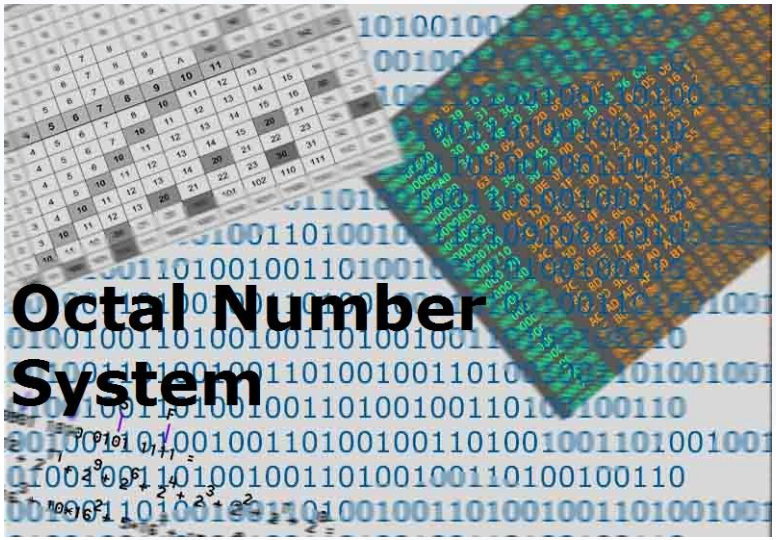
Introduction To Number Systems

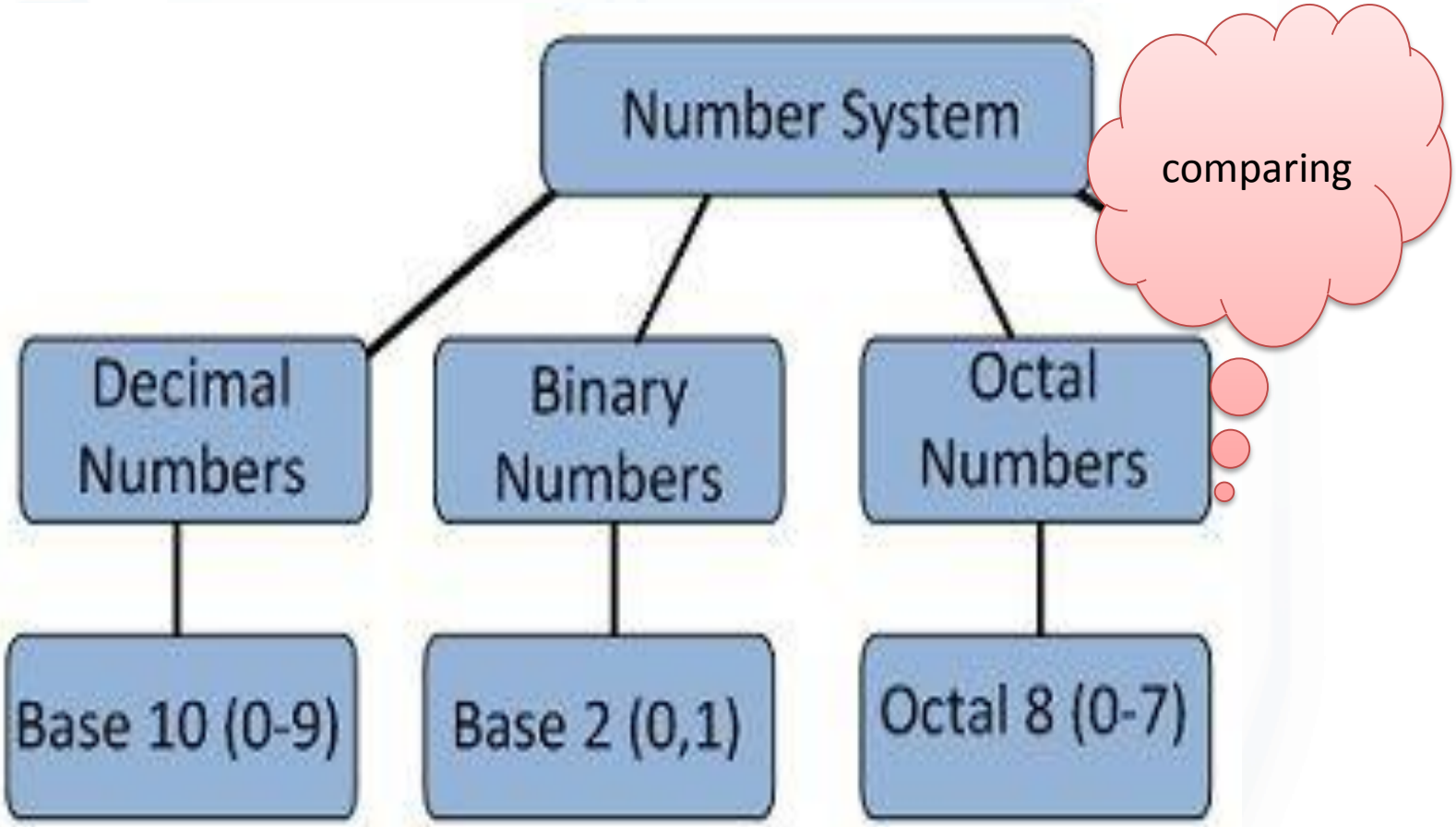
Octal System



Objectives:

The main purpose for this lesson is to introduce the following:

- Understand the concept of Octal system.
- Conversion any number decimal to the Octal number and converse.
- define how can conversion Octal Fractions.
- Relation between binary number system with octal number system.
- Conversion any binary number to the Octal number and converse.
- Use Arithmetic in the Octal System.


Number System
comparing
Decimal Numbers
Binary Numbers
Octal Numbers
Base 10 (0-9)
Base 2 (0,1)
Octal 8 (0-7)

Octal System

- The octal, or base 8, number system is a common system used with computers.
- Because of its relationship with the binary system, it is useful in programming some types of computers.
- Octal is fancy for Base Eight meaning eight symbols are used to represent all the quantities. They are 0, 1, 2, 3, 4, 5, 6, and 7.

Octal	0	1	2	3	4	5	6	7	10	11	12 …	17	20 …	30 …	77	100
Decimal	0	1	2	3	4	5	6	7	8	9	10 …	15	16 …	24 …	63	64

Octal to decimal conversion

An octal number can be converted to decimal by forming the ***sum of powers of 8*** of the coefficients.

Convert to decimal

Example 1:

$65_8 = 6 \times 8^1 + 5 \times 8^0 = 48 + 5 = 53_{10}$

Example 2:

$$764_8 = 7 \times 8^2 + 6 \times 8^1 + 4 \times 8^0 = 488 + 48 + 4 = 500_{10}$$

Example 3:

$$0.235_8 = 2 \times 8^{-1} + 3 \times 8^{-2} + 5 \times 8^{-3}$$
$$= 2 \times 0.125 + 3 \times 0.016 + 5 \times 0.002 = 0.307.$$

Conversion of decimal to octal (base 10 to base 8)

- To convert from a base-10 integer numeral to its base-8 , the number is divided by eight, and the remainder is the least-significant bit. The (integer) result is again divided by eight, its remainder is the next least significant bit. This process repeats until the quotient becomes zero.

Example 4:

convert $(177)_{10}$ to octal equivalent

	Quotient	Remainder
$177 \div 8 =$	22	1
$22 \div 8 =$	2	6
$2 \div 8 =$	0	2

Answer = 261

Note: the answer is read from bottom to top, the same as with the binary case.

Decimal to octal conversion

To convert a decimal fraction to octal, **multiply by 8; the integer part of the result is the first digit** of the octal fraction. Repeat the process with the fractional part of the result, until it is null or within acceptable error bounds.

Conversion of decimal fraction to octal fraction

Example 4:

Convert $(0.1640625)_{10}$ to octal:

	Integer	Fraction
0.1640625x8 =	1	0.3125
0.3125 x8 =	2	0.5
0.5x8 =	4	0

Therefore, $0.1640625_{10} = (\mathbf{0.124})_8$.

Example 5:

convert $(0.523)_{10}$ to octal equivalent up to 3 decimal places.

Solution

	Integer	Fraction
0.523 x 8 =	4	0.184
0.184 x 8 =	1	0.472
0.472 x 8 =	3	0.776

So the answer is $(\mathbf{0.413})_8$

Octal to binary conversion

To convert octal to binary, **replace each octal digit by its binary representation in 3 bits**, so **add zeros to the left if necessary.**

Example 6:

Convert 51_8 to binary:

Solution

$$5_8 = 101_2$$
$$1_8 = 001_2$$

Therefore, $51_8 = 101001_2$.

Decimal Base-10	Binary Base-2	Octal Base-8	Hexadecimal Base-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9

Binary to Octal conversion

In order to convert the Binary number into its equivalent octal numbers, split the given binary number into groups and each group should contain ***three binary bits (because $2^3=8$), add zeros to the left if necessary, and then converting each group into its equivalent octal number.***

Example 7:

convert binary **1010111100** to octal.

Solution:

$1010111100_2 = 1274_8$.

001	010	111	100
1	2	7	4

Example 7:

Convert binary **11100.01001** to octal:

Solution:

$11100.01001_2 = 34.22_8$.

011	100	.	010	010
3	4	.	2	2

Arithmetic in octal system

Octal Addition

Addition of the octal number is carried out in the same way as the decimal addition is performed. The steps are given below:

1. First, add the two digits of the unit column of the octal number in decimal.
2. This process is repeated for each larger significant digit of the octal number.
3. During the process of addition, if the sum is less than or equal to 7, then it can be directly written as an octal digit.
4. If the sum is greater than 7, then subtract 8 from the digit and carry 1 to the next digit position.
5. Note that in this addition the largest octal digit is 7.

Example 8:

Evaluate:

(i) $(162)_8 + (537)_8$

Solution:

```
  1 1        <---- carry
  1 6 2
+ 5 3 7
-------
  7 2 1
```

Therefore, sum = 721_8

(ii) $(136)_8 + (636)_8$

Solution:

1 <---- carry

1 3 6

6 3 6

7 7 4 <---- 6+6=12>8 in decimal, so in octal 6+6= 12-8=14 (4 and carry 1)

Therefore, sum = 774_8

(iii) $(25.27)_8 + (13.2)_8$

Solution:

1 <---- carry

25.27

13.2

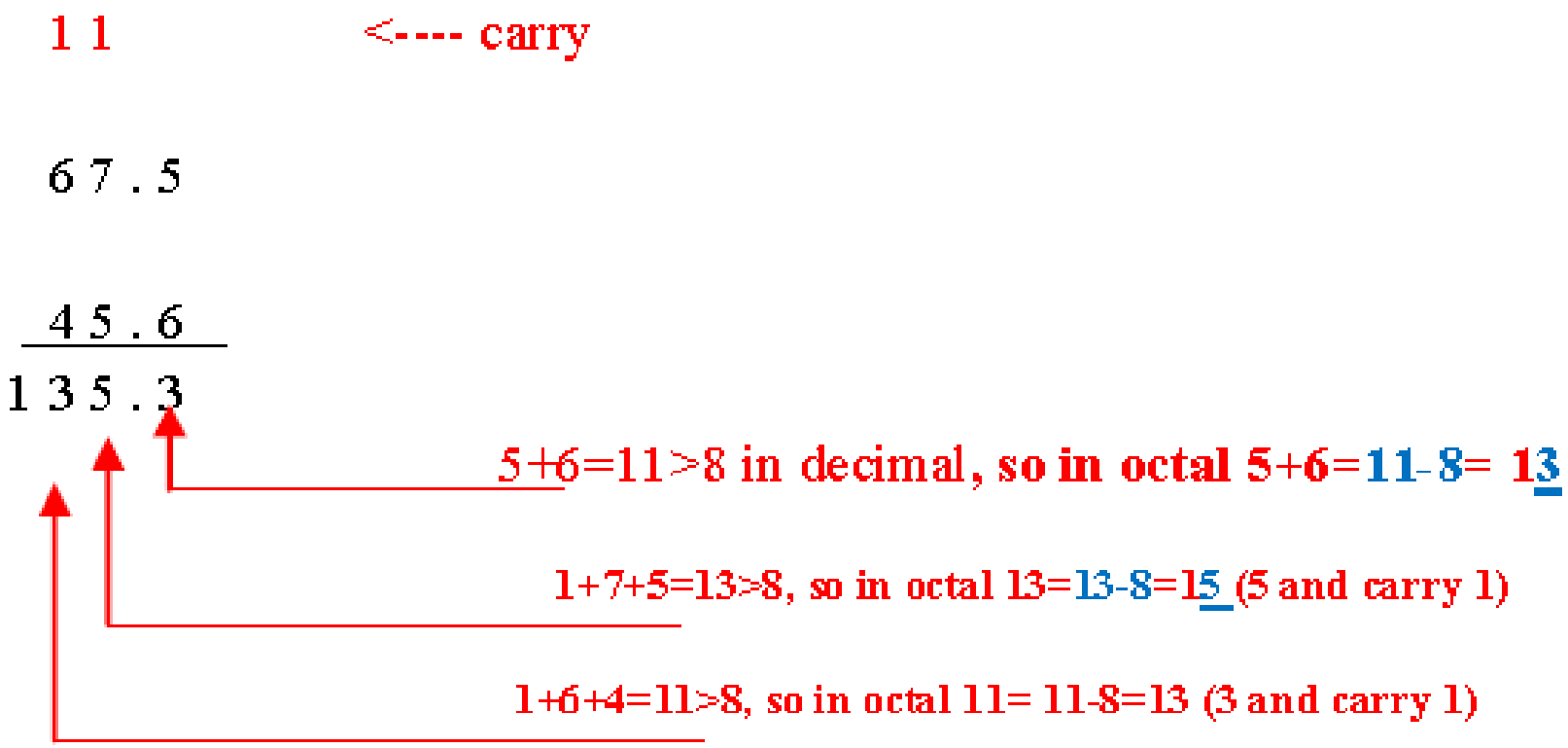
40.47

5+3= 8>7 in decimal, so in octal 5+3=8-8=10 (0 and carry 1)

Therefore, sum = $(40.47)_8$

(iv) $(67.5)_8 + (45.6)_8$

Solution:



Therefore, sum = $(135.3)_8$

Subtraction of Octal Numbers

The subtraction of octal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in the quantity of ***the borrow***.

In the decimal system, you had to borrow a group of 10_{10}.

In the binary system, you borrowed a group of 2_{10}.

In the octal system you will borrow a group of 8_{10}.

DECIMAL	BINARY	OCTAL
10_{10}	10_2	10_8
$-\ 1_{10}$	$-\ 1_2$	$-\ 1_8$
9_{10}	1_2	7_8

10	2	8	Borrow
$\not{1}0_{10}$	$\not{1}0_2$	$\not{1}0_8$	
$-\ 1_{10}$	$-\ 1_2$	$-\ 1_8$	
9_{10}	1_2	7_8	

Example 9

Subtract $532_8 - 174_8$

Solution:

$$\begin{array}{r} {\scriptstyle +8} \quad {\scriptstyle +8} \\ 4\ \not{5}\ \not{3}\ 2_8 \\ -\ 1\ 7\ 4_8 \\ \hline 3\ 3\ 6 \end{array}$$

Steps:

1. Since 2<4 then borrow 1 from 3 and add 8 to 2.
2. 2+8=10 in decimal so 10-4=6.
3. In the second column we have 2 after borrowing but 2<7, so we need to borrow 1 from 5 and add 8 to 2.
4. 2+8 = 10, 10-7=3
5. We have 4 after borrowing so we have 4-1=3.

Homework

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