Chapter 6: Simulation Using Spread-Sheets (Excel)

Refer to Reading Assignments

Simulation Using Spread-Sheets (Excel)

OBJECTIVES

To be able to …

- ***Generate*** random numbers within a spreadsheet environment.
- ***Modeling*** random variables with various distributional models within a spreadsheet.
- ***Perform*** simple simulation in a spreadsheet environment.
- ***Compute*** statistical quantities for a simulation model.
- ***Understand*** how to build the model logic.

Simulation Using Spread-Sheets (Excel)

WHY Use Spreadsheet Applications for Simulation?

- learn how to perform basic simulation methods within a spreadsheet environment.
- The simulation results in spreadsheet is immediate (you design and see the result immediately)
- Immediate results in spreadsheet environment help understanding of many methods discussed in Chapter 5.

Simulation Using Spread-Sheets (Excel)

WHY use spreadsheet applications for simulation?

- Learn about some functions available within spreadsheets useful for simulation
- Useful as modeling tool for simple to moderately complex spreadsheet simulations.
- Gives introduction to begin to appreciate the more advanced simulation models.

1. LCG Using Excel

- An LCG defines a sequence of integers, R_0, R_1, … between 0 and m − 1 according to the following recursive relationship, where i = 0, 1, 2, ...:

$$\boldsymbol{R_{i+1} = (aR_i + c) \bmod m \text{ for } i = 0, 1, 2, \ldots.}$$

- (m, a, c, R_0) *are integers with*
- $a > 0$, $c \geq 0$, $m > (a$, c , *and* R_0)

- To compute the corresponding pseudorandom uniform number, we use

$$U_i = \frac{R_i}{m}$$

1. LCG Using Excel

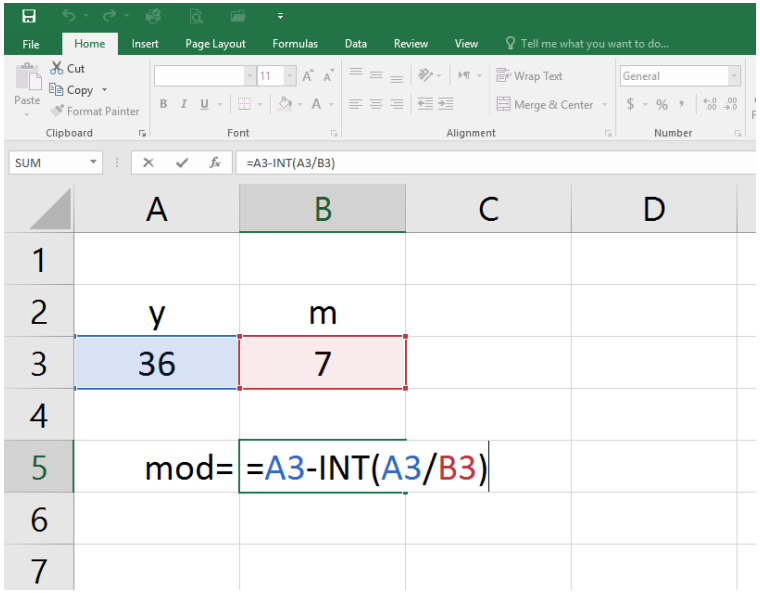
- to compute using the mod operator. The mod operator is defined as

$$z = y \bmod m$$

$$= y - m \left\lfloor \frac{y}{m} \right\rfloor$$

where $\lfloor \cdot \rfloor$ is the floor operator,

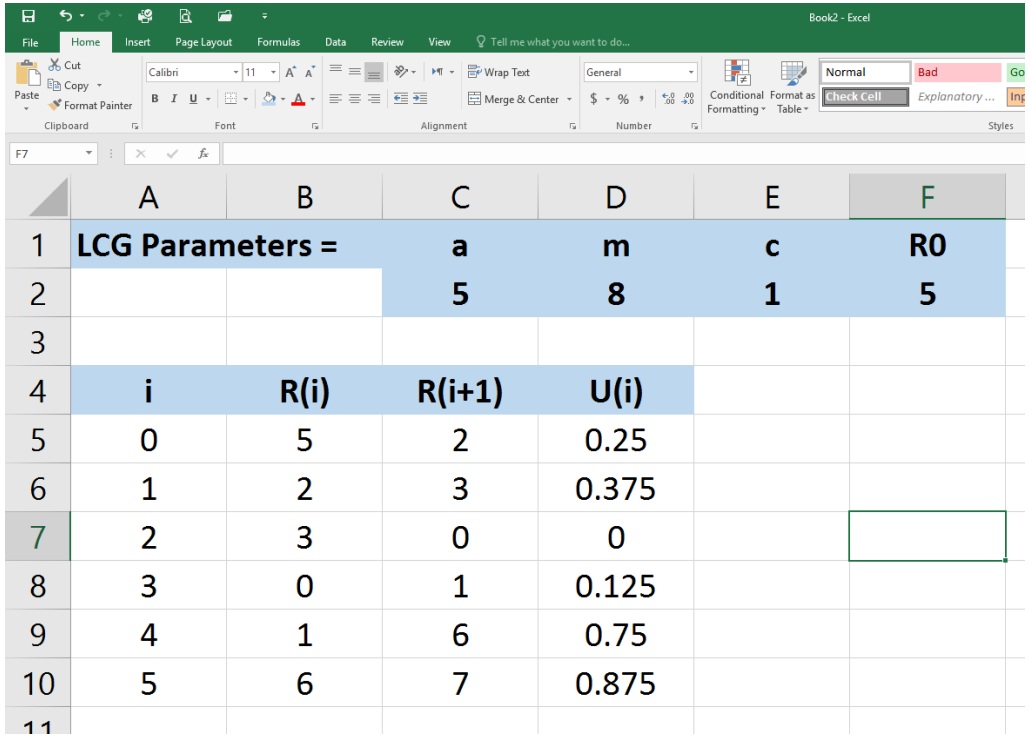
- Mod function in Excel



1. LCG Using Excel

EXAMPLE

Construct an Excel sheet for an LCG with parameters ($m = 8$, $a = 5$, $c = 1$, $R_0 = 5$).



	A	B	C	D	E	F
1	**LCG Parameters =**		**a**	**m**	**c**	**R0**
2			**5**	**8**	**1**	**5**
3						
4	**i**	**R(i)**	**R(i+1)**	**U(i)**		
5	0	5	2	0.25		
6	1	2	3	0.375		
7	2	3	0	0		
8	3	0	1	0.125		
9	4	1	6	0.75		
10	5	6	7	0.875		

1. LCG Using Excel

EXAMPLE

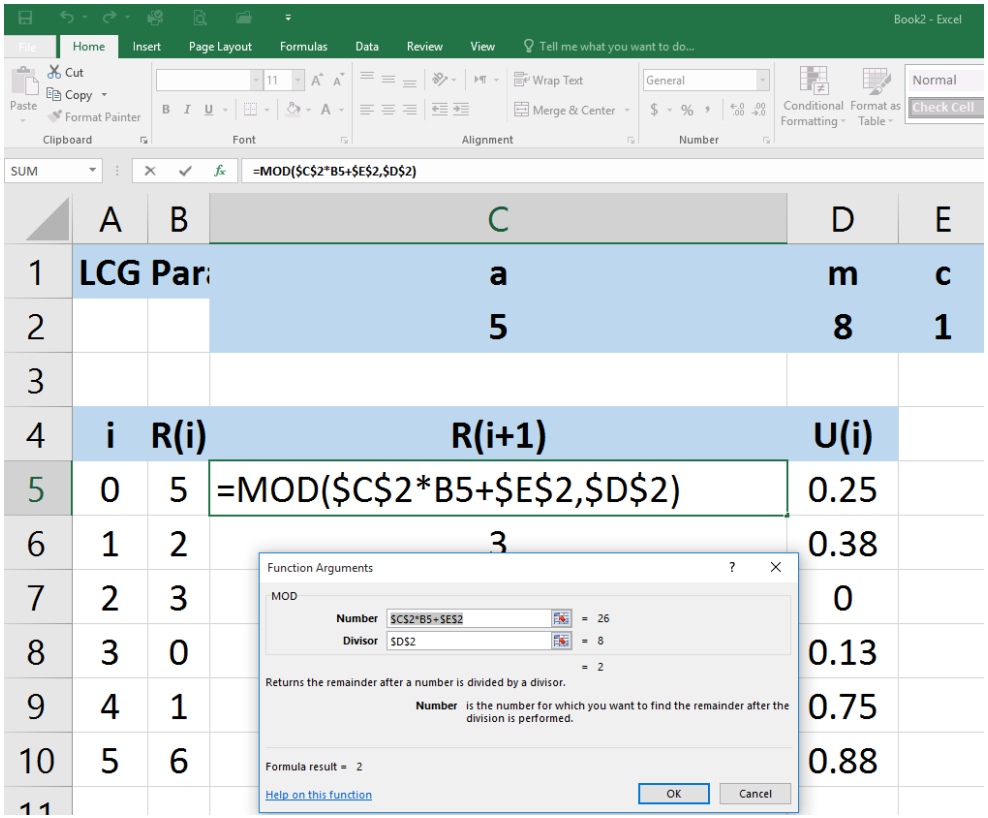
Construct an Excel sheet for an LCG with parameters ($m = 8$, $a = 5$, $c = 1$, $R_0 = 5$).

	A	B	C	D
1	**LCG Para**		**a**	**m**
2			**5**	**8**
3				
4	**i**	**R(i)**	**R(i+1)**	**U(i)**
5	0	=F2	=($C$2*B5+$E$2)-$D$2*INT(($C$2*B5+$E$2)/$D$2)	=C5/$D$2
6	1	=C5	=($C$2*B6+$E$2)-$D$2*INT(($C$2*B6+$E$2)/$D$2)	=C6/$D$2
7	2	=C6	=($C$2*B7+$E$2)-$D$2*INT(($C$2*B7+$E$2)/$D$2)	=C7/$D$2
8	3	=C7	=($C$2*B8+$E$2)-$D$2*INT(($C$2*B8+$E$2)/$D$2)	=C8/$D$2
9	4	=C8	=($C$2*B9+$E$2)-$D$2*INT(($C$2*B9+$E$2)/$D$2)	=C9/$D$2
10	5	=C9	=($C$2*B10+$E$2)-$D$2*INT(($C$2*B10+$E$2)/$D$2)	=C10/$D$2

1. LCG Using Excel

EXAMPLE

Construct an Excel sheet for an LCG with parameters ($m = 8$, $a = 5$, $c = 1$, $R_0 = 5$).



1. LCG Using Excel

Exercise

Consider the LCG generator with

a = 13, c= 1, m = 64, and seeds X_0 = 1,2,3,4

a) Using Excel, does this generator achieve its maximum period for these parameters?

b) Generate a period's worth of uniform random variables from each of the supplied seeds.

c) Using Excel fined all streams of LCG

2. Generating Uniform numbers in Excel

The Function RAND()

- The RAND() in Excel generates random numbers uniformly distributed on the interval from (0,1).
- Using RAND() many other random numbers can be generated from user defined distributions.
- The RAND() function is an “active” function: each time the worksheet is updated, a new value from RAND() will be returned.

2. Generating Uniform numbers in Excel

The Function RAND()

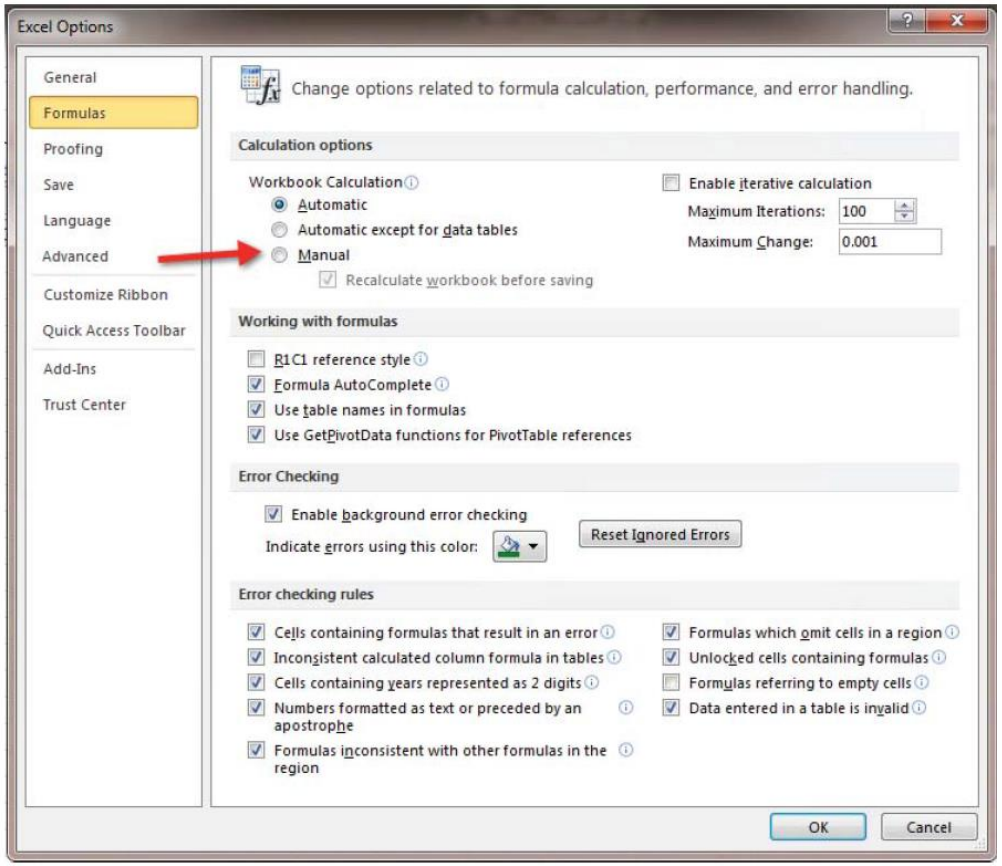
- To turn off automatic calculation, you can set the calculation option for the spreadsheet to manual

- When automatic calculation is turn off, use F9 to recalculate the random numbers.

2. Generating Uniform numbers in Excel

The Function RAND()

turn off automatic calculation



2. Generating Uniform numbers in Excel

The Function RAND()

Exercise

- Generate 5 samples of uniform random numbers between 0 and 1
- Each sample of size 100 number
- For each sample compute the:
  - Average
  - Standard Deviation
  - Min-Value and Max-Value
  - Draw the empirical distribution

3. Generating from Random Distributions in Excel

Generating Sequenced Integer

- Excel has the function RANDBETWEEN(a,b) to generate a random integers between [a,b].
- This function is also an active function.
- This function generates values between a and b ***<u>including a and b</u>***
- This function is used when integer values are in sequence and each value has the same probability.
- Do Not use it for integer values with different probabilities.

2. Generating Uniform numbers in Excel

<u>Exercise</u>

Consider demand on some product

Generate 10 samples of integer uniform random numbers between 3 and 10

- Each sample of size 150 number
- For each sample compute the:
  - Average
  - Standard Deviation
  - Min-Value and Max-Value
  - Draw the empirical distribution
- Compute the probability of each integer value

3. Generating from Random Distributions in Excel

Generating Continuous U[*a*,*b*]

Use the function :

a + (b-a) * RAND()

<u>Exercise</u>

- Generate 10 samples of uniform random numbers between 3 and 10 Each sample of size 150
- For each sample compute the:
  - Average and Standard Deviation
  - Min-Value and Max-Value
  - Draw the empirical distribution
- Compute the probability of each integer value

4. Inverse Transform using Excel

Investment Application:

- Consider an investment in a company. The annual return of investment on this company range between -1 and 1. past data shows that the return of investment (say x) follow the following pdf:

$$f(x) = \begin{cases} \dfrac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

- Assume that you have a 10 years investment plan of in this company with a starting budget of 100,000 SR. Using simulation, what is the expect return after 10 years and how many years the investment is gaining or losing. Build 95% confidence interval
- What is the average percentage of gain of the company is gaining.
- What is the average percentage of losing of the company is losing.

4. Inverse Transform using Excel

Investment Application:

- Return of investment probability density function:

$$f(x) = \begin{cases} \frac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The CDF :

For $x < -1$, $\quad F(x) = 0$

For $-1 \le x \le 1$, $\quad F(x) = \frac{1}{2}(x^3 + 1)$

For $x > 1$, $\quad F(x) = 1$

Inverse Transform:

Let $u \sim U[0,1]$ then $\quad u = F(x)$

$$u = F(x) = \frac{1}{2}(x^3 + 1)$$

$$2u = (x^3 + 1)$$

$$2u - 1 = x^3$$

$$\sqrt[3]{2u - 1} = x$$

Then ,

$$F^{-1}(u) = \sqrt[3]{2u - 1}$$

4. Inverse Transform using Excel

Investment Application:

- Excel Formulation

$$f(x) = \begin{cases} \dfrac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

4. Inverse Transform using Excel

Investment Application:

- What happen if the return distribution become U[-1,1]

4. Inverse Transform using Excel

Highway Accidents:

Accidents on a belt line highway happen according to an exponential distribution. Distance between every two accidents is exponentially distribution with average distance of 10 km. If an accident accrue,

- A random time between 10 to 15 minutes needed to clear the road (uniform)
- A random time minutes needed for the Police officer to resolve the problem (Erlnag (α=3,λ=5))

1. Simulate this system for 1000 km
2. From simulation what is the distribution of number accidents in each 10 km

4. Inverse Transform using Excel

Exponential Distribution

- Now the inverse of the CDF can be derived by setting $u = F(x)$

$$u = 1 - e^{-\lambda x}$$

$$x = \frac{-1}{\lambda} \ln\ (1 - u) = F^{-1}(u)$$

- Suppose that $\lambda = 0.75$ and we have $u = 0.7$, then the generated x would be

$$x = (-1/0.75)\ln\ (1 - 0.7) = 1.6053$$

4. Inverse Transform using Excel

Uniform (*a*,*b*)

- The uniform distribution over an interval (a, b)
- Used when the ***analyst does not have much information***
- Assume outcomes are equally likely over a range of values.

$$X \sim \text{Uniform}(a, b)$$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{E}[X] = \frac{a+b}{2} \qquad \text{Var}[X] = \frac{(b-a)^2}{12}$$

4. Inverse Transform using Excel

Uniform (*a*,*b*)

- Derive the inverse CDF for the $U(a, b)$ distribution.

$$F(x) = \begin{cases} 0.0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1.0 & x > b \end{cases}$$

$$u = \frac{x-a}{b-a}$$

$$u(b-a) = x - a$$

$$x = a + u(b-a) = F^{-1}(u)$$

4. Inverse Transform using Excel

Weibull Distribution

- probability density function:

$$f(x) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}}$$

$$E[X] = \frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right) \qquad Var(X) = \frac{\beta^2}{\alpha^2}\left\{2\Gamma\left(\frac{2}{\alpha}\right) - \frac{1}{\alpha}\left[\Gamma\left(\frac{1}{\alpha}\right)\right]^2\right\}$$

- CDF

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{(-x/\beta)^{\alpha}} & \text{if } x \geq 0 \end{cases}$$

4. Inverse Transform using Excel (Discrete Dist.)

- Given a PMF for a discrete variable X

X	a_1	a_2	a_3	a_4	a_5
P{X}	p_1	p_2	p_3	p_4	p_5

With

$$0 \le p_i \le 1$$

$$p_1+p_2+p_3+p_4+p_5 = 1$$

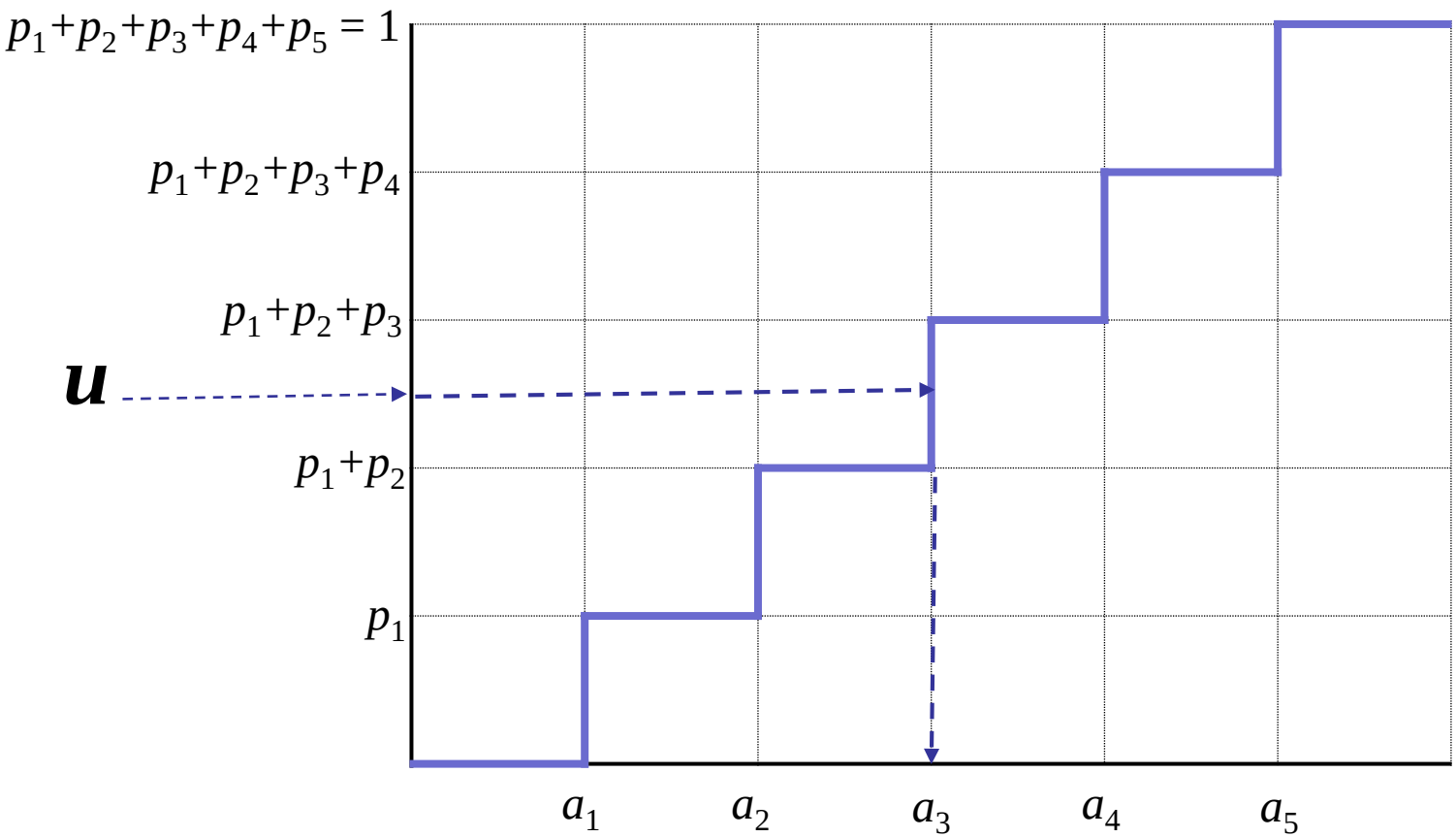
- The CDF of X is:

X	a_1	a_2	a_3	a_4	a_5
P{X≤ a_i}	p_1	p_1+p_2	$p_1+p_2+p_3$	$p_1+p_2+p_3+p_4$	$p_1+p_2+p_3+p_4+p_5$

- We need to have for any u~ U[0,1] a value from X.

4. Inverse Transform using Excel (Discrete Dist.)

- The CDF graph



4. Inverse Transform using Excel (Discrete Dist.)

- From the CDF graph: for $u \sim U[0,1]$

If $0 \le u \le p_1$

Return X = a_1

If $p_1 \le u \le p_1+p_2$

Return X = a_2

If $p_1+p_2 \le u \le p_1+p_2+p_3$

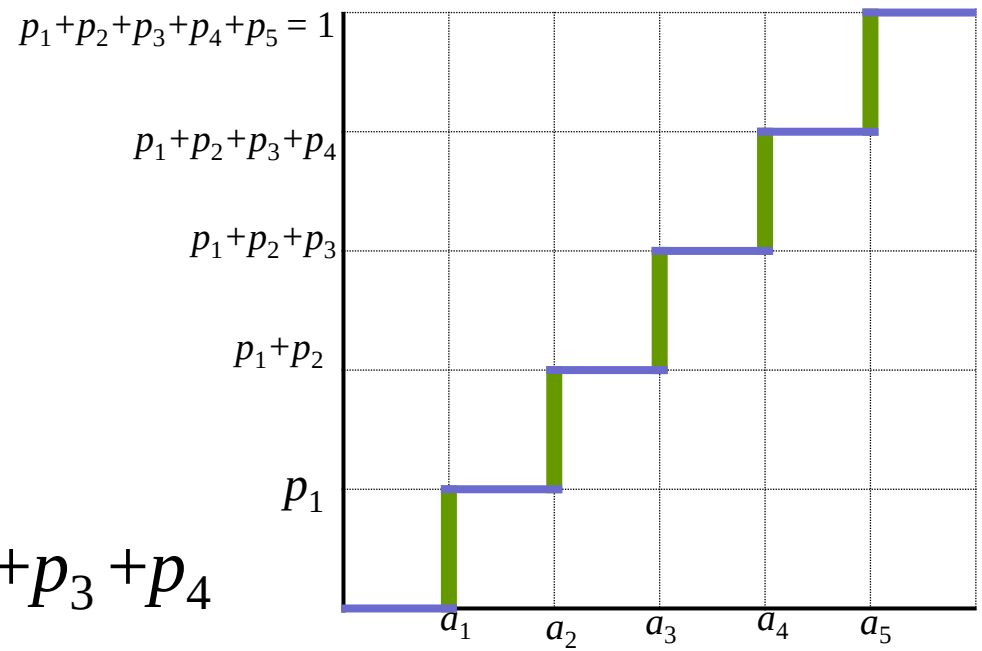
Return X = a_3

If $p_1+p_2+p_3 \le u \le p_1+p_2+p_3+p_4$

Return X = a_4

If $p_1+p_2+p_3+p_4 \le u \le 1$

Return X = a_5



4. Inverse Transform using Excel (Discrete Dist.)

Example:

The functional form of the PMF and CDF are given as follows:

$$P\{X = x\} = \begin{cases} 0.4 & x = 1 \\ 0.3 & x = 2 \\ 0.2 & x = 3 \\ 0.1 & x = 4 \end{cases} \qquad F(x) = \begin{cases} 0.0 & \text{if } x < 1 \\ 0.4 & \text{if } 1 \le x < 2 \\ 0.7 & \text{if } 2 \le x < 3 \\ 0.9 & \text{if } 3 \le x < 4 \\ 1.0 & \text{if } x \ge 4 \end{cases}$$

3. Simulating Simple Models in Excel

- Some examples independent simulation
- How to collect data
- How to make decisions
- Confidence intervals
- How to compute probabilities from simulation runs in excel
- Get easi examples and find probabilities
  - Pr{N=k} in Poisson arrival from simulation
  - Something like that

4. Simulating Advanced Models in Excel

- Car wash
- Inventory
- ATM
- Coffee Shop
- Balance
- Investment: a portfoiloi with k companies each has a markove chain Up Dn