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Applicability of time conformable derivative to Wick-fractional-stochastic PDEs

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Abstract Fractional-stochastic quadratic-cubic nonlinear Schrödinger equation (QC-NLSE) describing propagation of solitons through optical fibers is analyzed. Hermite transforms, white noise analysis and an improved computational method are used to investigate uncertain solutions for QC-NLSE. Specifically, Hermite transformation is applied to convert fractional-stochastic differential equations by Wick-type into deterministic fractional differential equations with an integral term. Furthermore, inverse Hermite transformation is employed to obtain stochastic solutions in the white noise space. Characteristics of presented equations are shown by using some specific values of physical arguments on obtained solutions.

1. Introduction

The fractional calculus consisting of any real number order derivative and integral operators became a significant tool and has applications in many different disciplines. Some of recent studies in this area may be seen in [1–10]. An equation involving uncertain, random or noisy terms besides having fractional-order operators is known as fractional-stochastic differential equation (FSDE). Researching on FSDEs is quite active and important area nowadays. Because models using integer valued derivative and integral operators do not take into account previous historical effects of models at every time step but the models considering also past effects may be more accurate and suitable in the computational science. On the other hand, fractional-order operators works locally and takes into account historical effects, which is considered as strength

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of these models over the models generated by integer valued operators. There are many researchers connected to stochastic fractional partial differential equations [11–16]. Some further engineering and scientific applications of fractional-order models may be seen in the references: [28–49].

Researching with FSDE is more complicated, due to uncertainties and its extra random locations [11]. Stochastic models have a significant act in a range of implementation fields of science [11–16]. A model may include some uncertain, or noisy or random terms due to faults in empirical states, experiments or measured data [17]. Random models have application areas in applied science including physics, biology, chemistry, finance, economics, medicine and so on.

There have been many types of research and developments on analysis and solutions of NLSE in the history of research [18–22]. NLSE has been a vital role in various fields of engineering and science. It studies in many applied areas, including fluid dynamics, plasma physics, nonlinear optics, and protein chemistry. We focus on stochastic QC-NLSE with Wick-type in this article due to its efficient power in modeling. This paper will study quadratic-cubic law nonlinearity of QC-NLSE. Present model gives soliton solutions (both spatial and temporal) with applications in ultra-fast soliton switches, soliton lasers, logic gate devices, and optical communications [22–25]. NLSE with quadratic cubic nonlinearity is given by

$$ip_{\rho}(\kappa,\rho) + ap_{\kappa\kappa}(\kappa,\rho) - b_1 p(\kappa,\rho)|p(\kappa,\rho)| + b_2 p(\kappa,\rho)|p(\kappa,\rho)|^2 = 0, \tag{1.1}$$

where

$i = \sqrt{-1}.$

In Eq. (1.1), κ and ρ are the independent variables representing the temporal and spatial variables, respectively. $p(\kappa,\rho)$ is dependent variable. Real-valued constant represents group velocity dispersion (GVD), while b_1 and b_2 are real-valued constants. In [24], analytical self-similar wave solutions of equation were constructed. In [25], optical solitons are obtained by using a modified extended direct algebraic method.

We can rewrite Eq. (1.1) by using conformable derivatives (CDs) as: [7]

$$iD_{\rho}^{\alpha}p(\kappa,\rho) + a(\rho)D_{\rho}^{2\alpha}p(\kappa,\rho) - b_1(\rho)p(\kappa,\rho)|p(\kappa,\rho)| + b_2(\rho)p(\kappa,\rho)|p(\kappa,\rho)|^2 = 0, \tag{1.2}$$

$(\kappa,\rho) \in \mathbb{R}\times\mathbb{R}_+, 0 < \alpha \le 1,$

where $a(\rho), b_1(\rho), b_2(\rho)$ are measurable in $\mathbb{R}_+$.

CD was first expressed in [26]. Presented derivative is important since it eliminates shortcomings of previous derivative terms. Furthermore, fractional derivative operator with order $\eta \in (0,1]$ is an efficient operator since it may include any η.

For order $\eta \in (0,1)$, the CD of a $g : (0,\infty) \to \mathbb{R}$ is defined as

$${}_tD^{\eta}g(t) = \lim_{\varepsilon\to 0}\frac{g(t+\varepsilon t^{1-\eta}) - g(t)}{\varepsilon}.$$

This definition of CD describes a natural normal derivatives as well. This expression for $0 \leqslant \eta < 1$ describes with usual expressions on polynomials (up to a constant).

Some characteristics [26,27] of CD may be given as:

a) ${}_tD^{\eta}t^{\alpha} = \alpha t^{\alpha-\eta}, \ \forall\eta \in \mathbb{R},$

b) ${}_tD^{\eta}(fg) = f_tD^{\eta}g + g_tD^{\eta}f,$

c) ${}_tD^{\eta}(fog) = t^{1-\eta}g'(t)f'(g(t)),$

d) ${}_tD^{\eta}\left(\frac{f}{g}\right) = \frac{g_tD^{\eta}f - f_tD^{\eta}g}{g^2}.$

Advantage of this derivative is that it is easy to apply. There have been several studies about fractional computations by using CDs, see e.g. [27–31].

Eq. (1.1) with Wick type by using CDs is:

$$iD_{\rho}^{\alpha}P + A(\rho)\diamondsuit D_{\kappa}^{2\alpha}P - B_1(\rho)\diamondsuit P\diamondsuit|P| + B_2(\rho)\diamondsuit P\diamondsuit|P|^2 = 0. \tag{1.3}$$

in which $\diamondsuit$ is Wick term in Kondratiev space$(V)_{-1}$, and $A(\rho), B_1(\rho), B_2(\rho)$ are $(V)_{-1}$-type maps.

In this article we will apply an improved computational method [28] to find the traveling wave solutions of Eq. (1.3). This method is important for solving physical events and comparing the accuracy of the results. In particular, there are very few studies to find the traveling wave solutions of these type differential equations.

We aim to obtain explicit solutions of QC-NLSE via time fractional conformable maps. We apply an improved computational method [28], white noise theory and Hermite transformation to get traveling wave solutions (TWS) for QC-NLSE by using CDs. Additionally, wave solutions of stochastic QC-NLSE by CDs are obtained via inverse Hermite transformation. We also express how stochastic solutions can be given in terms of functional solutions in Brownian motion type.

2. Wick-type Stochastic fractional QC-NLSE equation

Suppose that $F(z) = \sum_{\alpha}a_{\alpha}H_{\alpha}$ with $a_{\alpha} \in \mathbb{R}^n$ and $z = (z_1,\ldots,z_n) \in \mathbb{C}^n$ are taken from $(V)_{-1}^n$.

2.1. Exact solutions of Eq. (1.3)

Wick product of $F = \sum_{\alpha}a_{\alpha}H_{\alpha}$ and $G = \sum_{\beta}b_{\beta}H_{\beta} \in (V)_{-1}^n$ is given as

$$F\diamondsuit G = \sum_{\alpha,\beta}(a_{\alpha},b_{\beta})H_{\alpha+\beta}.$$

HE of $F = \sum_{\alpha}a_{\alpha}H_{\alpha} \in (V)_{-1}^n$, $\widetilde{F}(z) = H(F)$ is

$$H(F) := \widetilde{F}(z) = \sum_{\alpha}a_{\alpha}z^{\alpha} \in \mathbb{C}^n,$$

in which $z := (z_1,z_2,\ldots) \in \mathbb{C}^n, z^{\alpha} := (z_1^{\alpha_1}, z_2^{\alpha_2},\ldots)$.

It is not hard to prove that, for $F,G \in (V)_{-1}^n$, if $\widetilde{F}(z)$ and $\widetilde{G}(z)$ exist, then $F\widetilde{\diamondsuit}G = \widetilde{F}(z)\,\widetilde{G}(z)$.

$$z^{\alpha} := \left(z_1^{\alpha_1}, z_2^{\alpha_2},\ldots,z_n^{\alpha_n}\right) = \sum_{k=1}^{n}z_k^{\alpha_1}z_k^{\alpha_2},$$

where $z_k^{\alpha_i} \in \mathbb{C}^n$.

By taking HE of Eq. (1.3),

$$iD_{\rho}^{\alpha}\widetilde{P}(\kappa,\rho,z) + \widetilde{A}(\rho)\diamondsuit D_x^{2\alpha}\widetilde{P}(\kappa,\rho,z) - \widetilde{B}_1(\rho)\diamondsuit\widetilde{P}(\kappa,\rho,z)\diamondsuit\left|\widetilde{P}(\kappa,\rho,z)\right|$$

$$+\widetilde{B}_2(\rho)\lozenge\widetilde{P}(\kappa,\rho,z)\lozenge\left|\widetilde{P}(\kappa,\rho,z)\right|^{\lozenge 2}=0. \tag{2.1}$$

For TWS of Eq. (2.1), let

$$\widetilde{A}(\rho,z):=u(\rho,z),\ \widetilde{B}_1(\rho,z):=b_1(\rho,z),\ \widetilde{B}_2(\rho,z):=b_2(\rho,z)$$

$$\widetilde{P}(\kappa,\rho,z):=p(\kappa,\rho,z):=p(\xi(\kappa,\rho,z))e^{i\phi(\rho,z)}$$

with

$$\xi(\kappa,\rho,z)=k\left(\frac{x^\alpha}{\alpha}\right)+\varpi\int_0^t\frac{\theta(\rho,z)}{\rho^{1-\alpha}}d\rho \tag{2.2}$$

in which k,ϖ are constants. Then, applying the transformation above Eq. (2.1) can be transformed to nonlinear ordinary differential equation as follows:

$$-i\varpi\theta(\rho,z)\frac{dp}{d\xi}+A(\rho,z)k^2\frac{d^2p}{d\xi^2}-\phi(\rho,z)p-B_1(\rho,z)p^2+B_2(\rho,z)p^3=0, \tag{2.3}$$

Let us assume that the solution to Eq. (2.3) is a series expansion solution. We can express this as follows,

$$p(\xi)=\sum_{i=0}^{N}\alpha_i(\rho,z)\left\{\frac{G(\xi)}{H(\xi)}\right\}^i, \tag{2.4}$$

where $\alpha_i(\rho,z)$ are unknown functions $\alpha_N(\rho,z)\neq 0$ and $G(\xi)$ and $H(\xi)$ satisfy the system with variable coefficients as follows:

$$G'(\xi)=g(\rho)G(\xi), \tag{2.5}$$
$$H'(\xi)=g(\rho)G(\xi)+h(\rho)H(\xi),$$

Now, the Eqs. (2.5) accepts the following expression:

$$\frac{G(\xi)}{H(\xi)}:=\frac{-h(\rho)+g(\rho)}{-h(\rho)e^{\xi(h(\rho)-g(\rho))}+g(\rho)}, \tag{2.6}$$

where $g(\rho)\neq 0$ and $h(\rho)\neq 0$ are integrable functions for factor ρ with $g(\rho)\neq h(\rho)$.

By balancing p^3 with $\frac{d^2p}{d\xi^2}$ in Eq. (2.3), is determined $1=N$. Then, Eq. (2.3) has a solution:

$$p(\xi)=\alpha_0(\rho,z)+\alpha_1(\rho,z)\frac{G(\xi)}{H(\xi)}, \tag{2.7}$$

where $G(\xi)$ and $H(\xi)$ satisfied Eq. (2.5).

By writing (2.7) and (2.5) in (2.3), by equating coefficients in $\frac{G(\xi)}{H(\xi)}$ to zero, some solutions are obtained. One of the six groups of these solutions is obtained as follow;

$$\begin{aligned}
\alpha_0(\rho,z)&=\frac{B_1(\rho,z)B_2(\rho,z)+\sqrt{2}\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(g(\rho)-h(\rho))^2}}{2B_2(\rho,z)^2},\\
\alpha_1(\rho,z)&=\frac{\sqrt{2}A(\rho,z)B_2(\rho,z)k^2g(\rho)(g(\rho)-h(\rho))}{\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(g(\rho)-h(\rho))^2}},\\
\theta(\rho,z)&=-\frac{iB_1(\rho,z)\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(g(\rho)-h(\rho))^2}}{\sqrt{2}B_2(\rho,z)^2\varpi(g(\rho)-h(\rho))},\\
\phi(\rho,z)&=-\frac{B_1(\rho,z)^2+2A(\rho,z)B_2(\rho,z)k^2(g(\rho)-h(\rho))^2}{4B_2(\rho,z)}.
\end{aligned} \tag{2.8}$$

By substitutions (2.6) and (2.7) into the solution $\widetilde{P}(\kappa,\rho,z):=p(\kappa,\rho,z):=p(\xi(\kappa,\rho,z))e^{i\phi(\rho,z)}$, hence; via (2.8), we provide new solutions of Eq. (2.1):

$$\begin{aligned}
&\widetilde{P}(\kappa,\rho,z)\\
&=e^{-i\frac{B_1(\rho,z)^2+2A(\rho,z)B_2(\rho,z)k^2(g(\rho)-h(\rho))^2}{4B_2(\rho,z)}}\\
&\frac{B_2(\rho,z)\quad\times B_1(\rho,z)+\sqrt{2}\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(-h(\rho)+g(\rho))^2}}{2B_2(\rho,z)^2}\\
&-\frac{\sqrt{2}A(\rho,z)B_2(\rho,z)k^2g(\rho)(g(\rho)-h(\rho))}{\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(-h(\rho)+g(\rho))^2}}\\
&-\frac{-h(\rho)+g(\rho)}{-h(\rho)+g(\rho)e^{Y(h(\rho)-g(\rho))}}\\
&+k\frac{x^\alpha}{\alpha}+\varpi\int_0^t\frac{iB_1(\rho,z)\sqrt{-A(\rho,z)B_2(\rho,z)^3k^2(g(\rho)-h(\rho))^2}}{\sqrt{2}B_2(\rho,z)^2}d\rho.
\end{aligned} \tag{2.9}$$

Next, we are concerned with Eq. (1.3) by using Theorem 4.1.1 [12] and inverse Hermite transformation. The term $p(\kappa,\rho,z)$ in Eq. (2.1) exists on $\widetilde{G}\subset\mathbb{R}\times\mathbb{R}_+, a<\infty, b>0$, all fractional derivatives in Eq. (2.1) for $(\kappa,\rho,z)\in\widetilde{G}\times K_a(b)$, for $(\kappa,\rho)\in\widetilde{G}$ for $\forall\ z\in\widetilde{G}\times K_a(b)$. These derivatives are analytic with respect to $z\in K_a(b)$, where $(\kappa,\rho)\in\widetilde{G}$. By [12], solution of Eq. (1.3) is $P(\kappa,\rho)$ $(P(\kappa,\rho)\in(V)_{-1})$, where $p(\kappa,\rho,z)=\widetilde{P}(\kappa,\rho)(z)$ for $\forall(\kappa,\rho,z)\in\widetilde{G}\times K_a(b)$. Then, we obtain solutions of Eq. (1.3) for $A(\rho)>0, B_1(\rho)>0, B_2(\rho)>0$ with the aid of the inverse Hermite transform on Eqs. (2.9), as follows;

$$\begin{aligned}
\widetilde{P}(\kappa,\rho)=&e^{-i\frac{B_1(\rho)^{\lozenge 2}+2A(\rho)\lozenge B_2(\rho)\lozenge k^2(g(\rho)-h(\rho))^2}{4B_2(\rho)}}\\
&+\frac{B_1(\rho)\lozenge B_2(\rho)+\sqrt{2}\sqrt{-A(\rho)\lozenge B_2(\rho)^{\lozenge 3}k^2(g(\rho)-h(\rho))^2}}{2B_2(\rho)^{\lozenge 2}}\\
&+\frac{\sqrt{2}A(\rho)\lozenge B_2(\rho)k^2g(\rho)(g(\rho)-h(\rho))}{\sqrt{-A(\rho)\lozenge B_2(\rho,z)^{\lozenge 3}k^2(g(\rho)-h(\rho))^2}}\\
&+\frac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)e^{L(h(\rho)-g(\rho))}}L
\end{aligned} \tag{2.10}$$

where

$$L=k\frac{x^\alpha}{\alpha}+\varpi\int_0^t\frac{-\frac{iB_1(\rho)\lozenge\sqrt{-A(\rho)\lozenge B_2(\rho)^{\lozenge 3}k^2(g(\rho)-h(\rho))^2}}{\sqrt{2}B_2(\rho)^{\lozenge 2}\lozenge\varpi(g(\rho)-h(\rho))}}{\rho^{1-\alpha}}d\rho.$$

3. Examples

We give specific application examples to show the suitability of our conclusions. We can say that solutions of Eq. (1.3) depend strongly on change of $A(\rho)$, $B_1(\rho)$ and $B_2(\rho)$ values. So, different solutions of Eq. (1.3) may be examined for values of $A(\rho)$, $B_1(\rho)$ and $B_2(\rho)$ in Eq. (2.10). For illustrating this, we can give the following examples.

Suppose $A(\rho)=\sigma B_2(\rho), B_2(\rho)=\mu B_1(\rho)$ and $B_1(\rho)=f(\rho)+\lambda W_\rho$, where σ,μ,λ are parameters, $f(\rho)$ is a measurable in $\mathbb{R}_+$. Hermite transformation of W_ρ is $\widetilde{W}_\rho(z)=\sum_{i=0}^{\infty}z_i\int_0^\rho\Psi_i(t)dt$ [29]. By employing $\widetilde{W}_\rho(z)$, Eq. (2.10) yields the white noise functional solution of Eq. (1.3):

$$P(\kappa,\rho)=e^{-i\frac{(f(\rho)+\lambda W_\rho)^2+2\sigma\mu(f(\rho)+\lambda W_\rho)k^2(g(\rho)-h(\rho))^2}{4}}\left(\begin{array}{c}\mu\big(f(\rho)+\lambda W_\rho\big)^2\\ \dfrac{+\sqrt{2}\sqrt{-\sigma k^2(g(\rho)-h(\rho))^2}}{2}+\dfrac{\sqrt{2}\sigma k^2 g(\rho)}{\sqrt{-\sigma k^2}}\left(\dfrac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)\exp\left\{\dfrac{(h(\rho)-g(\rho))(k\left(\frac{x^\alpha}{\alpha}\right)}{-\frac{i\sqrt{-\sigma k^2}}{\sqrt{2}}\left\{\int_0^\rho\frac{f(t)}{\rho^{1-\alpha}}dt+\lambda\left(B_\rho-\frac{\rho^2}{2}\right)\right\}+c)}\right\}}\right)\end{array}\right). \qquad (3.1)$$

3.1. Case 1

When $\sigma=-1,\mu=-\frac{2}{5}$ and $\lambda=0.3$, Eq. (3.1) give the white noise functional solution of Eq. (1.3).

$$P(\kappa,\rho)=e^{-i\frac{(f(\rho)+0.3W_\rho)^2+\frac{2}{5}(f(\rho)+0.3W_\rho)k^2(g(\rho)-h(\rho))^2}{4}}\left(\begin{array}{c}-\frac{2}{5}\left(f(\rho)+0.3W_\rho\right)^2\\ \dfrac{+\sqrt{2}\sqrt{k^2(g(\rho)-h(\rho))^2}-\sqrt{2}kg(\rho)\left(\frac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)}\right))\exp\left\{\dfrac{(h(\rho)-g(\rho))(k\left(\frac{\kappa^\alpha}{\alpha}\right)}{-\frac{i\sqrt{-\sigma k^2}}{\sqrt{2}}\left\{\int_0^\rho\frac{f(\rho)}{\rho^{1-\alpha}}d\rho+\lambda\left(B_\rho-\frac{\rho^2}{2}\right)\right\}+c)}\right\}.}{2}\end{array}\right) \qquad (3.2)$$

3.2. Case 2

When $\sigma=\frac{1}{2},\mu=2$ and $\lambda=-1$, Eq. (3.1) give solution of Eq. (1.3).

$$P(\kappa,\rho)=e^{-i\frac{(f(\rho)-W_\rho)^2+2(f(\rho)-W_\rho)k^2(g(\rho)-h(\rho))^2}{4}}\left(\begin{array}{c}2\big(f(\rho)-W_\rho\big)^2\\ \dfrac{+\sqrt{2}\sqrt{-\frac{1}{2}k^2(h(\rho)-g(\rho))^2}}{2}+\dfrac{\sqrt{2}\frac{1}{2}kg(\rho)}{\sqrt{-\frac{1}{2}}}\left(\dfrac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)\exp\left\{\dfrac{(h(\rho)-g(\rho))(k\left(\frac{x^\alpha}{\alpha}\right)}{-\frac{i\sqrt{-\frac{1}{2}k^2}}{\sqrt{2}}\left\{\int_0^\rho\frac{f(t)}{\rho^{1-\alpha}}dt-\left(B_\rho-\frac{\rho^2}{2}\right)\right\}+c)}\right\}}\right)\end{array}\right). \qquad (3.3)$$

3.3. Case 3

When $\sigma = 1, \mu = -1$ and $\lambda = 0$, Eq. (3.1) give white noise functional solution of (1.3):

$$P(\kappa,\rho) = e^{-i\frac{f(\rho)^2-2f(\rho)k^2(g(\rho)-h(\rho))^2}{4}}\left(\frac{-f(\rho)^2+\sqrt{2}\sqrt{-k^2(g(\rho)-h(\rho))^2}}{2}+\frac{\sqrt{2}k^2 g(\rho)}{\sqrt{-k^2}}\left(\frac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)\exp\left\{\frac{(h(\rho)-g(\rho))\left(k\left(\frac{\kappa^\alpha}{\alpha}\right)\right)}{-\frac{i\sqrt{-k^2}}{\sqrt{2}}\left\{\int_0^\rho \frac{f(t)}{\rho^{1-\alpha}}dt\right\}+c)}\right\}}\right)\right). \tag{3.4}$$

In particular, Figs. 1–4 are plotted for without white noise effect and with white noise effect. The white noise effect is generated in the part (a) as time changes, the white noise effect is generated in the part (b) as time and x changes and without white noise functional, the noise is disappearing as time increases in the part (c).

3.4. Case 4

When $\sigma = \mu = \lambda = -2$, Eq. (3.1) give solution of Eq. (1.3).

$$P(\kappa,\rho) = e^{-i\frac{(f(\rho)-2W_\rho)^2+8(f(\rho)-2W_\rho)k^2(g(\rho)-h(\rho))^2}{4}}\left(\frac{-2\left(f(\rho)-2W_\rho\right)^2+\sqrt{2}\sqrt{2k^2(g(\rho)-h(\rho))^2}}{2}-2kg(\rho)\left(\frac{g(\rho)-h(\rho)}{g(\rho)-h(\rho)\exp\left\{\frac{(h(\rho)-g(\rho))(k\left(\frac{\kappa^\alpha}{\alpha}\right)}{-ik\left\{\int_0^\rho \frac{f(\rho)}{\rho^{1-\alpha}}dt-2\left(B_\rho-\frac{\rho^2}{2}\right)\right\}+c)}\right\}}\right)\right) \tag{3.5}$$

In Fig. 5, we can say that fractional-order has an effect on solution of Eq. (1.3). Fig. 5 show the change of wave motions for exact solutions (3.2)–(3.5) with distinct α.

We have already used the following relation [29] in the above examples.

$$\exp^{\diamond}(B_\rho) = \exp\left(B_\rho - \frac{\rho^2}{2}\right).$$

4. Physical reviews

In this part, we present some graphics to explain behaviors of solutions obtained in previous section for Eq. (1.3).

In above figures, we use $g(\rho) = -1, h(\rho) = 0.2, k = 1, \alpha = 0.5, c = 1, B_1(\rho) = f(\rho) + \lambda W_\rho, f(\rho) = \sinh(2\rho)$. Furthermore, we shown characteristic behaviors of stochastic Eq. (1.3). We thought Brownian motion $B_\rho = 2\sin(0.5\rho), B_\rho = [0,1] \times \text{Sinh}2\rho$ and $B_\rho = i, i = 1, 2,$ in these figures. In (c), we can say that the behaviors of stochastic Eq. (1.3) unaffected by stochastic term W_ρ.

5. Summary and discussion

One of the important issues of random PDEs is random waves. In this paper, we focused on a Wick-type stochastic QC-NLSE. To obtain exact or explicit solutions of fractional-random QC-NLSE, we discussed Wick-type fractional-uncertain QC-NLSE (1.3) expressed by Gaussian white noise. Besides that, we solved deterministic fractional QC-NLSE. There is an important connection between white noise Gaussian and Poisson spaces. Thus, in case of functions $A(\rho), B_1(\rho), B_2(\rho)$ are functions of type of Poisson white noise as in Eq. (1.3), one may reproduce some Poisson type solutions by using this connection. As $\alpha \rightarrow 1$, we can obtain explicit TWS and Wick-type stochastic QC-NLSE with integer derivatives and variable coefficients. Then, Eq. (2.5) has different solutions for different values of $g(\rho)$ and $h(\rho)$. We used Matlab and Mathematica in our computations and generating figures.

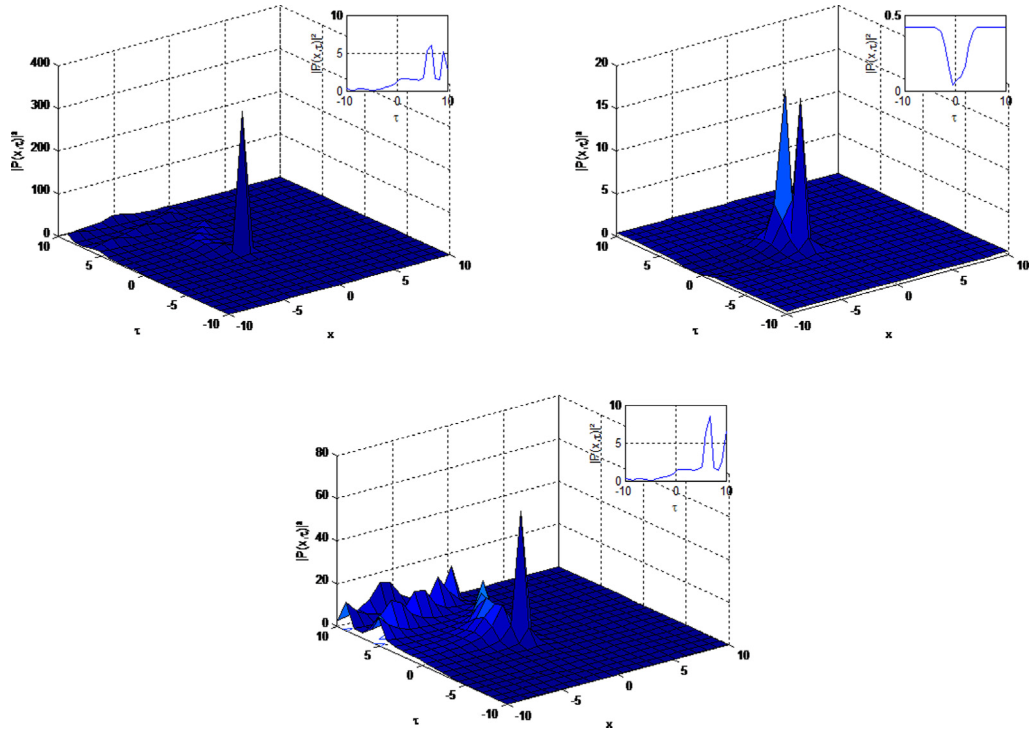

Fig. 1 Wave profiles of solution (3.2) for fractional-stochastic QC-NLSE (1.3) with Wick-type $\left(\sigma = -1, \mu = -\frac{2}{5}, \lambda = 0.3\right)$ (a) for $B_\rho = 2\sin(0.5\rho)$, (b) for $B_\rho = \text{rand } om[0,1]x\sinh 2\rho$, (c) for $B_\rho = 1.2$.

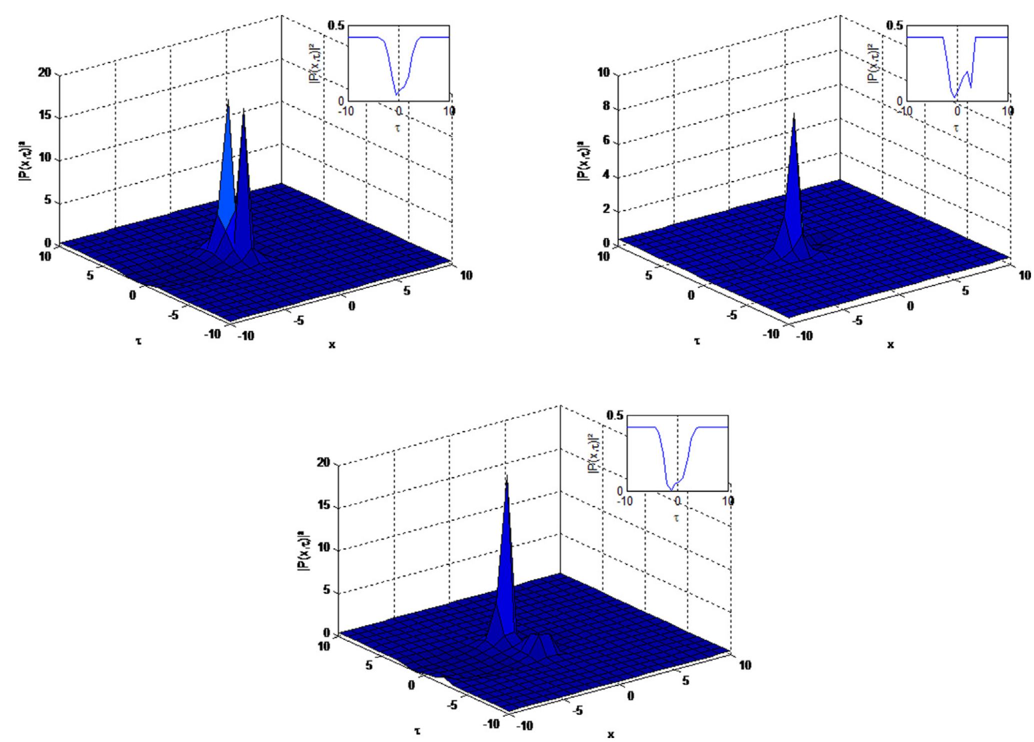

Fig. 2 Wave profiles of solution (3.3) for stochastic fractional QC-NLSE (1.3) with Wick-type $\left(\sigma = \frac{1}{2}, \mu = 2, \lambda = -1\right)$ (a) for $B_\rho = 2\sin(0.5\rho)$, (b) for $B_\rho = \text{rand}om\ [0,1]x\sinh 2\rho$, (c) for $B_\rho = 1.2$.

We have generated several graphics for different cases. Obtained results indicate stability of method. In a future extension of the present research work, we will apply the method to some other fractional-stochastic differential equations having Wick product, a Levy noise or a jump process. Furthermore, we will employ many different types of fractional-order deriva-

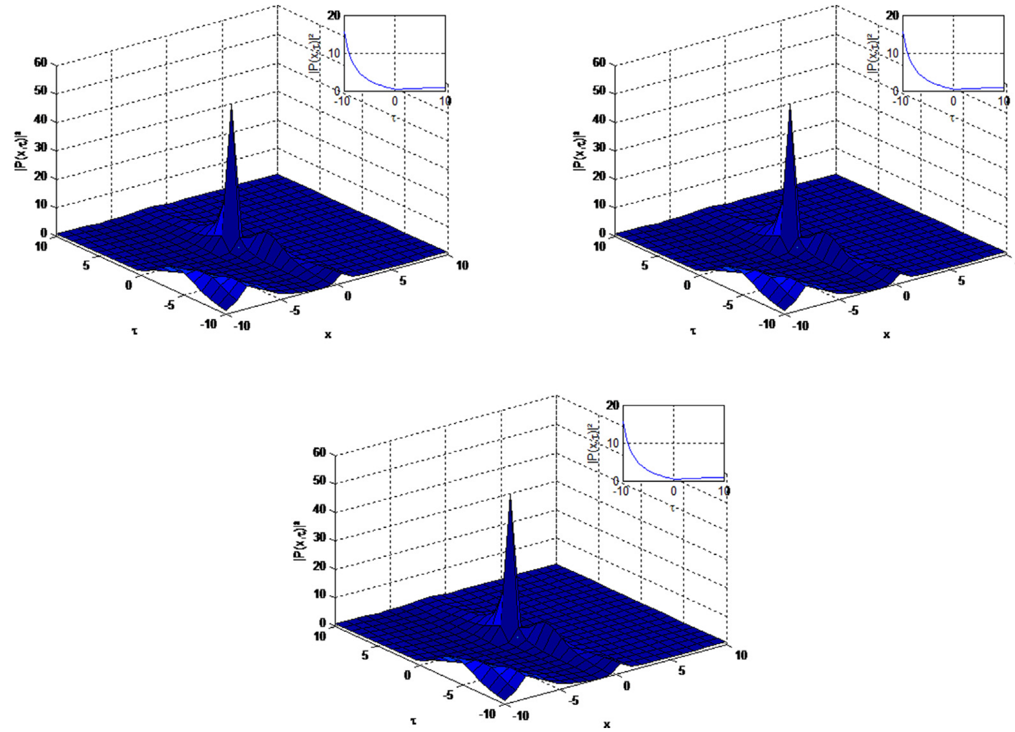

Fig. 3 Wave profiles of solution (3.4) for stochastic fractional QC-NLSE (1.3) with Wick-type $(\sigma = 1, \mu = -1, \lambda = 0)$ (a) for $B_\rho = 2\sin(0.5\rho)$, (b) for $B_\rho = \text{rand } om[0,1]x \sinh 2\rho$, (c) for $B_\rho = 1.2$.

tive operator to determine the most suitable one for these types of computations. We will investigate the stability, consistency and convergence of the method and its comparisons with some of the existing methods in the literature in a separate paper. In this paper, we indicated the strength and stability of the method via computational results and Figures.

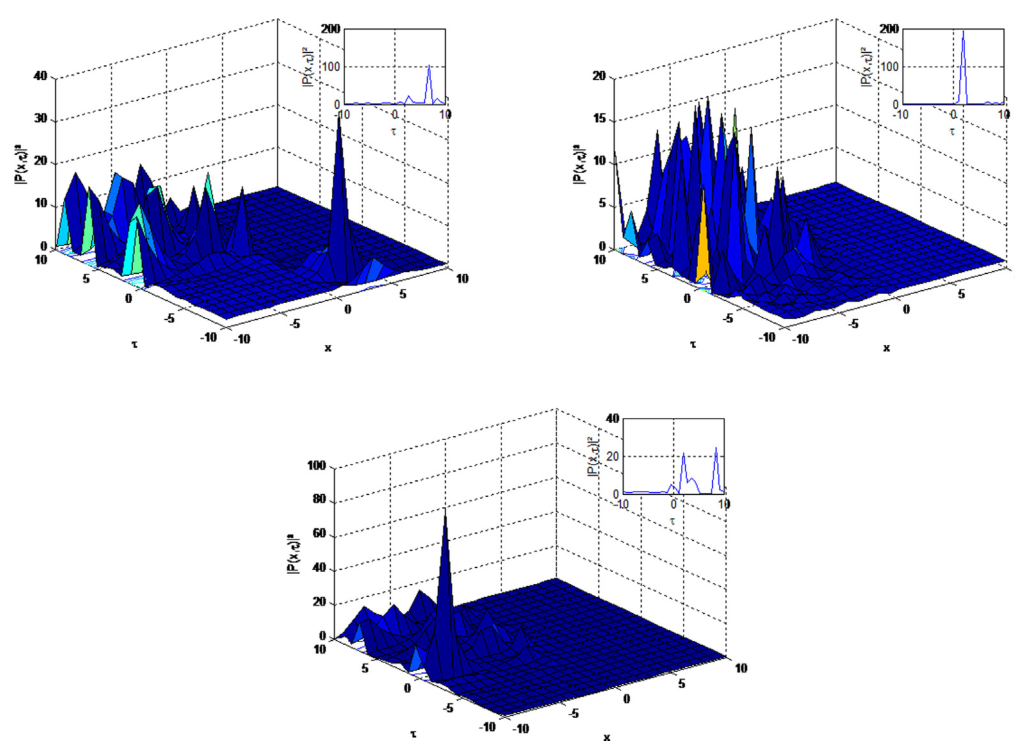

Fig. 4 Wave profiles of solution (3.5) for stochastic fractional QC-NLSE (1.3) with Wick-type $(\sigma = \mu = \lambda = -2)$ (a) for $B_\rho = 2\sin(0.5\rho)$, (b) for $B_\rho = \text{rand } om[0,1]x \sinh 2\rho$, (c) for $B_\rho = 1.2$.

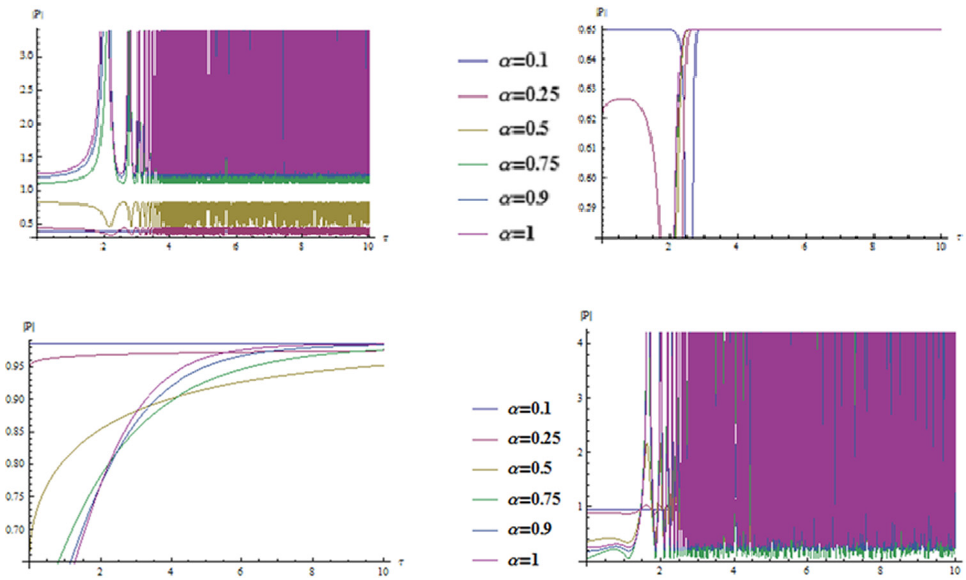

Fig. 5 Comparison of the 2D graphics for Wick-type stochastic fractional QC-NLSE (1.3) with distinct α. $(g(\rho) = -1, h(\rho) = 0.2, k = 1, c = 1, B_1(\rho) = f(\rho) + \lambda W_\rho, f(\rho) = \sinh(2\rho), B_\rho = \text{rand } om[0,1]x \sinh 2\rho)$. (a) for solution (3.2), (b) for solution (3.3), (c) for solution (3.4), (d) for solution (3.5).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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