Page 176 :

10) $y'' - 4y' + 4y = e^{2x} x^{-4}$; $x>0$

$y_G = y_c + y_p$

y_c : $y'' - 4y' + 4y = 0$

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m_1 = 2$, $m_2 = 2$

$y_c = c_1 e^{2x} + c_2 x e^{2x}$

y_p : variation of parameters

$y_p = u_1 y_1 + u_2 y_2$

$y_1 = e^{2x}$, $y_2 = x e^{2x}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & 2x e^{2x} + e^{2x} \end{vmatrix} = e^{4x}$

$W_1 = \begin{vmatrix} 0 & y_2 \\ \frac{e^{2x}}{x^4} & y_2' \end{vmatrix} = -\frac{e^{2x}}{x^4} \cdot x e^{2x} = -\frac{e^{4x}}{x^3}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & \frac{e^{2x}}{x^4} \end{vmatrix} = e^{2x} \cdot \frac{e^{2x}}{x^4} = \frac{e^{4x}}{x^4}$

$u_1' = \frac{W_1}{W} = -\frac{e^{4x}}{x^3} \cdot \frac{1}{e^{4x}} = -\frac{1}{x^3}$

$u_1 = \int \frac{-1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2}$

$u_2' = \frac{W_2}{W} = \frac{e^{4x}}{x^4} \cdot \frac{1}{e^{4x}} = \frac{1}{x^4}$

$u_2 = \int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{x^{-3}}{-3}$

$y_p = \frac{e^{2x}}{2x^2} - \frac{1}{3x^3} e^{2x} = \frac{e^{2x}}{2x^2} - \frac{e^{2x}}{3x^3}$

$y_G = c_1 e^{2x} + c_2 x e^{2x} + \frac{e^{2x}}{2x^2} - \frac{e^{2x}}{3x^3}$

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20) $x^2 y'' + 4x y' - 4y = x^{\frac{1}{2}} \ln x$, $x>0$

$y_G = y_c + y_p$

y_c : $x^2 y'' + 4x y' - 4y = 0$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

$x = e^t$, $y = x^m$

$[m(m-1) + 4m - 4] x^m = 0$

$m^2 + 3m - 4 = 0$ ($x^m \neq 0$ لأنه)

$(m+4)(m-1) = 0$

$m_1 = -4$, $m_2 = 1$

$y_c = c_1 x^{-4} + c_2 x^1$

by variation of parameters

$y_p = u_1 y_1 + u_2 y_2$

$y_1 = x^{-4}$, $y_2 = x^1$

(Complete)

$x^2 y'' + 4x y' - 4y = x^{\frac{1}{2}} \ln x$

$x^{\frac{1}{2}} \ln x$

$t e^{\frac{1}{2}t} = e^t$

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25) $y'' + 4y' = \sec^2(2x)$

by variation of parameters