Partial Fraction Integrations

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1. Partial Fraction Integrations

We will focus in this section on integration of rational function i.e. $q(x) = \frac{P(x)}{Q(x)}$, where both $P(x)$ and $Q(x)$ are polynomials and the *degree* of $P(x)$ is smaller than the degree of $Q(x)$.

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Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\ldots+\frac{A_k}{(ax+b)^k},\ k=1,2,\ldots$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\ldots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

To integrate the function $f(x)=\frac{P(x)}{Q(x)}$, we have to factorize the detonator completely, then use in each factor, the suitable partial fraction from above.

Example:

1. $\int \frac{3x+11}{x^2-x-6}\,dx.$
2. $\int \frac{x^2+4}{3x^3+4x^2-4x}\,dx.$
3. $\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx.$
4. $\int \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2}\,dx.$
5. $\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx.$
6. $\int \frac{x^2}{x^2-1}\,dx.$