2.2:

(4) $xy^2\,dx = -(y+1)e^{-x}\,dy$

$x\,dx = -\frac{(y+1)}{y^2}e^{-x}\,dy$

$xe^x\,dx = -\frac{(y+1)}{y^2}\,dy$ (separable)

$\int xe^x\,dx = \int -\frac{(y+1)}{y^2}\,dy$

(by parts)

$xe^x - e^x + C = -\int y^{-2}(y+1)\,dy$

$xe^x - e^x + C = -\int (y^{-2} + y^{-1})\,dy$

$xe^x - e^x + C = -\left(\frac{y^{-1}}{-1} + \frac{y^{-2}}{-2}\right)$

$xe^x - e^x + C = \frac{1}{y} + \frac{1}{2y^2}$

8) $yy' = y^2x^3 + y^2x$

$y\frac{dy}{dx} = y^2x^3 + y^2x$

$y\,dy = (y^2x^3 + y^2x)\,dx$

$y\,dy = y^2(x^3+x)\,dx$

$\frac{1}{y}\,dy = (x^3+x)\,dx$ (separable)

$\int \frac{1}{y}\,dy = \int (x^3+x)\,dx$

$\ln|y| = \frac{x^4}{4} + \frac{x^2}{2} + C$ → implicit solution

explicit solution $y = \phi(x)$

(14) $dy = \frac{xy+3x-y-3}{xy-2x+4y-8}\,dx$

$dy = \frac{xy-y+3x-3}{xy+4y-2x-8}\,dx$

$dy = \frac{y(x-1)+3(x-1)}{y(x+4)-2(x+4)}\,dx$

$dy = \frac{(x-1)(y+3)}{(x+4)(y-2)}\,dx$

$\frac{y-2}{y+3}\,dy = \frac{x-1}{x+4}\,dx$ (separable)

$\int \frac{y-2}{y+3}\,dy = \int \frac{x-1}{x+4}\,dx$

long division

$y+3 \overline{)\,y-2}$ → $1 - \frac{5}{y+3}$

long division

$x+4 \overline{)\,x-1}$ → $1 - \frac{5}{x+4}$

$\int \left(1 - \frac{5}{y+3}\right)dy = \int \left(1 - \frac{5}{x+4}\right)dx$

$y - 5\ln|y+3| = x - 5\ln|x+4| + C$ general solution

(20) $\frac{dy}{dx} = x\sqrt{1-y^2}$

$dy = x\sqrt{1-y^2}\,dx$

$\frac{1}{\sqrt{1-y^2}}\,dy = x\,dx$ (separable) $y \neq 1, y \neq -1$

$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int x\,dx$

$\sin^{-1}(y) = \frac{x^2}{2} + C$ (general solution)

$\sin^{-1}(1) = \frac{x^2}{2} + C$ $y = 1$

$\frac{\pi}{2} = \frac{x^2}{2} + C$ $\frac{dy}{dx} = x\sqrt{1-y^2}$

$\frac{\pi}{2} - \frac{x^2}{2} = C$ $\frac{dy}{dx} = 0$ ✓