10) $y_G = y_c + y_p$

y_c : $y'' - 4y' + 4y = 0$

$m^2 - 4m + 4 = 0$ (c.e)

$(m-2)(m-2) = 0$

$m = 2$ (repeated)

$y_c = c_1 e^{2x} + c_2 x e^{2x}$

y_p : By variation of parameters

$y_p = u_1 y_1 + u_2 y_2$

$= u_1 e^{2x} + u_2 x e^{2x}$ $\quad u_1 = ?, u_2 = ?$

$u_1' = \frac{W_1}{W}$, $u_2' = \frac{W_2}{W}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = e^{4x}$

$W_1 = \begin{vmatrix} 0 & y_2 \\ \frac{e^{2x}}{x^2} & y_2' \end{vmatrix} = -xe^{2x} \cdot \frac{e^{2x}}{x^2} = -\frac{e^{4x}}{x^3}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & \frac{e^{2x}}{x^2} \end{vmatrix} = \frac{e^{4x}}{x^2}$

$u_1' = -\frac{e^{4x}}{x^3} \cdot \frac{1}{e^{4x}} = -\frac{1}{x^3} \Rightarrow u_1 = \int \frac{-1}{x^3} dx = \frac{1}{2x^2}$

$u_2' = \frac{e^{4x}}{x^2} \cdot \frac{1}{e^{4x}} = \frac{1}{x^2} \Rightarrow u_1 = \int \frac{1}{x^2} dx = \frac{x^{-1}}{-1}$

$y_p = \frac{1}{2x^2} \cdot e^{2x} - \frac{1}{3x^3} \cdot xe^{2x}$

$= \frac{1}{2x^2} e^{2x} - \frac{1}{3x^2} e^{2x}$

$y_G = c_1 e^{2x} + c_2 x e^{2x} + \frac{1}{2x^2} e^{2x} - \frac{1}{3x^2} e^{2x}$

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20) $y_G = y_c + y_p$

y_c : $x^2 y'' + 4xy' - 4y = 0$

$m^2 + 4m - 4 = 0$ ✗

$x^2 y'' + 4xy' - 4y = 0$ (Cauchy-Euler Eq)

$y = x^m \rightarrow y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$m(m-1)x^m + 4mx^m - 4x^m = 0$

$[m^2 - m + 4m - 4]x^m = 0$

$m^2 + 3m - 4 = 0$: x^m yazınca

$(m+4)(m-1) = 0$

$m = -4, m = 1$

$y_c = c_1 e^{-4x} + c_2 e^x$ ✗

$y_c = c_1 x^{-4} + c_2 x^1$

y_p : $y_p = u_1 y_1 + u_2 y_2$

$= u_1 x^{-4} + u_2 x^1$ $\quad u_1 = ?$ $\quad u_2 = ?$

$u_1' = \frac{W_1}{W}$, $u_2' = \frac{W_2}{W}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^{-4} & x \\ -4x^{-5} & 1 \end{vmatrix} = 5/x^4$

$W_1 = \begin{vmatrix} 0 & y_2 \\ \frac{x^{\frac{1}{4}} \ln x}{x^4} & y_2' \end{vmatrix} = -\frac{x^{\frac{1}{4}} \ln x}{x} = -\frac{\ln x}{x^{\frac{3}{4}}}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & \frac{x^{\frac{1}{4}} \ln x}{x^4} \end{vmatrix} = \frac{x^{\frac{1}{4}} \ln x}{x^3} = \frac{\ln x}{x^{\frac{11}{4}}}$

$u_1' = -\frac{\ln x}{x^{\frac{3}{4}}} \cdot \frac{x^4}{5} = -\frac{\ln x \cdot x^{\frac{13}{4}}}{5}$

$u_1 = \int -\frac{\ln x \cdot x^{\frac{13}{4}}}{5} dx = -\frac{4x^{\frac{17}{4}}[17 \ln x - 4]}{1445}$

$u_2' = \frac{\ln x}{x^{\frac{3}{4}}} \cdot \frac{x^4}{5} = \frac{\ln x \cdot x^{\frac{13}{4}}}{5}$

$u_2 = \int -\frac{\ln x}{5} x^{-\frac{3}{4}} dx = \frac{4}{15} x^{-\frac{3}{4}} \ln x + \frac{16}{45} x^{-\frac{3}{4}}$

$y_p = \ldots\ldots$

$y_G = \ldots\ldots + \ldots\ldots$