Chapter 3:
Simulation By Hand

LEARNING OBJECTIVES

- To be able to understand simple systems
- To be able construct mathematical relations between components of the system.
- To be able to identify the states and the events of the system
- To be able to carry on the calculation of the system.

Simulation By Hand

- It is important to be able to do the simulation of the system in order to understand the behavior of the system.
- You will be able to validate the model by doing simulation by hand

Hand Simulation Application#1

ATM-Machine

Consider cars arrive at a drive-through to a single ATM to have some cash. The machine serve only on car at a time. Assume that cars arrive to the ATM at some random pattern. Also, the time that a car takes at the ATM is random.

The bank was able to collect the data for the car arrival and service time for 20 cars.

Do the hand simulation for this system

Hand Simulation Application#1

ATM-Machine

Arrival times

Service times

car #	Arrival time (min)	Service time (min)
1	2.00	1.00
2	4.00	3.00
3	5.00	1.00
4	7.00	4.00
5	12.00	8.00
6	13.00	5.00
7	23.00	4.00
8	26.00	4.00
9	32.00	2.00
10	34.00	7.00
11	57.00	3.00
12	59.00	2.00
13	61.00	5.00
14	66.00	1.00
15	67.00	2.00
16	72.00	2.00
17	73.00	3.00
18	76.00	5.00
19	78.00	5.00
20	86.00	10.00

Hand Simulation Application#1

ATM-Machine

1. Compute the dynamics of each car on time.
2. You need to decide what data you will collect:
   - Determine the goal of your simulation
   - More measures = more details you need to compute

3. Measures to compute:
   - When the service start for each car?
   - How much each car waits for service?
   - When each car leave the ATM with service?
   - How much time the ATM remains Idle?

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 1:

Arrival time = 2.00 Service Time = 1.00

Is the car Wait? **No** Service Start = **2.00**

Waiting Time = **0.00**

Departure Time = **2.00+1.00=3.00**

Total ATM idle time = **Car#1 Service start time – last dep.**

= 2.00 – 0.00 = 2.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 2:

Arrival time = 4.00 Service Time = 3.00

Is the car Wait? **No** Service Start = **4.00**

Waiting Time = **0.00**

Departure Time = **4.00 + 3.00 =7.00**

Total ATM idle time = **Car#2 Service start time – last dep.**

= 4.00 – 3.00 = 1.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 3:

Arrival time = 5.00 Service Time = 1.00

Is the car Wait? **Yes**

Service Start = **Dep Car#2** = **7.00**

Waiting Time = **Dep Car#2 – Car#3 Arrival Time**

= **7.00 – 5.00 = 2.00**

Departure Time = **5.00 + 1.00 + 2.00 =8.00**

Total ATM idle time = **Car#3 Service start time – last dep.**

= 7.00 – 7.00 = 0.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 4:

Arrival time = ??? Service Time = ??

Is the car Wait? **??**

Service Start = **???**= **??**

Waiting Time = **??? – ????**

= **?? – ?? = ???**

Departure Time = **??? + ??? + ??? =???**

Total ATM idle time = **??? – ???**

= ?? – ?? = ???

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 4:

Arrival time = 7.00 Service Time = 4.00

Is the car Wait? **Yes**

Service Start = **Dep Car#3** = **8.00**

Waiting Time = **Dep Car#3 – Car#4 Arrival Time**

= **8.00 – 7.00 = 1.00**

Departure Time = **7.00 + 4.00 + 1.00 = 12.00**

Total ATM idle time = **Car#4 Service start time – last dep.**

= 8.00 – 8.00 = 0.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 5:

Arrival time = 12.00 Service Time = 8.00

Is the car Wait? **No**

Service Start = **12.00** = **12.00**

Waiting Time = **??? – ????**

= **?? – ?? = 0.00**

Departure Time = **12.00 + 8.00 = 20.00**

Total ATM idle time = **??? – ???**

= 12.00 – 12.00 = 0.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 6:

Arrival time = 13.00 Service Time = 5.00

Is the car Wait? **Yes**

Service Start = **last Dep** = **20.00**

Waiting Time = **??? – ????**

= **20.00 – 13.00 = 7.00**

Departure Time = **13.00 + 5.00 + 7.00 = 25.00**

Total ATM idle time = **??? – ???**

= 20.00 – 20.00 = 0.00

Hand Simulation Application#1

ATM-Machine Hand Simulation Steps

ATM clock start at time 0.00

Car # 7:

Arrival time = ??? Service Time = ??

Is the car Wait? **??**

Service Start = **???**= **??**

Waiting Time = **??? – ????**

= **?? – ?? = ???**

Departure Time = **??? + ??? + ??? =???**

Total ATM idle time = **??? – ???**

= ?? – ?? = ???

Hand Simulation Application#1

ATM-Machine

car #	Arrival time	Service time	Service start	WITE?	Wait Time	Departure time	ATM Idle time
1	2.00	1.00					
2	4.00	3.00					
3	5.00	1.00					
4	7.00	4.00					
5	12.00	8.00					
6	13.00	5.00					
7	23.00	4.00					
8	26.00	4.00					
9	32.00	2.00					
10	34.00	7.00					
11	57.00	3.00					
12	59.00	2.00					
13	61.00	5.00					
14	66.00	1.00					
15	67.00	2.00					
16	72.00	2.00					
17	73.00	3.00					
18	76.00	5.00					
19	78.00	5.00					
20	86.00	10.00					

Hand Simulation Application#1

ATM-Machine

car #	Arrival time	Service time	Service start	WITE?	Wait Time	Departure time	ATM Idle time
1	2.00	1.00	2.00	0	0	3.00	2.00
2	4.00	3.00	4.00	0	0.00	7.00	1.00
3	5.00	1.00	7.00	1	2.00	8.00	0.00
4	7.00	4.00	8.00	1	1.00	12.00	0.00
5	12.00	8.00	12.00	0	0.00	20.00	0.00
6	13.00	5.00	20.00	1	7.00	25.00	0.00
7	23.00	4.00	25.00	1	2.00	29.00	0.00
8	26.00	4.00	29.00	1	3.00	33.00	0.00
9	32.00	2.00	33.00	1	1.00	35.00	0.00
10	34.00	7.00	35.00	1	1.00	42.00	0.00
11	57.00	3.00	57.00	0	0.00	60.00	15.00
12	59.00	2.00	60.00	1	1.00	62.00	0.00
13	61.00	5.00	62.00	1	1.00	67.00	0.00
14	66.00	1.00	67.00	1	1.00	68.00	0.00
15	67.00	2.00	68.00	1	1.00	70.00	0.00
16	72.00	2.00	72.00	0	0.00	74.00	2.00
17	73.00	3.00	74.00	1	1.00	77.00	0.00
18	76.00	5.00	77.00	1	1.00	82.00	0.00
19	78.00	5.00	82.00	1	4.00	87.00	0.00
20	86.00	10.00	87.00	1	1.00	97.00	0.00

Hand Simulation Application#1

ATM-Machine

Define System States:
Number of Cars in ATM

car #	Arrival time	Service time	Service start	WITE?	Wait Time	Departure time	ATM Idle time
1	2.00	1.00	2.00	0	0	3.00	2.00
2	4.00	3.00	4.00	0	0.00	7.00	1.00
3	5.00	1.00	7.00	1	2.00	8.00	0.00
4	7.00	4.00	8.00	1	1.00	12.00	0.00
5	12.00	8.00	12.00	0	0.00	20.00	0.00
6	13.00	5.00	20.00	1	7.00	25.00	0.00
7	23.00	4.00	25.00	1	2.00	29.00	0.00
8	26.00	4.00	29.00	1	3.00	33.00	0.00
9	32.00	2.00	33.00	1	1.00	35.00	0.00
10	34.00	7.00	35.00	1	1.00	42.00	0.00
11	57.00	3.00	57.00	0	0.00	60.00	15.00
12	59.00	2.00	60.00	1	1.00	62.00	0.00
13	61.00	5.00	62.00	1	1.00	67.00	0.00
14	66.00	1.00	67.00	1	1.00	68.00	0.00
15	67.00	2.00	68.00	1	1.00	70.00	0.00
16	72.00	2.00	72.00	0	0.00	74.00	2.00
17	73.00	3.00	74.00	1	1.00	77.00	0.00
18	76.00	5.00	77.00	1	1.00	82.00	0.00
19	78.00	5.00	82.00	1	4.00	87.00	0.00
20	86.00	10.00	87.00	1	1.00	97.00	0.00

Hand Simulation Application#1

ATM-Machine

System Evaluation (Output Analysis)

- Average Waiting Time
- Average Service Speed (per hr)
- Average Arrival Rate (per hr)
- Utilization of the ATM

Hand Simulation Application#1

ATM-Machine

Define System States:

Number of Cars in ATM

- Any Arrival ***increase*** the state by +1
- Any Departure ***decrease*** the state by – 1

car #	Arrival time	Change in System	Departure time	Change in System
1	2.00		3.00	
2	4.00		7.00	
3	5.00		8.00	
4	7.00		12.00	
5	12.00		20.00	
6	13.00		25.00	
7	23.00		29.00	
8	26.00		33.00	
9	32.00		35.00	
10	34.00		42.00	
11	57.00		60.00	
12	59.00		62.00	
13	61.00		67.00	
14	66.00		68.00	
15	67.00		70.00	
16	72.00		74.00	
17	73.00		77.00	
18	76.00		82.00	
19	78.00		87.00	
20	86.00		97.00	

Hand Simulation Application#1

ATM-Machine

Define System States:

Number of Cars in ATM

- Any Arrival ***increase*** the state by +1
- Any Departure ***decrease*** the state by – 1

car #	Arrival time	Change in System	Departure time	Change in System
1	2.00	+1	3.00	-1
2	4.00	+1	7.00	-1
3	5.00	+1	8.00	-1
4	7.00	+1	12.00	-1
5	12.00	+1	20.00	-1
6	13.00	+1	25.00	-1
7	23.00	+1	29.00	-1
8	26.00	+1	33.00	-1
9	32.00	+1	35.00	-1
10	34.00	+1	42.00	-1
11	57.00	+1	60.00	-1
12	59.00	+1	62.00	-1
13	61.00	+1	67.00	-1
14	66.00	+1	68.00	-1
15	67.00	+1	70.00	-1
16	72.00	+1	74.00	-1
17	73.00	+1	77.00	-1
18	76.00	+1	82.00	-1
19	78.00	+1	87.00	-1
20	86.00	+1	97.00	-1

Hand Simulation Application#1

ATM-Machine

Define System

Clock for

Number of Cars in ATM

- Sort the time of Arrivals and Departures in one column

Time	Change in System
0	0
2	+1
3	-1
4	+1
5	+1
7	-1
7	+1
8	-1
12	-1
12	+1
13	+1
20	-1
23	+1

Hand Simulation Application#1

<u>ATM-Machine</u>

Define System

Clock for

Number of Cars in ATM

- Sort the time of Arrivals and Departures in one column

Time	Change in System
0	1
2	-1
3	1
4	1
5	1
7	-1
7	-1
8	1
12	-1
12	1
13	-1
20	1
23	-1
25	1
26	-1
29	1
32	1
…	…
97	-1

Hand Simulation Application#1

ATM-Machine

Define System Clock for Number of Cars in ATM

- Table of details

System Clock From	To	Change in System	Number in System	Interval
0	2	0	0	2
2	3	+1	1	1
3	4	-1	0	1
4	5	+1	1	1
5	7	+1	2	2
7	7	-1	1	0
7	8	+1	2	1
8	12	-1	1	4
12	12	-1	0	0
12	13	+1	1	1
13	20	+1	2	7
20	23	-1	1	3
23	25	+1	2	2
25	26			
26	29			
29	32			
32				

Hand Simulation Application#1

ATM-Machine

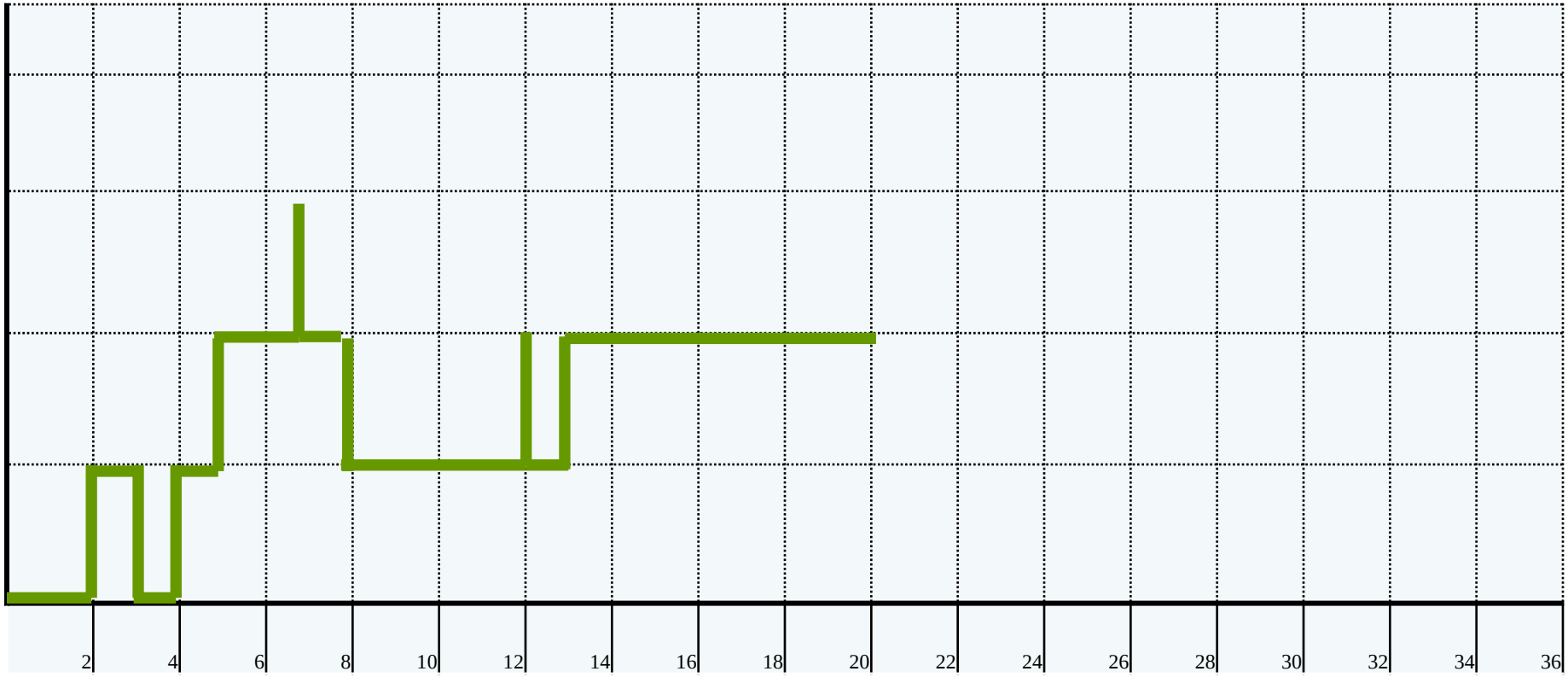
Define System Clock for Number of Cars in ATM

- Sort the time of Arrivals and Departures in one column

System Clock From	To	Change in System	Number in System	Interval
0	2	0	**0**	2
2	3	1	**1**	1
3	4	-1	**0**	1
4	5	1	**1**	1
5	7	1	**2**	2
7	7	1	**3**	0
7	8	-1	**2**	1
8	12	-1	**1**	4
12	12	1	**2**	0
12	13	-1	**1**	1
13	20	1	**2**	7
20	23	-1	**1**	3
23	25	1	**2**	2
25	26	-1	**1**	1
26	29	1	**2**	3
29	32	-1	**1**	3
32	33	1	**2**	1
...	...		**...**	...
...	...			
97	100	-1	**0**	3

Hand Simulation Application#1

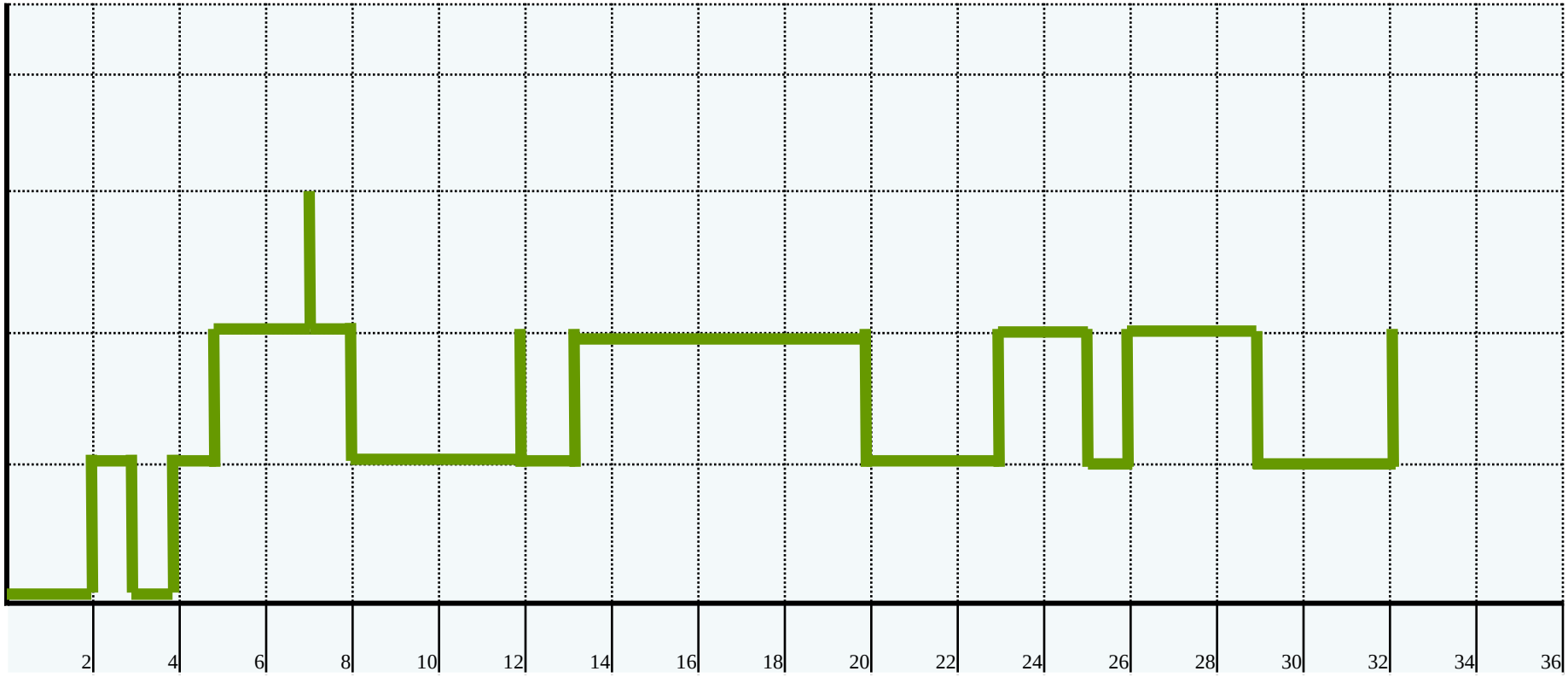
Number Cars at ATM



System Clock Time

Hand Simulation Application#1

Number Cars at ATM



System Clock Time

Hand Simulation Application#1

ATM-Machine

System Evaluation

Output Analysis

- Average Waiting Time
- Average Service Speed (per hr)
- Average Arrival Rate (per hr)
- Utilization of the ATM

System Clock From	To	Change in System	Number in System	Interval
0	2	0	**0**	2
2	3	1	**1**	1
3	4	-1	**0**	1
4	5	1	**1**	1
5	7	1	**2**	2
7	7	1	**3**	0
7	8	-1	**2**	1
8	12	-1	**1**	4
12	12	1	**2**	0
12	13	-1	**1**	1
13	20	1	**2**	7
20	23	-1	**1**	3
23	25	1	**2**	2
25	26	-1	**1**	1
26	29	1	**2**	3
29	32	-1	**1**	3
32	33	1	**2**	1
...	...		**...**	...
...	...			
97	100	-1	**0**	3

Hand Simulation Application#1

System Clock From	To	Change in System	Number in System	Interval
0	2	0	**0**	2
2	3	1	**1**	1
3	4	-1	**0**	1
4	5	1	**1**	1
5	7	1	**2**	2
7	7	1	**3**	0
7	8	-1	**2**	1
8	12	-1	**1**	4
12	12	1	**2**	0
12	13	-1	**1**	1
13	20	1	**2**	7
20	23	-1	**1**	3
23	25	1	**2**	2
25	26	-1	**1**	1
26	29	1	**2**	3
29	32	-1	**1**	3
32	33	1	**2**	1
33	34	-1	**1**	1
34	35	1	**2**	1
35	42	-1	**1**	7

System Clock From	To	Change in System	Number in System	Interval
42	57	-1	**0**	15
57	59	1	**1**	2
59	60	1	**2**	1
60	61	-1	**1**	1
61	62	1	**2**	1
62	66	-1	**1**	4
66	67	1	**2**	1
67	67	1	**3**	0
67	68	-1	**2**	1
68	70	-1	**1**	2
70	72	-1	**0**	2
72	73	1	**1**	1
73	74	1	**2**	1
74	76	-1	**1**	2
76	77	1	**2**	1
77	78	-1	**1**	1
78	82	1	**2**	4
82	86	-1	**1**	4
86	87	1	**2**	1
87	97	-1	**1**	10
97	100	-1	**0**	3

Hand Simulation Application#1

<u>ATM-Machine</u>

System Evaluation (Output Analysis)

- Computing the Distribution of State of the system.

This means : what percentage of time that the system will be in any of the states.

State of the system is

number of cars at the ATM

Pr{state=0} = ?? , Pr{state=1} = ?? , Pr{state=2} = ?? , …

Hand Simulation Application#2

Milk Inventory

Consider a minimarket that sells milk on a daily basis. Customers arrive to the super market at random. Data for customers arrival is collected. The demand on milk by any customer is randomly **1, 2 or 3** bottles. The manager places the order for delivery when there is **0** bottles on the shelve. The new order of milk arrives by the next morning. Therefore, any demand when there is 0 bottles is lost. The manager orders **50** bottles every time he makes a new order. Minimarket open daily for **12** work hours. If customer demand less than what is available he gets what is availed only.

Do the hand simulation for this system

Hand Simulation Application#2

Milk Inventory

Cust. #	Arrival time	Demand
1	29	1
2	113	3
3	199	2
4	253	1
5	295	3
6	312	2
7	377	1
8	387	2
9	398	3
10	421	2
11	429	3
12	462	3
13	495	1
14	503	3
15	515	3
16	528	2

Cust. #	Arrival time	Demand
17	548	2
18	554	2
19	555	1
20	671	3
21	693	3
22	719	1
23	725	3
24	742	3
25	745	3
26	757	1
27	819	1
28	827	2
29	929	1
30	955	1

Hand Simulation Application#2

Milk Inventory

What to Consider???

1. Arrival time (day)
2. Arrival time (min. in a day)
3. Demand of a customer
4. The available milk ***before*** the demand of a customer
5. The available milk ***after*** the demand of a customer
6. During any day the manger must decide to order new order or not.

Hand Simulation Application#2

Milk Inventory

Table of Simulation

Cust. #	Arrival time	Day	Time	Demand	starting inventory	ending inventory	Place New Order	Shortage
1	29							
2	113							
3	199							
4	253							
5	295							
6	312							
7	377							
8	387							
9	398							
10	421							
11	429							
12	462							

Hand Simulation Application#2

Milk Inventory

Table of Simulation

Cust. #	Arrival time	Day	Time	Demand	starting inventory	ending inventory	Place New Order	Shortage
1	29	1	29	1	50	49	NO	0
2	113	1	113	3	49	46	NO	0
3	199	1	199	2	46	44	NO	0
4	253	1	253	1	44	43	NO	0
5	295	1	295	3	43	40	NO	0
6	312	1	312	2	40	38	NO	0
7	377	1	377	1	38	37	NO	0
8	387	1	387	2	37	35	NO	0
9	398	1	398	3	35	32	NO	0
10	421	1	421	2	32	30	NO	0
11	429	1	429	3	30	27	NO	0
12	462	1	462	3	27	24	NO	0
13	495	1	495	1	24	23	NO	0
14	503	1	503	3	23	20	NO	0
15	515	1	515	3	20	17	NO	0

Hand Simulation Application#2

Milk Inventory

Table of Simulation

Cust. #	Arrival time	Day	Time	Demand	starting inventory	ending inventory	Place New Order	Shortage
15	515	1	515	3	20	17	NO	0
16	528	1	528	2	17	15	NO	0
17	548	1	548	2	15	13	NO	0
18	554	1	554	2	13	11	NO	0
19	555	1	555	1	11	10	NO	0
20	671	1	671	3	10	7	NO	0
21	693	1	693	3	7	4	NO	0
22	719	1	719	1	4	3	NO	0
23	725	2	5	3	3	0	YES	0
24	742	2	22	3	0	0	YES	3
25	745	2	25	3	0	0	YES	3
26	757	2	37	1	0	0	YES	1
27	819	2	99	1	0	0	YES	1
28	827	2	107	2	0	0	YES	2
29	929	2	209	1	0	0	YES	1
30	955	2	235	1	0	0	YES	1