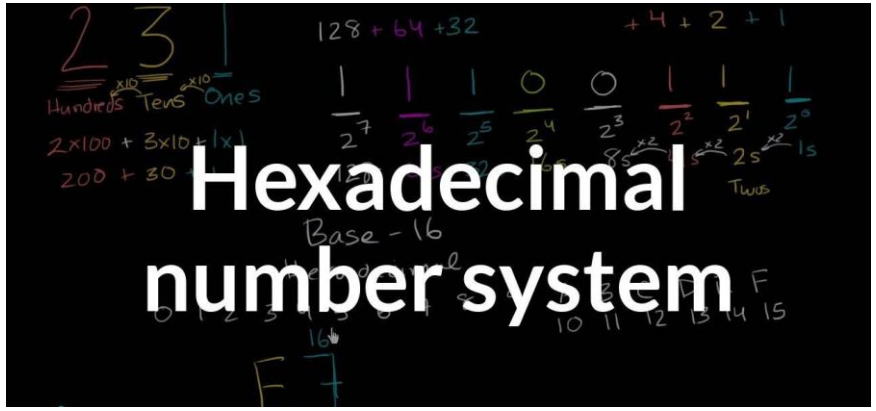
Introduction To Number Systems

Hexadecimal System



Objectives:

- **The main purpose for this lesson is to introduce the following:**

➢Understand the concept of Hexadecimal system.

➢Conversion any number decimal to the Hexadecimal number and converse.

➢define how can conversion Hexadecimal Fractions.

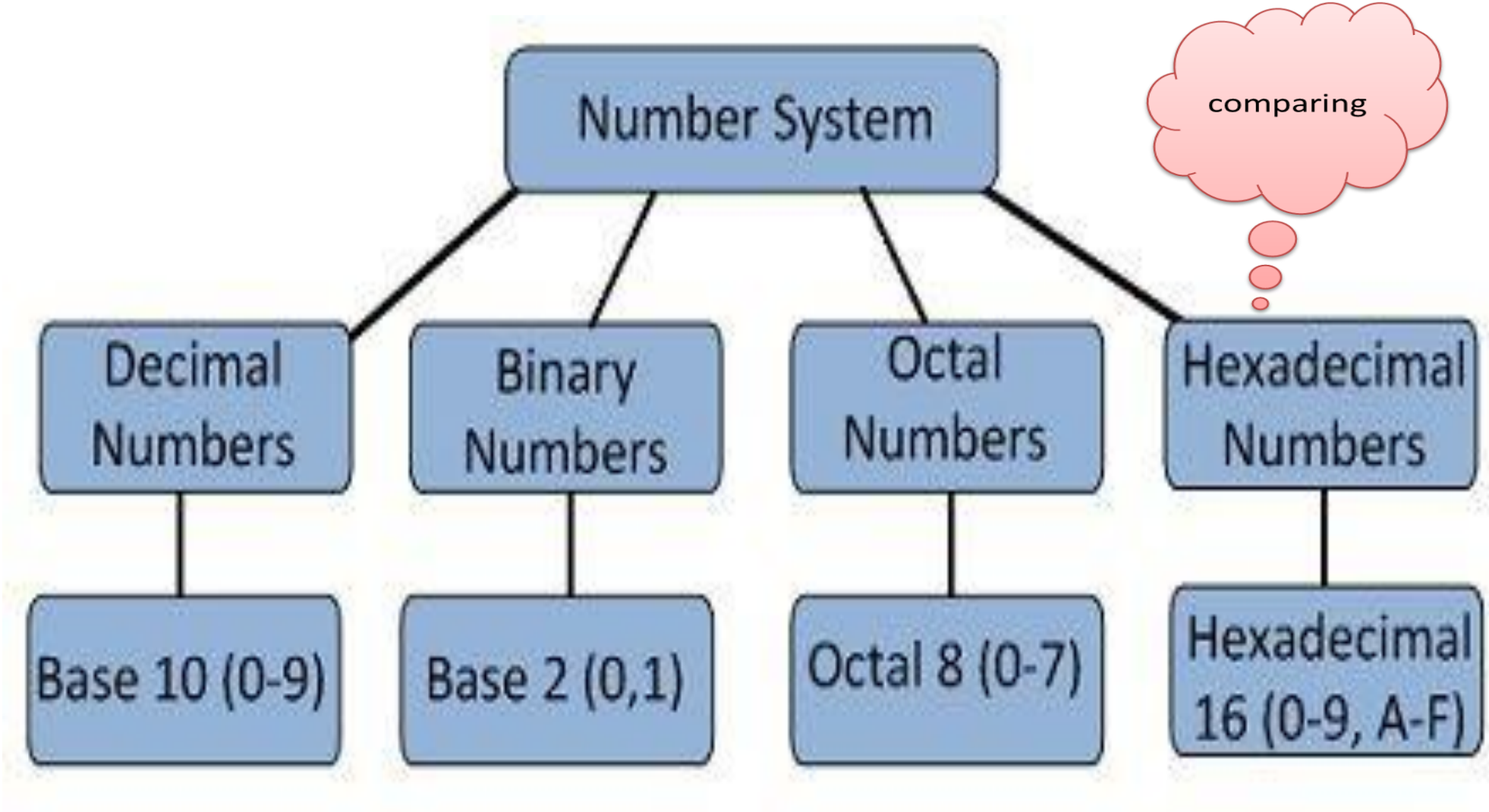
➢Relation between binary number system with Hexadecimal number system.

➢Conversion any binary number to the Hexadecimal number and converse.

➢Relation between octal number system with Hexadecimal number system.

➢Conversion any octal number to the Hexadecimal number and converse.

➢Use Arithmetic in the Hexadecimal System.


Number System
comparing
Decimal Numbers
Binary Numbers
Octal Numbers
Hexadecimal Numbers
Base 10 (0-9)
Base 2 (0,1)
Octal 8 (0-7)
Hexadecimal 16 (0-9, A-F)

Hexadecimal System

- Hexadecimal is the name given to a special number system which uses "16" as a base.
- In a hexadecimal (base 16) system, we need 16 single digits.
- We could use 0-9, then invent six more.
- More conveniently, we use the letters A - F for the remaining digits

(where A = 10, B = 11, C = 12, D = 13, E = 14 and F = 15).

Hexadecimal to Decimal Conversion

The principle of converting a base 16 number to decimal is the same as previously discussed, except that each column now represents an increasing power of 16 .

Example 1

Convert D30C to decimal.

Solution

$D30C = 13x16^3 + 3x16^2 + 0x16^1 + 12x16^0$

$= 13x4096 + 3x256 + 0 + 12$

$=53248 + 768 + 12$

$= 54028$

Decimal To Hexadecimal Conversion

Similarly, any decimal number can be converted to hexadecimal by successive divisions by 16, keeping track of the remainder.

Example 2

Convert **2563_{10} to base 16,**

Solution

		Quotient		Remainder
1.	2,563 ÷ 16 =	160		3
2.	160 ÷ 16 =	10		0
3.	10 ÷ 16 =	0		10

The process stops when the quotient becomes zero. The answer is found by reading "up" from the bottom.
Therefore, $\mathbf{2,563_{10}} = \mathbf{A03_{16}}$

Binary To Hexadecimal Conversion

Hexadecimal has another important property.

Since there are *exactly 16 hexadecimal digits, it* ***requires exactly 4 bits*** *to represent every hexadecimal digit (since* $2^4 = 16$*).*

- ***In order to convert the Binary number into its equivalent octal numbers, split the given binary number into groups and each group should contain four binary bits*** *(because* $2^4=16$*),*
- ***add zeros to the left if necessary****,*
- *and* ***then converting each group into its equivalent octal number.***

This can be shown by the following table on the right:

Binary To Hexadecimal Conversion

Example 3:

Convert $(100000111001110)_2$ to hexadecimal.

Solution:

0100|0001|1100|1110 = $\mathbf{41CE_{16}}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Hexadecimal To Binary Conversion

To convert Hexadecimal to binary, replace each Hexadecimal digit by its binary representation in 4 bits, so add zeros to left if necessary.

Example 4:

Convert **$F2D3_{16}$ to binary.**

Solution:

$$\mathbf{F2D3}_{16} = 1111\ 0010\ 1101\ 0011_2$$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Octal to Hexadecimal Conversion

When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal.

Example 5

Convert **345** octal into hex.

Solution:

Octal = 3 4 5

Binary = 011 100 101 = 011100101

Drop any leading zeros or pad with leading zeros to get groups of four binary digits (bits):

Binary 011100101 = 1110 0101

Then, look up the groups in a table to convert to hexadecimal digits.

Binary = 1110 | 0101

Hexadecimal = **E5**.

Hexadecimal To Octal Conversion

When converting from hexadecimal to octal, it is often easier to first convert the hexadecimal number into binary and then from binary into octal.

Example 6 :

Convert **A2DE** hex. into octal.

Solution:

Hexadecimal = **A 2 D E**

Binary = **1010 0010 1101 1110**

So, we can write

Binary = 001|010 |001|0 11|01 1|110

Octal = 1 2 1 3 3 6 = $\mathbf{121336}_8$

Hexadecimal Addition:

Use the following steps to perform hexadecimal addition:

1. Add one column at a time.

2. Convert to decimal and add the numbers.

3a. If the result of step two is 16 or larger subtract the result from 16 and carry 1 to the next column.

3b. If the result of step two is less than 16, convert the number to hexadecimal.

Example 7:

Add: **AC5A9+ED694**

Solution:

Carry Over:
1. Add one column at a time 2. Convert to decimal & add (9 + 4 = **13**) 3. Follow less than 16 rule Decimal **13** is hexadecimal **D**

A	C	5	A	9
E	D	6	9	4
				D

Carry Over:
1. Add next column 2. Convert to decimal & add (**10** + **9** = **19**) 3. Follow 16 or larger than 16 rule (**19** – 16 = **3** carry a **1**)

		1		
A	C	5	**A**	9
E	D	6	**9**	4
			3	D

Carry Over:
1. Add next column 2. Convert to decimal & add (**1** + 5 + 6 = **12**) 3. Follow less than 16 rule, convert to hex Decimal 12 is hexadecimal **C**

		1		
A	C	**5**	A	9
E	D	**6**	9	4
		C	3	D

	Carry Over:
1.	Add next column
2.	Convert to decimal & add (12 + 13 = 25)
3.	Follow 16 or larger than 16 rule (25 – 16 = 9 carry a 1)

	1				
A	C	5	A	9	
E	D	6	9	4	
	9	C	3	D	

	Carry Over:
1.	Add next column
2.	Convert and add (1 + 10 + 11 = 22)
3.	Follow 16 or larger than 16 rule (22 – 16 = 6 carry a 1)

	1				
A	C	5	A	9	
B	D	6	9	4	
6	9	C	3	D	

	Carry Over:
1.	Add next column
2.	Convert and add (1 + 0 + 0 = 1)
3.	Follow less than 16 rule

	1					
0	A	C	5	A	9	
0	B	D	6	9	4	
1	6	9	C	3	D	

Hexadecimal Subtraction

Subtraction in hexadecimal works similar to subtraction in decimal except that we occasionally have a digit larger than 9.

Example:

Subtract A8D2 - 3EAC (hexadecimal)

Solution:

- We'll align our numbers:

```
  A 8 D 2
- 3 E A C
  ----------
```

- Now in the ones place, we can't subtract C (12) from 2 so we borrow 1 from the sixteens place.

```
    12  18
  A 8 D 2
- 3 E A C
  ----------
        6
```

(D = 13 minus the 1 we borrowed) and gives us 18 ones (2 plus the 16 we got from the borrow), then subtract 18-12 = 6.

- Now we don't need to borrow because we can subtract

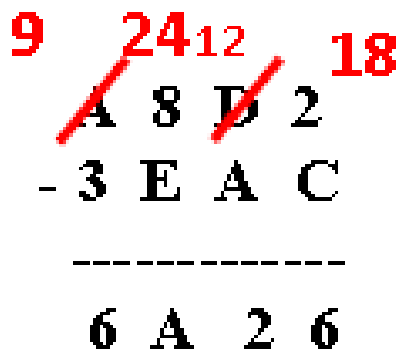
10 (A) from 12:

```
      12 18
   A  8 D 2
 - 3  E A C
 ------------
      2  6
```

- In the 256's place, we again need to borrow. We'll borrow 1 from the 4096's place and exchange it for sixteen 256's (one 4096 equals sixteen 256's). This leaves us 9 in the 4096's place (A = 10 minus the 1 that we borrowed), and gives us 24 in the 256's place (8 plus the 16 from the borrow). We then can subtract 24-14 = 10 = A. So we have:

```
 9  24 12 18
   A  8 D 2
 - 3  E A C
 ------------
    A  2  6
```

- **Finally, we subtract 9-3 = 6 in the 4096's place:**



Homework

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