14) $y_2 = y_1 \cdot \int \frac{e^{-\int p(x)\,dx}}{y_1^2}\,dx$ (Formula)

$p(x) = \frac{1}{x}$ $\left(p(x) = \frac{y' \text{ term}}{y'' \text{ term}}\right)$

$y_2 = \ln x \cdot \int \frac{e^{-\int \frac{1}{x} dx}}{(\ln x)^2}\,dx$

$= \ln(x) \cdot \int \frac{e^{-\ln x}}{(\ln x)^2}\,dx$

$= \ln x \cdot \int \frac{1}{x(\ln x)^2}\,dx$

$= \ln x \cdot -\frac{1}{\ln x}$

$= -1$

هامش:

$\int \frac{1}{x}(\ln x)^{-2}\,dx$

$= \frac{(\ln x)^{-1}}{-1}$

$\int \frac{\frac{1}{x}}{(\ln x)^2}\,dx$

General solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 \cdot \ln x + c_2(-1)$

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23) Assume $y = y_1 u$ is a solution of non-homog. eq.

$y = \sin x \cdot u$???

$y' = \sin x \cdot u' + \cos x \cdot u$

$y'' = \sin x \cdot u'' + \cos x \cdot u' + \cos x \cdot u' - \sin x \cdot u$

$= u'' \sin x + 2u' \cos x - \sin x \cdot u$

$u'' \sin x + 2u' \cos x - \cancel{u \sin x} + \cancel{u \sin x} = \sec(x)$

$u'' \sin x + 2u' \cos x = \sec(x)$

Assume $w = u' \longrightarrow w' = u''$

$w' \sin x + 2w \cos x = \sec(x)$ between w & x (Linear) (1st order DE)

$p(x) = 2\cos x / \sin x$

$\mu = e^{\int p(x)\,dx} = e^{\int 2\frac{\cos x}{\sin x}dx} = e^{2\ln(\sin x)} = \sin^2 x$

$\frac{d}{dx}(\mu \cdot w) = \mu \cdot \sec(x)$

$\mu \cdot w = \int \sin^2 x \cdot \sec x\,dx + c_1$

$\sin^2 x \cdot w = \int \sin^2 x \cdot \frac{1}{\cos x}\,dx + c_1$

$\sin^2 x \cdot w = \int \frac{1 - \cos^2 x}{\cos x}\,dx + c_1$

$\sin^2 x \cdot w = \int (\sec x - \cos x)\,dx + c_1$

$\sin^2 x \cdot w = \left[\ln|\sec x + \tan x| - \sin x\right] + c_1$

$u' = w = \frac{\ln|\sec x + \tan x|}{\sin^2 x} - \csc(x) + \frac{c_1}{\sin^2 x}$

$u = \int \frac{\ln|\sec x + \tan x|}{\sin^2 x}\,dx - \int \csc(x)\,dx + c_1 \int \csc^2(x)\,dx$

$u = -\cot x \ln|\sec x + \tan x| + \not{} - \not{}$ $\quad -c_1 \cot x + c_2$

General solution is

$y = \sin x\left[-\cot x \ln|\sec x + \tan x| - c_1 \cot x + c_2\right]$

$= -\cos x \ln|\sec x + \tan x| - c_1 \cos x + c_2 \sin x$