17) $y = cx + c^2 + 1 \quad \ldots (*)$

$y' = c$

Go to $(*)$

$y = y' \cdot x + (y')^2 + 1$ (D.E)

20) $y = x^2 + c_1 e^{2x} + c_2 e^{3x}$

$y' = 2x + 2c_1 e^{2x} + 3c_2 e^{3x}$

$y'' = 2 + 4c_1 e^{2x} + 9c_2 e^{3x}$

$\left.\begin{aligned} y' - 2x &= c_1 \cdot 2e^{2x} + c_2 \cdot 3e^{3x} \\ y'' - 2 &= c_1 \cdot 4e^{2x} + c_2 \cdot 9e^{3x} \end{aligned}\right\}$ system, unknowns: c_1, c_2

$W = \begin{vmatrix} 2e^{2x} & 3e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} = 18e^{5x} - 12e^{5x} = 6e^{5x} \neq 0$

$W_1 = \begin{vmatrix} y' - 2x & 3e^{3x} \\ y'' - 2 & 9e^{3x} \end{vmatrix} = 9e^{3x}y' - 18xe^{3x} - 3e^{3x}y'' + 6e^{3x}$

$c_1 = \frac{W_1}{W} = \frac{9y'}{6e^{2x}} - \frac{18x}{6e^{2x}} - \frac{3y''}{6e^{2x}} + \frac{1}{e^{2x}}$

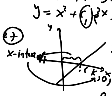
$W_2 = \begin{vmatrix} 2e^{2x} & y' - 2x \\ 4e^{2x} & y'' - 2 \end{vmatrix} = 2e^{2x}y'' - 4e^{2x} - 4e^{2x}y' + 8xe^{2x}$

$c_2 = \frac{W_2}{W} = \frac{2y''}{6e^{3x}} - \frac{4}{6e^{3x}} - \frac{4y'}{6e^{3x}} + \frac{8x}{6e^{3x}}$

نعوض c_1, c_2 :

$y = x^2 + c_1 e^{2x} + c_2 e^{3x}$

27)



$y = ax + b$

slope $= a$

$0 = a \cdot k + b$

$-\frac{b}{a} = k$

$\Rightarrow a = -\frac{b}{a}$

$a^2 = -b$

$-a^2 = b$

$\boxed{y = ax - a^2}$, a arbitrary constant

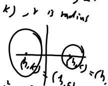
$y' = a$

$\boxed{y = y'x - (y')^2}$: D.E

30) any circle has the eq:

$(x-h)^2 + (y-k)^2 = r^2$

Center (h,k), r is radius



$(x-h)^2 + y^2 = r^2$, two arbitrary constants (h, r)

$2(x-h) + 2yy' = 0$

$2 + 2yy'' + 2y' \cdot y' = 0$

$\boxed{1 + yy'' + (y')^2 = 0}$ (D.E)

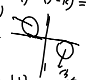
31) any circle has the eq

$(x-h)^2 + (y-k)^2 = r^2$

(h,k)

$k = r$

$k = -r$



r fixed not arbitrary

$(*) \ldots\ldots (x-h)^2 + (y-r)^2 = r^2$

$2(x-h) + 2(y-r)y' = 0$ one arbitrary constant: h

$x - h = -(y-r)y'$

Go to $(*)$:

$[-(y-r)y']^2 + (y-r)^2 = r^2$

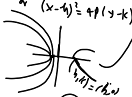
$\boxed{(y-r)^2(y')^2 + (y-r)^2 = r^2}$

34) Parabolas have the eqs:

$(y-k)^2 = 4p(x-h)$ ✓

or $(x-h)^2 = 4p(y-k)$

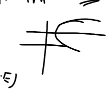
(h,k) vertix



$y^2 = 4p(x-h) = 4px - 4ph$ p, h

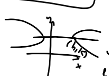
$2yy' = 4p$

$2yy'' + 2y' \cdot y' = 0$



$\boxed{yy'' + (y')^2 = 0}$ (D.E)

35)



$(y-k)^2 = 4p(x-h) = 4px - 4ph$

لدينا ثلاثة ثوابت اختيارية : h, k, p

نشتق ثلاث مرات

$2(y-k)y' = 4p$

$2(y-k)y'' + 2y' \cdot y' = 0$

$(y-k)y'' + (y')^2 = 0 \longrightarrow y - k = \frac{-(y')^2}{y''}$

$(y-k)y''' + y' \cdot y'' + 2(y')y'' = 0$

2.1:

11) $y' = \sqrt{y^2 - 9} = f(x,y)$

$(1,4)$

$f(x,y)$ is cts on $\{(x,y) \in \mathbb{R}^2 : y^2 - 9 \geq 0\}$

$= \{(x,y) \in \mathbb{R}^2 : y \geq 3 \text{ or } y \leq -3\}$

$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{y^2-9}} = \frac{y}{\sqrt{y^2-9}}$

$\frac{\partial f}{\partial y}$ is cts on $\{(x,y) \in \mathbb{R}^2 : y^2 - 9 > 0\}$

$= \{(x,y) \in \mathbb{R}^2 : y > 3 \text{ or } y < -3\}$

IVP has a unique solution on

$\{(x,y) \in \mathbb{R}^2 : y \geq 3 \text{ or } y \leq -3\} \cap \{(x,y) \in \mathbb{R}^2 : y > 3 \text{ or } y < -3\}$

$S = \{(x,y) \in \mathbb{R}^2 : y > 3 \text{ or } y < -3\}$

$(1,4) \in S \Rightarrow$ IVP has a unique solution