

Anatomy of a Cobb-Douglas Type Production/Utility Function in Three Dimensions

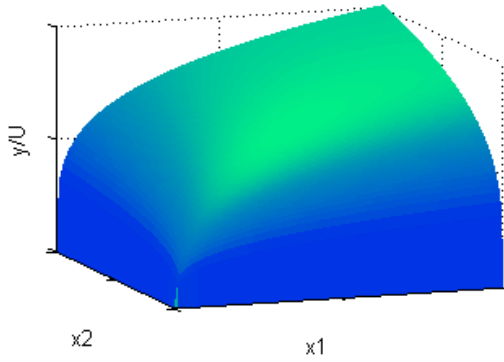
(A Visual Guide for Econ Majors)

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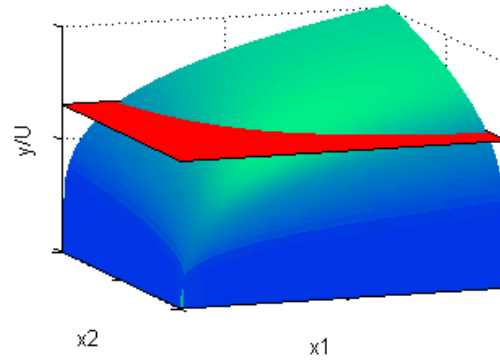
September 2006

Decreasing returns to scale (Strongly concave y/U)

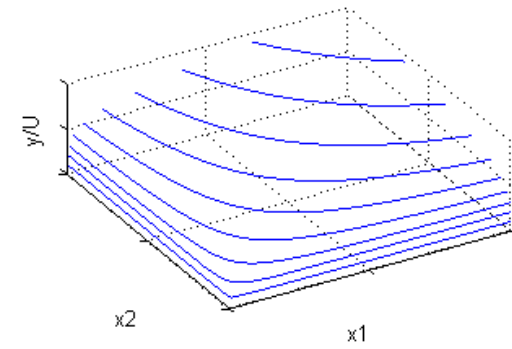
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



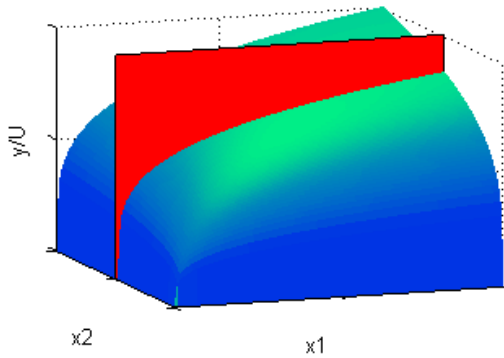
holding output/utility fixed



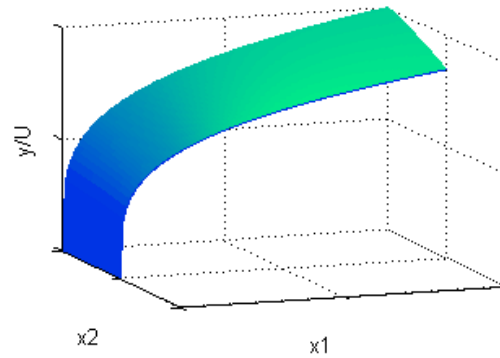
convex isoquants/indifference curves



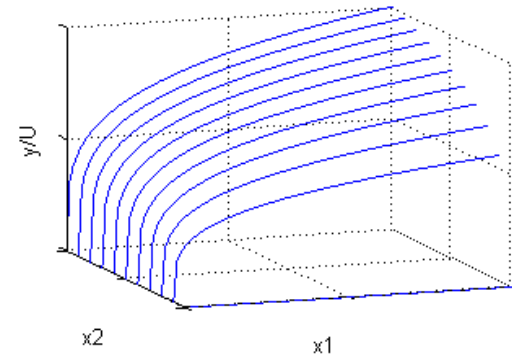
holding one input/good fixed



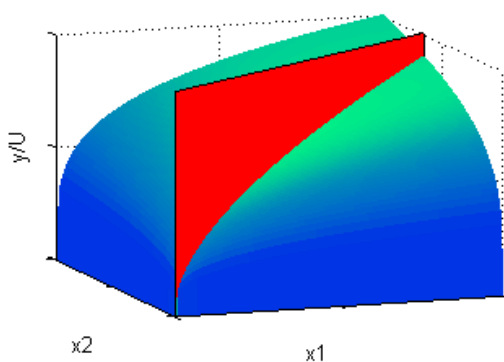
slice parallel to axis



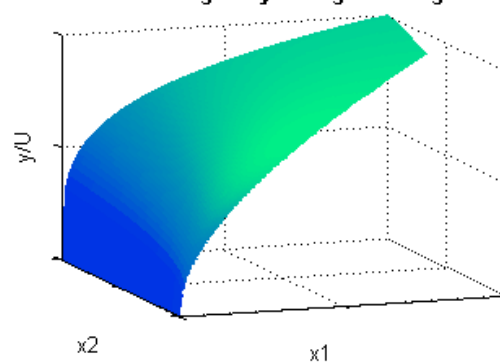
diminishing marginal product/utility



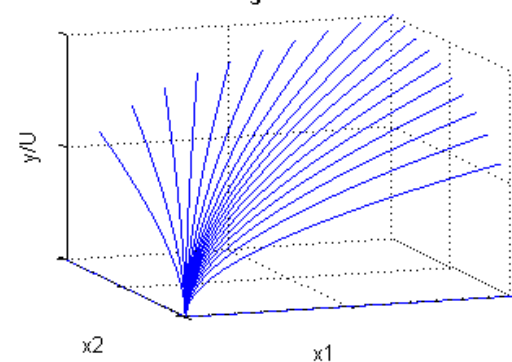
increasing both inputs/goods by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

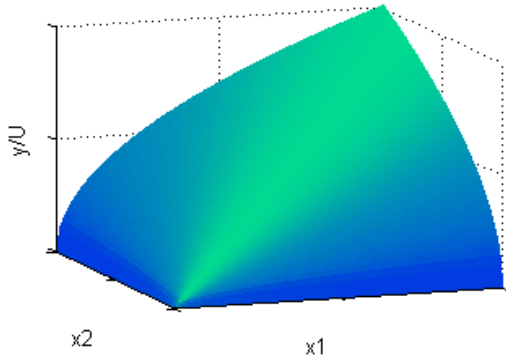


decreasing returns to scale

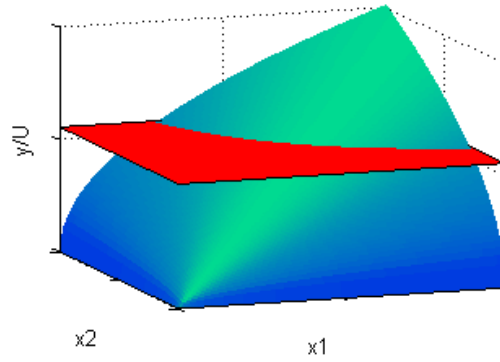


Constant returns to scale (Weakly concave y/U)

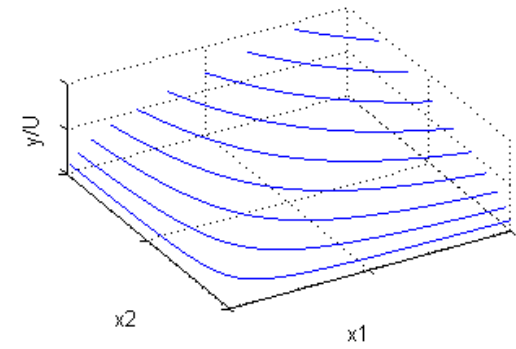
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



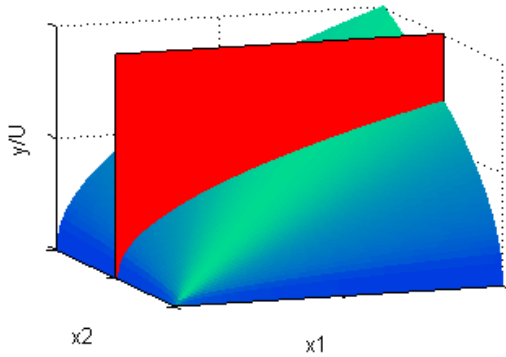
holding output/utility fixed



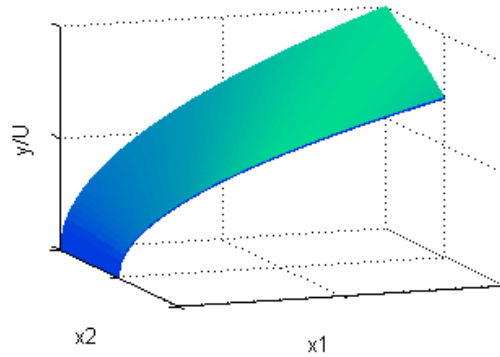
convex isoquants/indifference curves



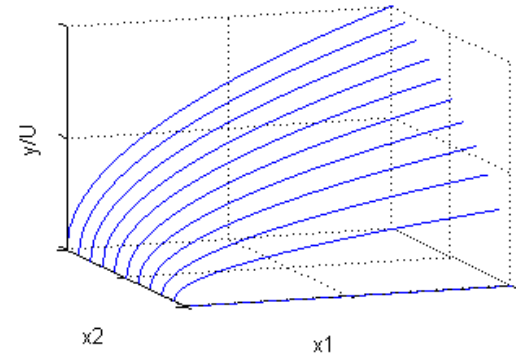
holding one input/good fixed



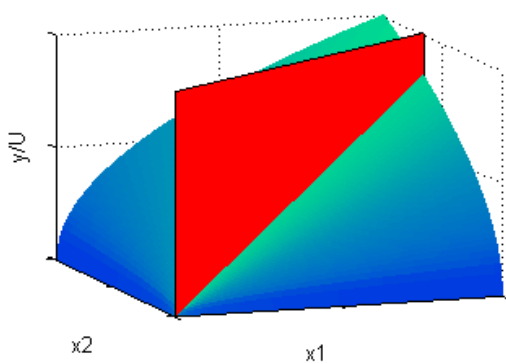
slice parallel to axis



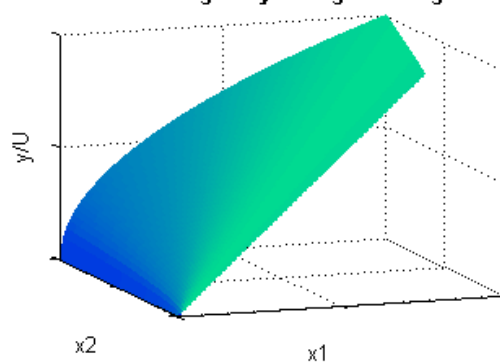
diminishing marginal product/utility



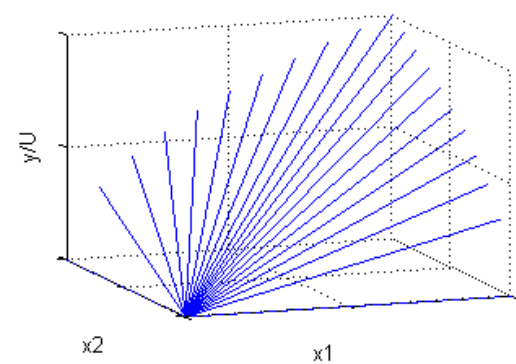
increasing both inputs/goods
 by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

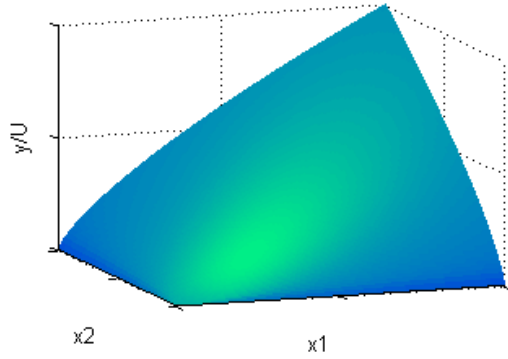


constant returns to scale

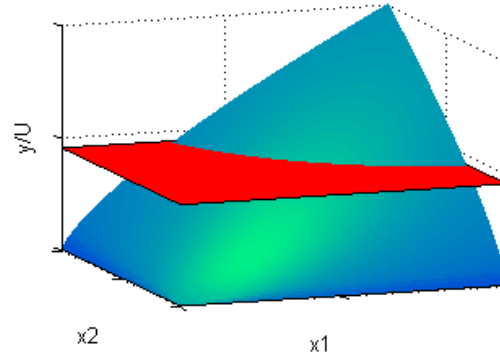


Increasing returns to scale with diminishing marginal product/utility (Quasiconcave y/U)

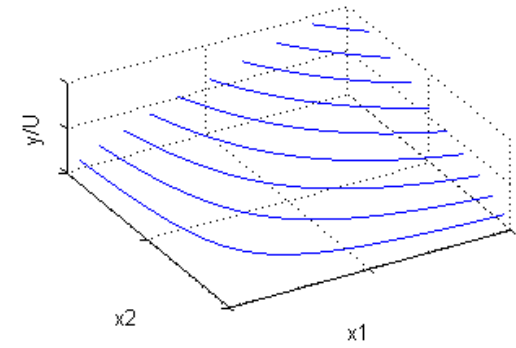
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.75$ - increasing returns to scale



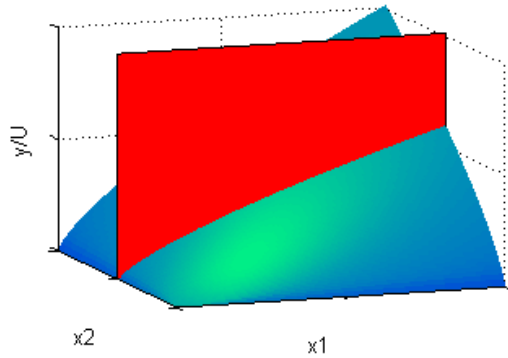
holding output/utility fixed



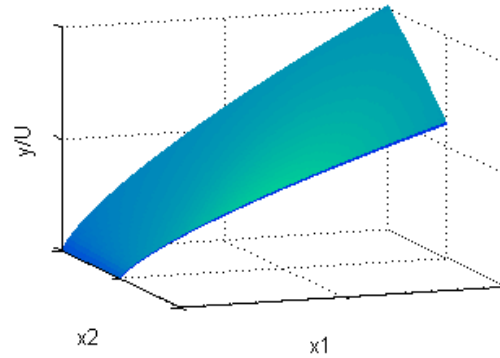
convex isoquants/indifference curves



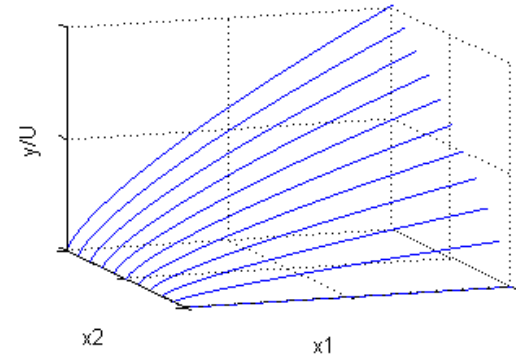
holding one input/good fixed



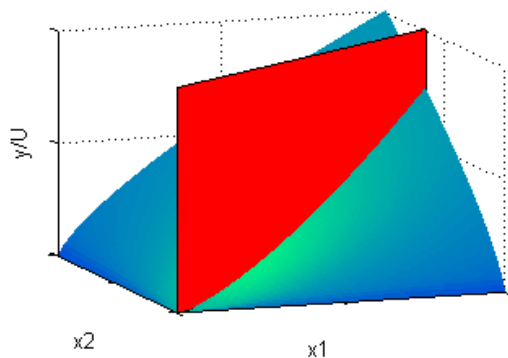
slice parallel to axis



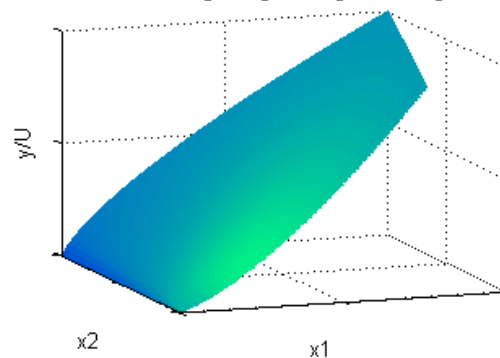
diminishing marginal product/utility



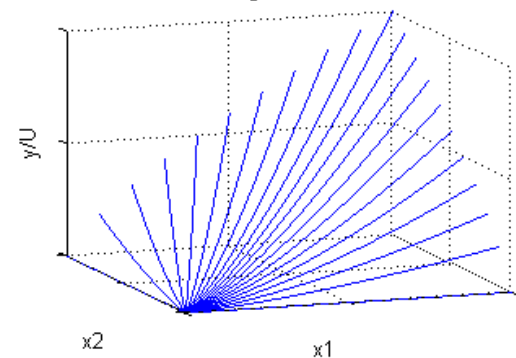
increasing both inputs/goods by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

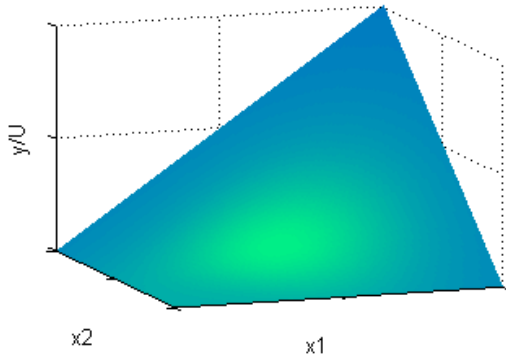


increasing returns to scale

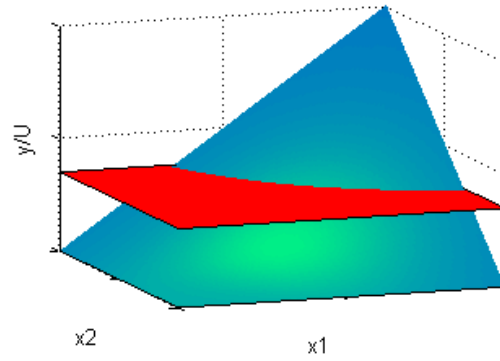


Increasing returns to scale with linear marginal product/utility (Quasiconcave y/U)

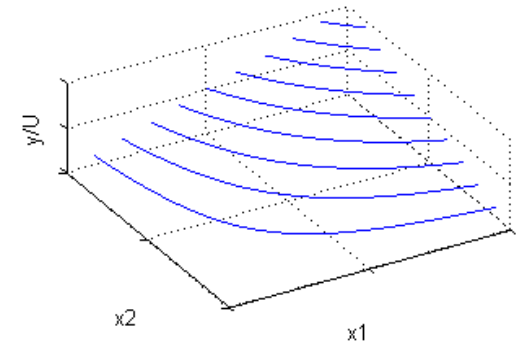
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=1$ - increasing returns to scale



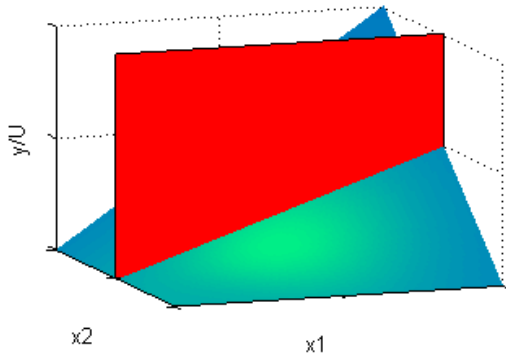
holding output/utility fixed



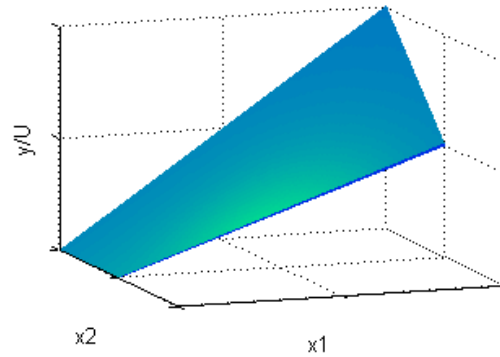
convex isoquants/indifference curves



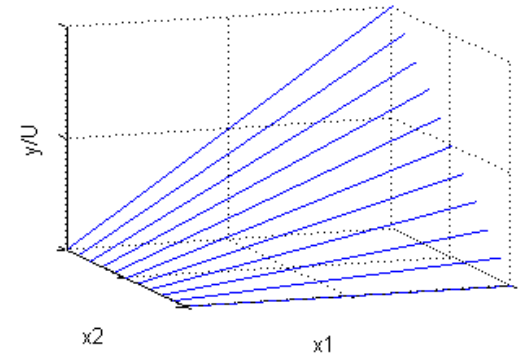
holding one input/good fixed



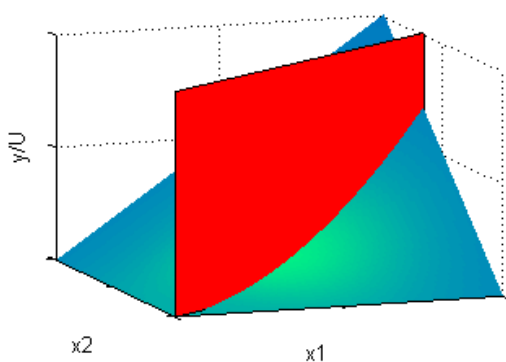
slice parallel to axis



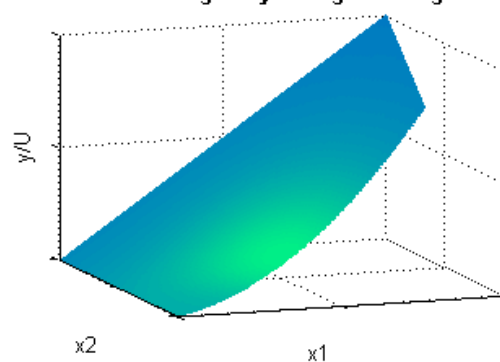
constant marginal product/utility



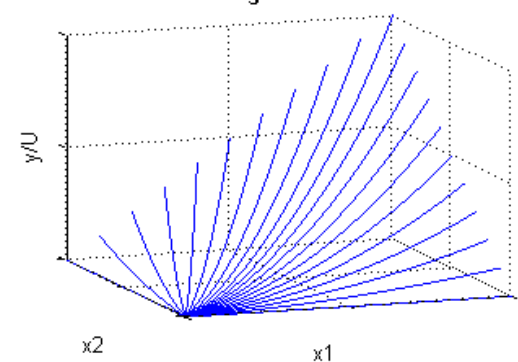
increasing both inputs/goods by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

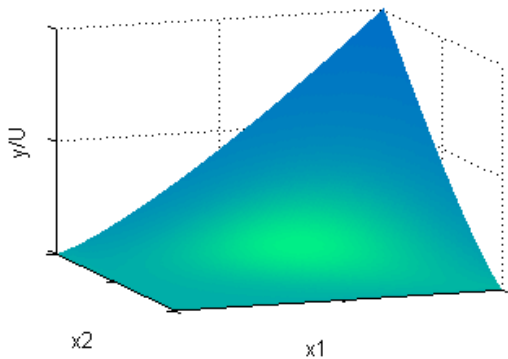


increasing returns to scale

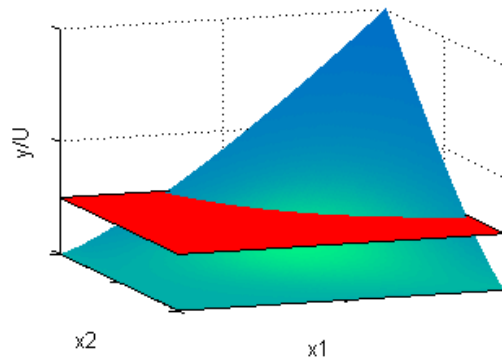


Increasing returns to scale with increasing marginal product/utility (Quasiconcave y/U)

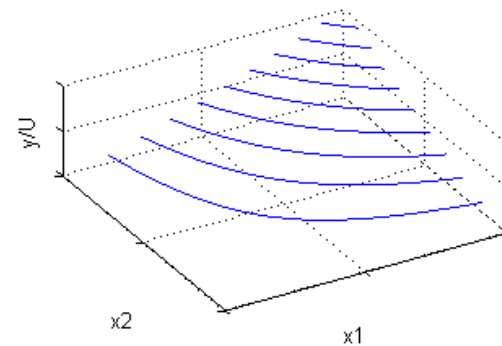
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=1.25$ - increasing returns to scale



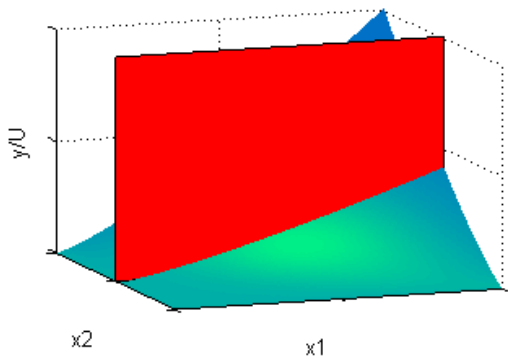
holding output/utility fixed



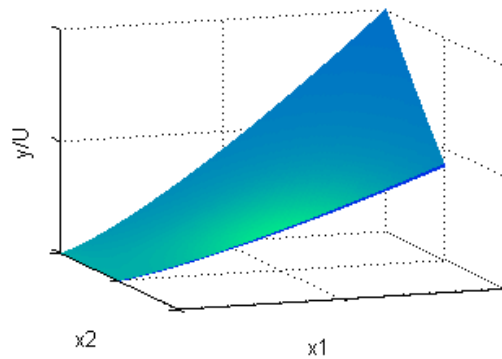
convex isoquants/indifference curves



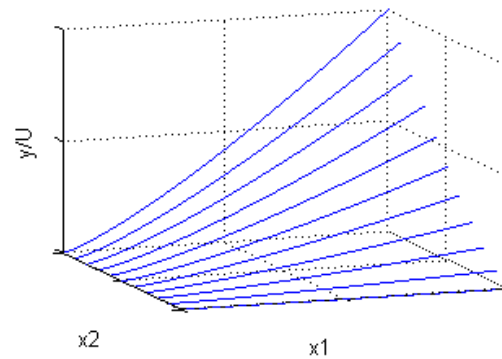
holding one input/good fixed



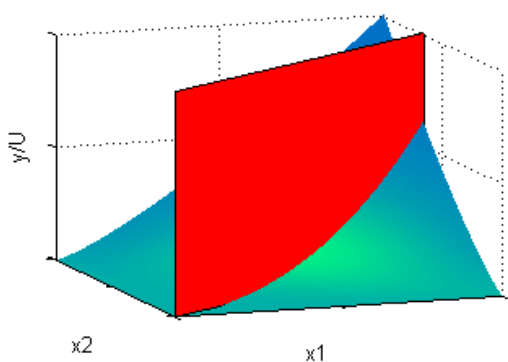
slice parallel to axis



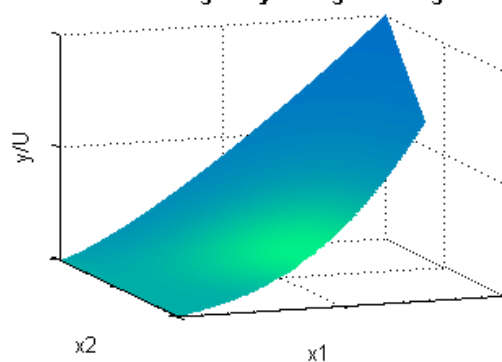
increasing marginal product/utility



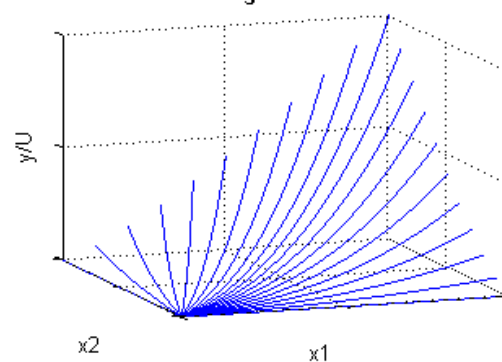
increasing both inputs/goods by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

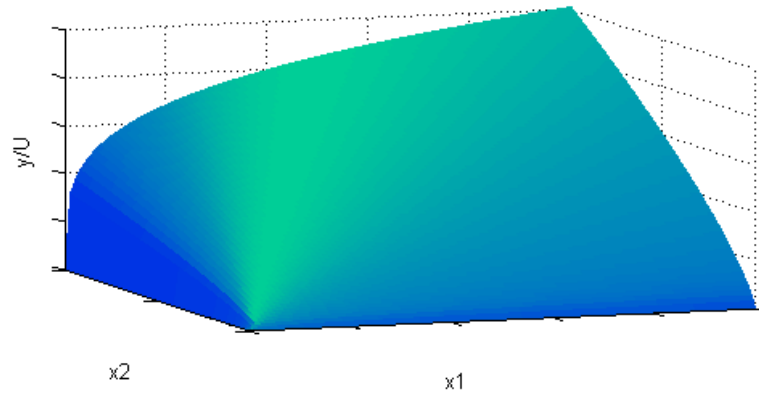


increasing returns to scale

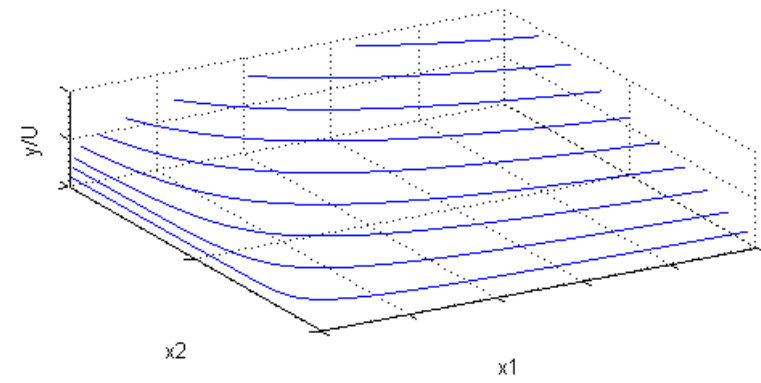


Non-symmetric production/utility function with constant returns to scale

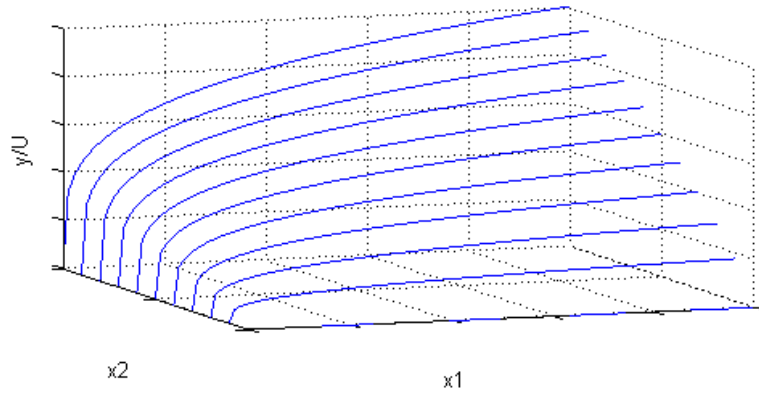
production/utility function: $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=0.75, \beta=0.25$



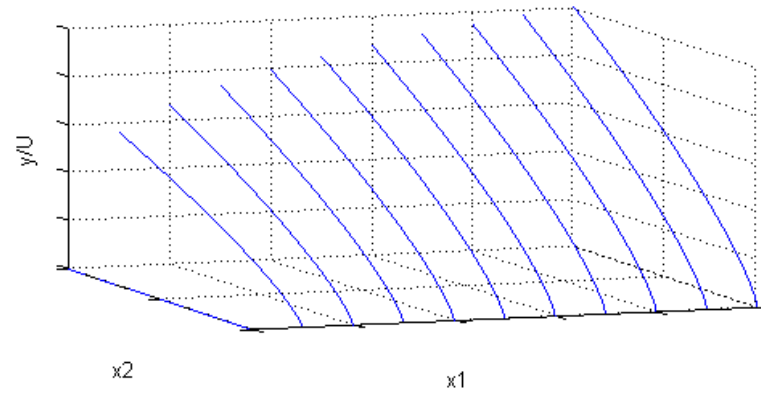
isoquants/indifference curves



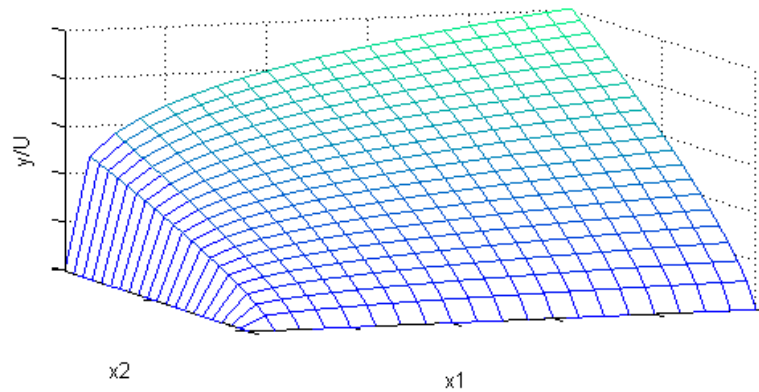
marginal properties (intensive margin)



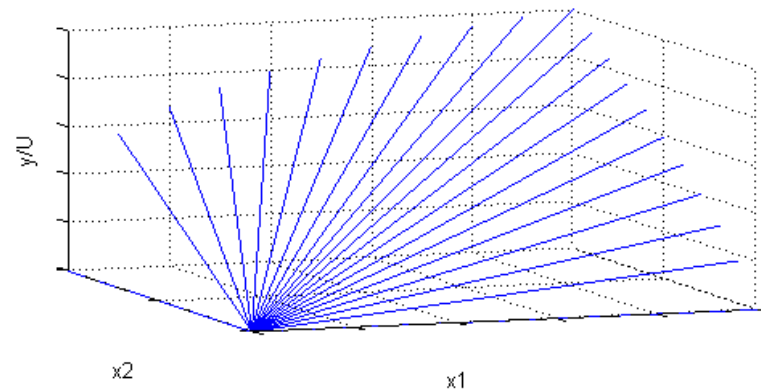
marginal properties (intensive margin)



meshgrid

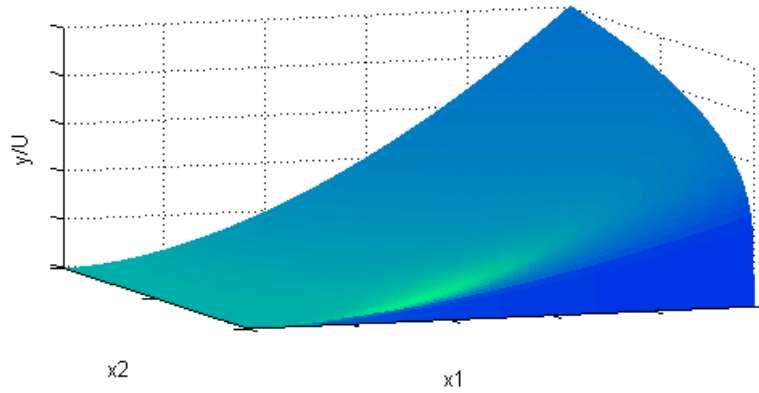


scale properties (extensive margin)

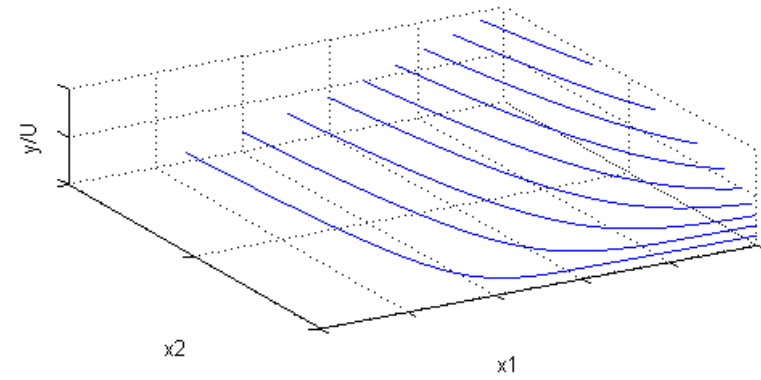


Non-symmetric production/utility function with increasing returns to scale

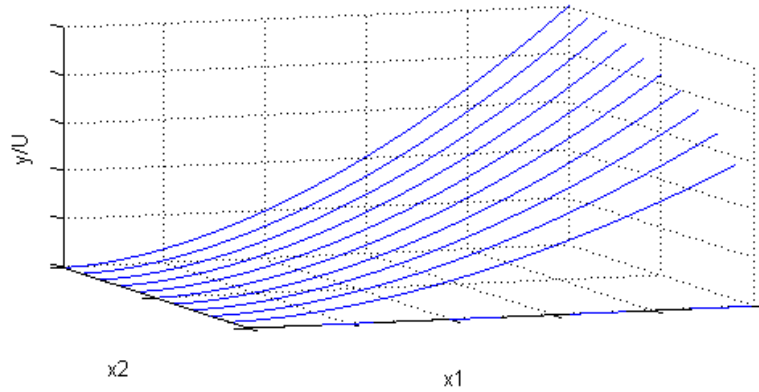
production/utility function: $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=0.25, \beta=1.75$



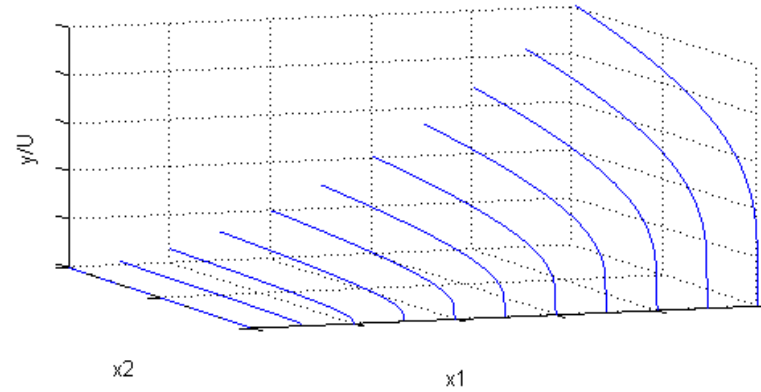
isoquants/indifference curves



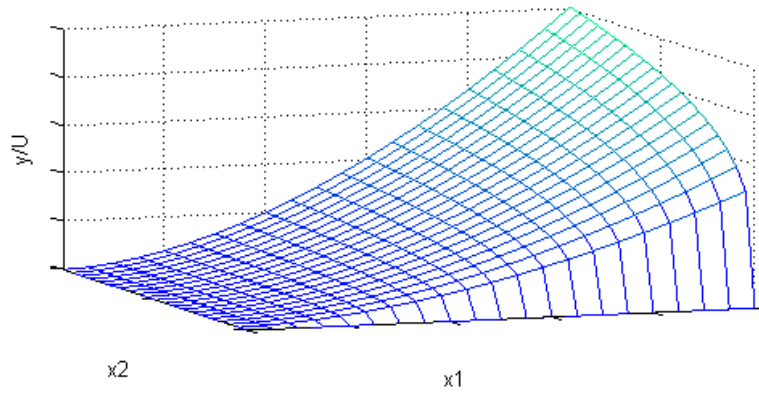
marginal properties (intensive margin)



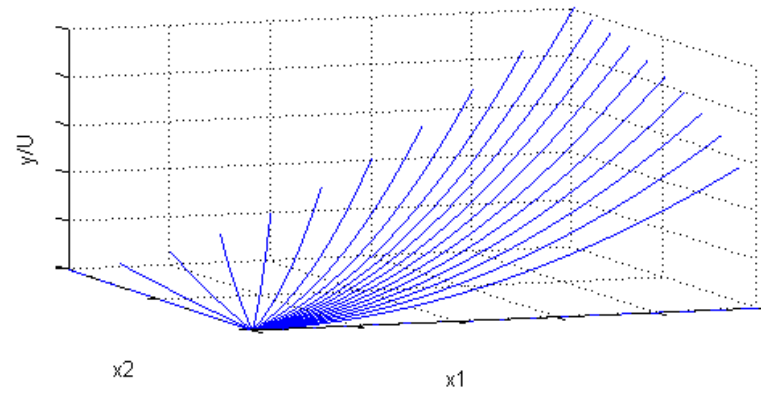
marginal properties (intensive margin)



meshgrid

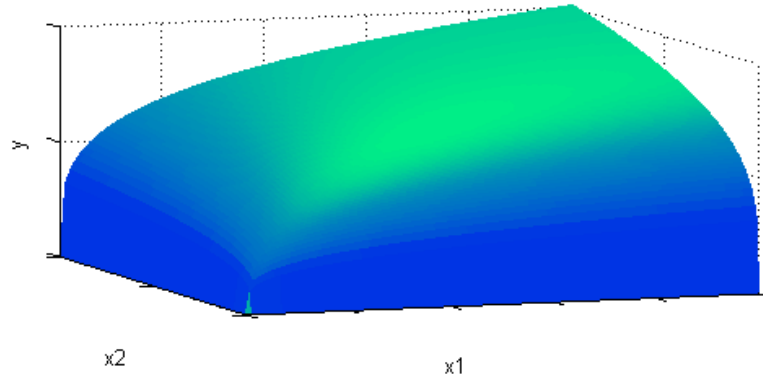


scale properties (extensive margin)

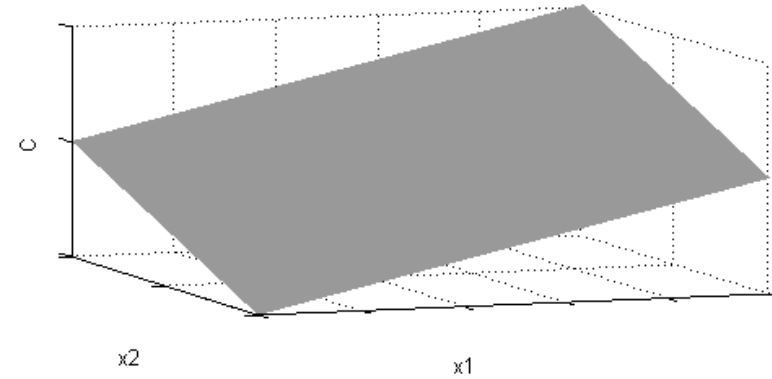


Profit maximization: production function with decreasing returns to scale

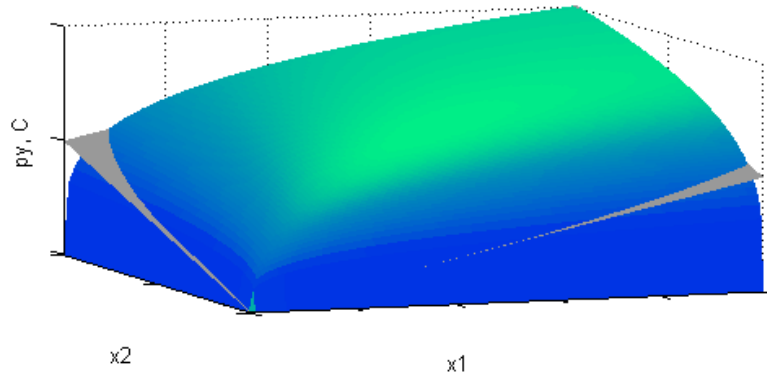
production function: $y = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



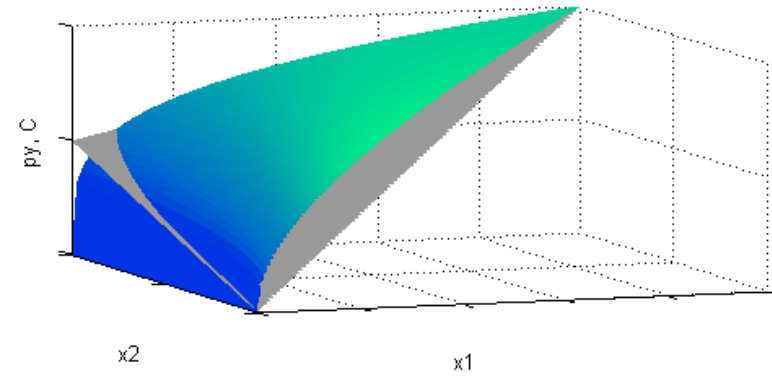
cost function: $C = w_1 x_1 + w_2 x_2$
 $w_1=w_2=1$



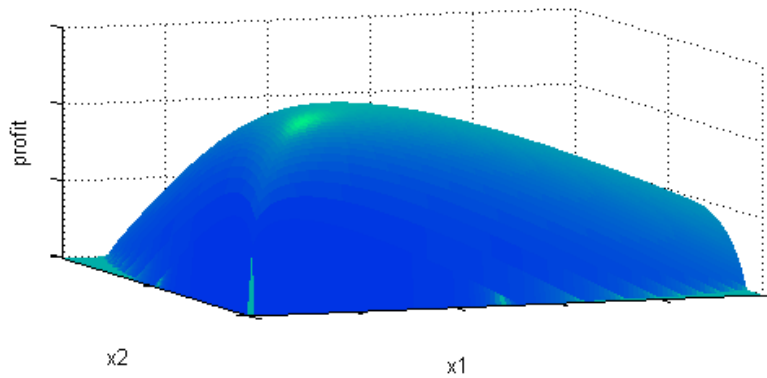
the two functions combined



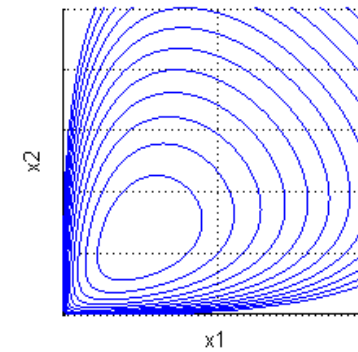
slice along a ray



the vertical difference: value of output - cost = profit
 only profit > 0 displayed

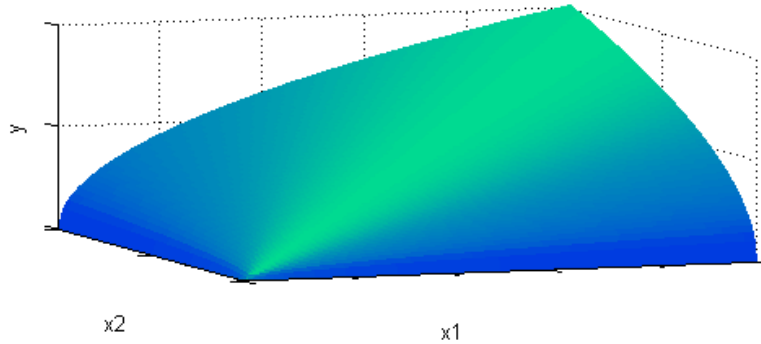


with decreasing returns to scale,
 an interior solution for maximum profit can be achieved

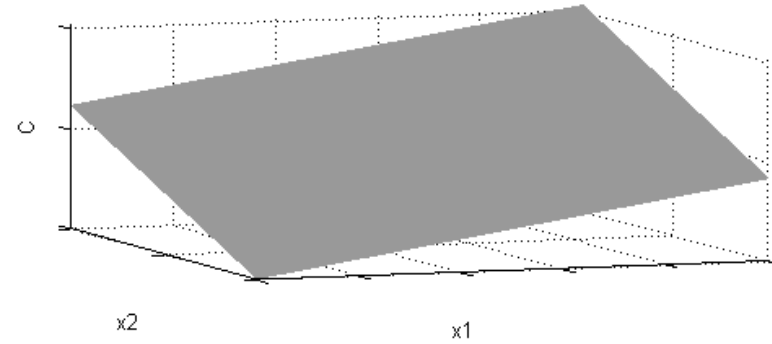


Profit maximization: production function with constant returns to scale

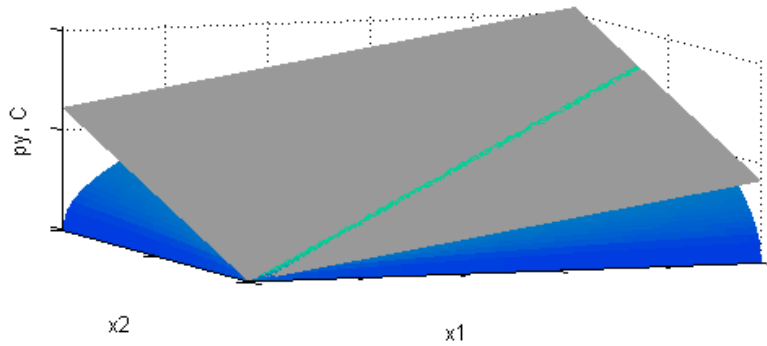
production function: $y = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



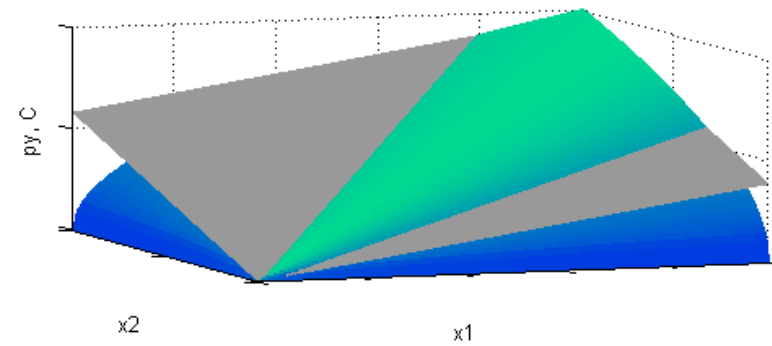
cost function: $C = w_1 x_1 + w_2 x_2$
 $w_1=2, w_2=3$



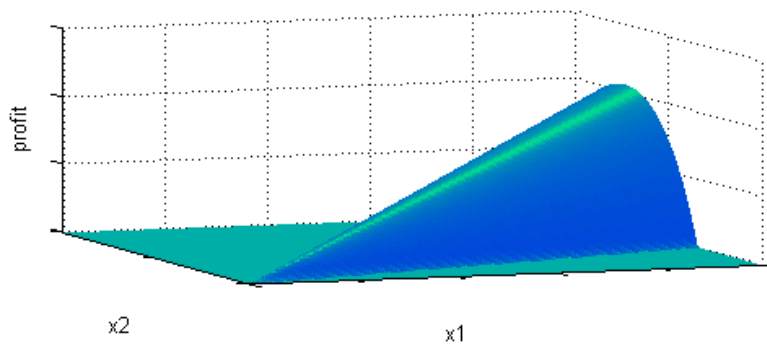
in a competitive market, price and wage adjust:
 $p = 2 \text{ sqrt}(w_1 w_2) = 2 \text{ sqrt}(6)$
 profit = value of output - cost = zero



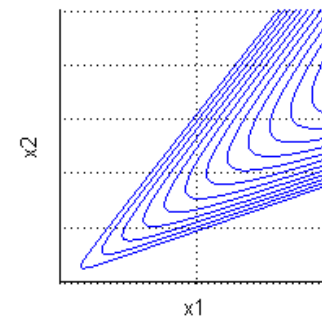
in a non-competitive market,
 we don't have to impose zero profit



profit in a non-competitive market
 profit = value of output - cost

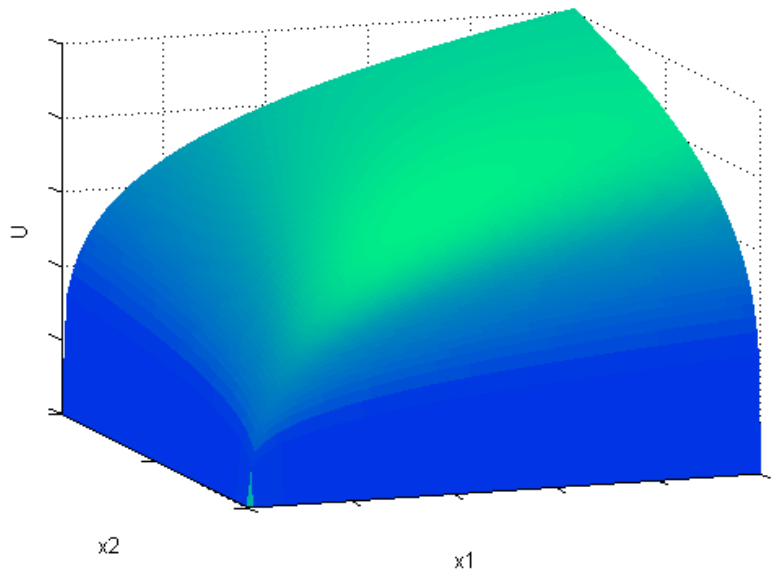


in a non-comp. mkt., profit could be infinite
 (but with increasing production,
 decreasing returns to scale eventually kick in)

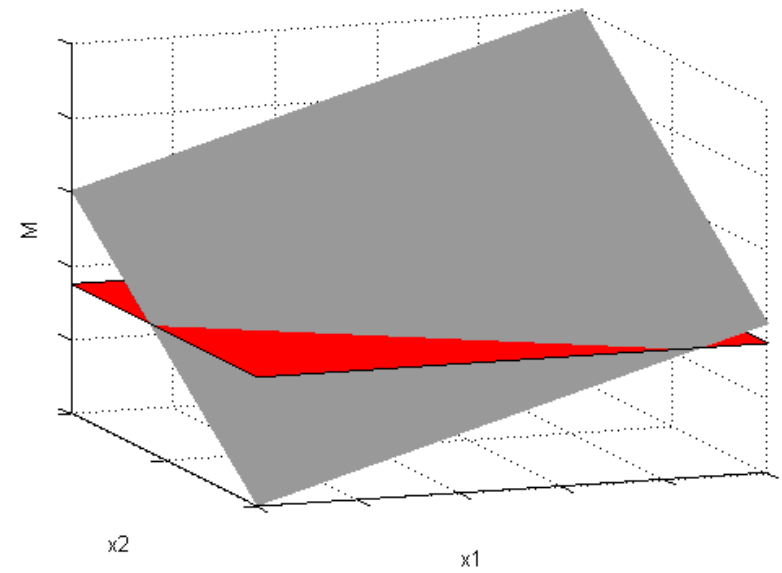


Utility maximization: utility function with decreasing returns to scale

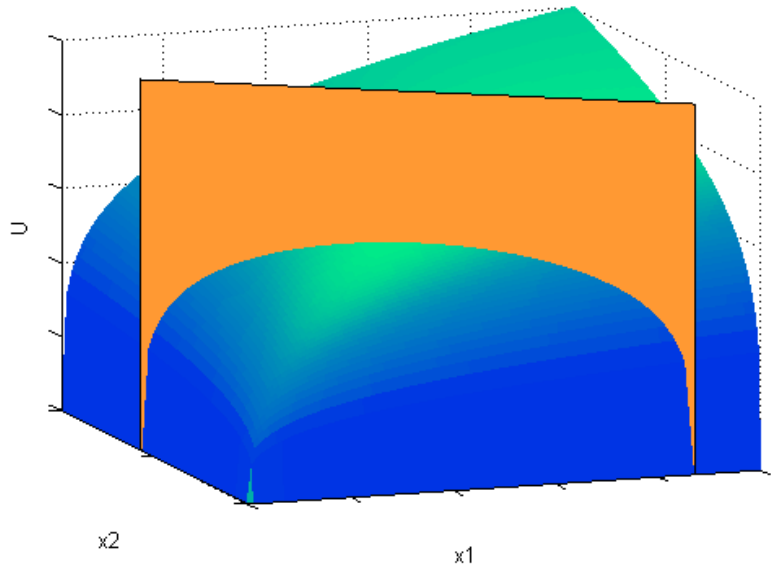
utility function: $U = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



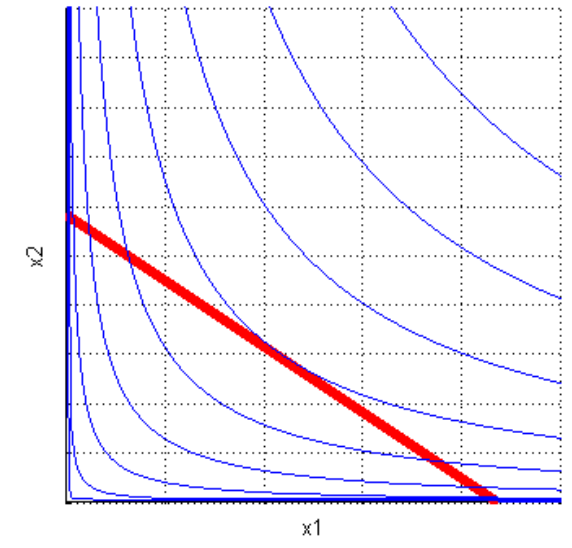
expenditure held constant at: $M^0 = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



maximize utility subject to expenditure constraint

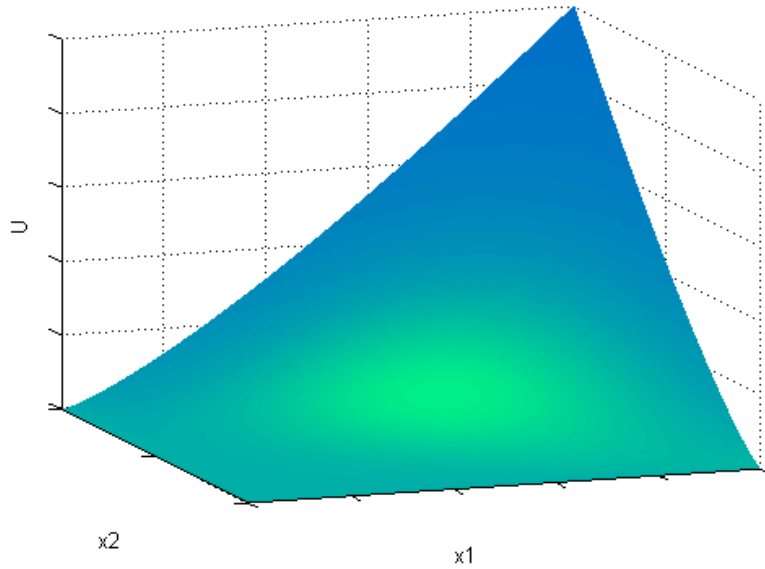


constrained maximum utility:
 the tangency point of the constraint
 and the indifference curve with the maximum value

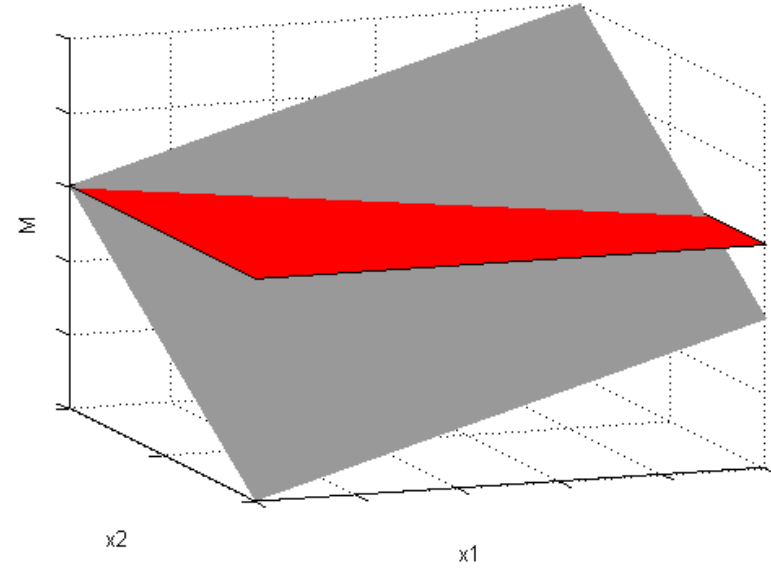


Utility maximization: utility function with increasing returns to scale

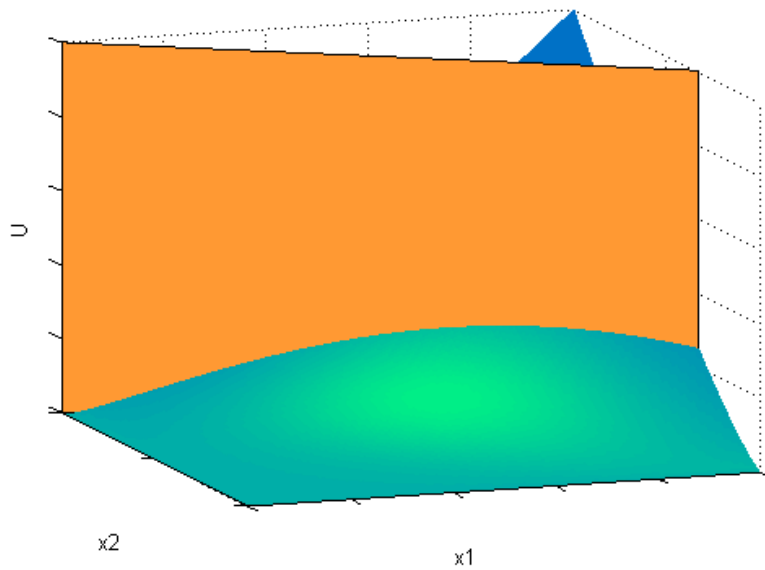
utility function: $U = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=1.25$ - increasing returns to scale



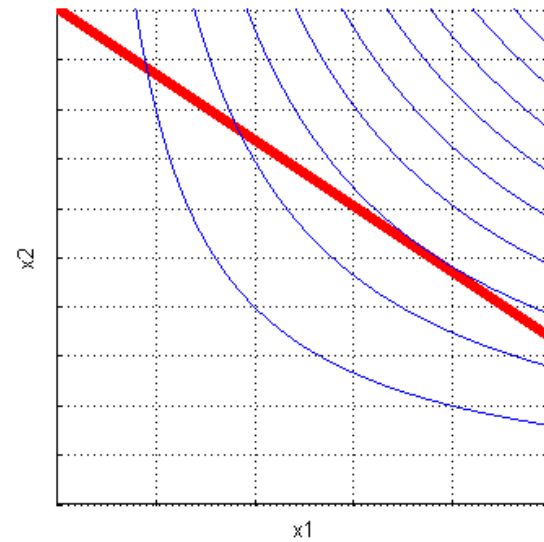
expenditure held constant: $M^0 = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



maximize utility subject to expenditure constraint

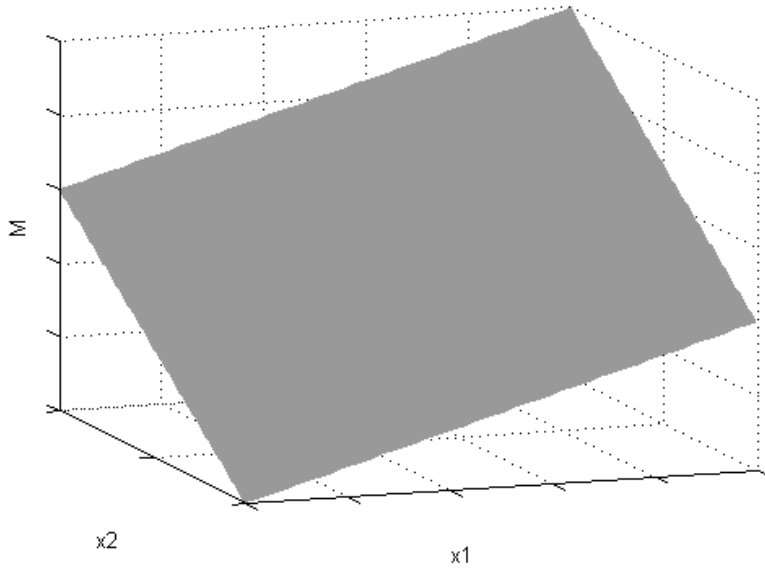


constrained maximum utility:
 the tangency point of the constraint
 and the indifference curve with the maximum value

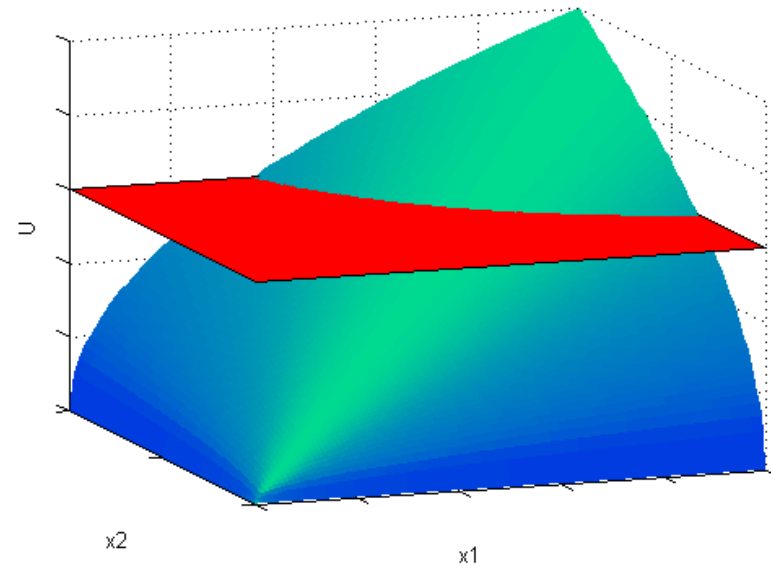


Expenditure minimization: utility function with constant returns to scale

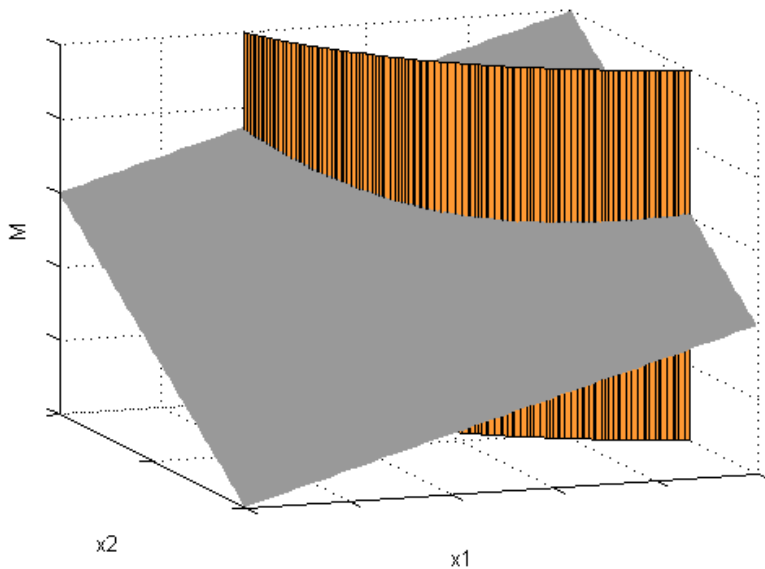
expenditure function: $M = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



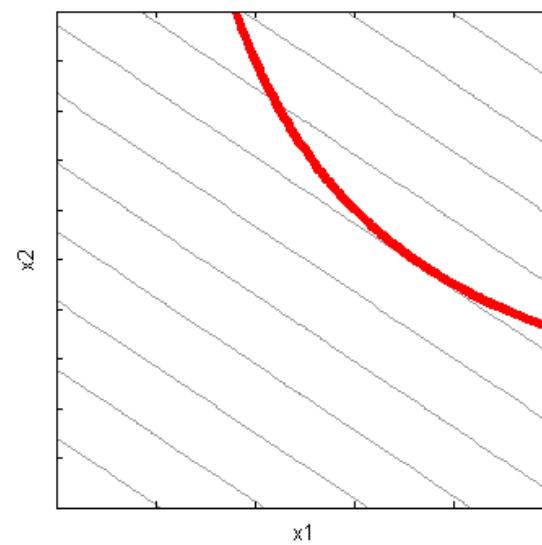
utility held constant at: $U^0 = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



minimize expenditure subject to utility constraint



constrained minimum expenditure:
 the tangency point of the constraint
 and the isocost curve with the minimum value



Literature and further reading:

The Structure of Economics, 3rd ed., Eugene Silberberg
MATLAB Documentation, MathWorks