find the electric field just above the middle of the sheet.**

$$E = \frac{\sigma}{2\epsilon_0} = \frac{9.00 \times 10^{-6}\ \text{C/m}^2}{2\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{508\ \text{kN/C, upward}}$$

**40. On a clear, sunny day, a vertical electric field of about 130 N/C points down over flat ground. What is the surface charge density on the ground for these conditions?**

From Gauss's Law, $EA = \frac{Q}{\epsilon_0}$

$$\sigma = \frac{Q}{A} = \epsilon_0 E = (8.85 \times 10^{-12})(-130) = -1.15 \times 10^{-9}\ \text{C/m}^2 = \boxed{-1.15\ \text{nC/m}^2}$$

**42. A solid copper sphere of radius 15.0 cm carries a charge of 40.0 nC. Find the electric field (a) 12.0 cm, (b) 17.0 cm, and (c) 75.0 cm from the center of the sphere. (d) What If? How would your answers change if the sphere were hollow?**

(a) All of the charge sits on the surface of the copper sphere at radius 15 cm. The field inside is $\boxed{\text{zero}}$.

(b) The charged sphere creates field at exterior points as if it were a point charge at the center:

$$\mathbf{E} = \frac{k_e q}{r^2} \text{ away} = \frac{\left(8.99\times10^{9}\ \text{Nm}^2\right)\left(40\times10^{-9}\ \text{C}\right)}{\text{C}^2(0.17\ \text{m})^2} \text{ outward} = \boxed{1.24\times10^{4}\ \text{N/C outward}}$$

(c) $$\mathbf{E} = \frac{\left(8.99\times10^{9}\ \text{Nm}^2\right)\left(40\times10^{-9}\ \text{C}\right)}{\text{C}^2(0.75\ \text{m})^2} \text{ outward} = \boxed{639\ \text{N/C outward}}$$

(d) All three answers would be the same.