Anatomy of a Cobb-Douglas Type Production/Utility Function in Three Dimensions

(A Visual Guide for Econ Majors)

Peter Fuleky
Department of Economics, University of Washington

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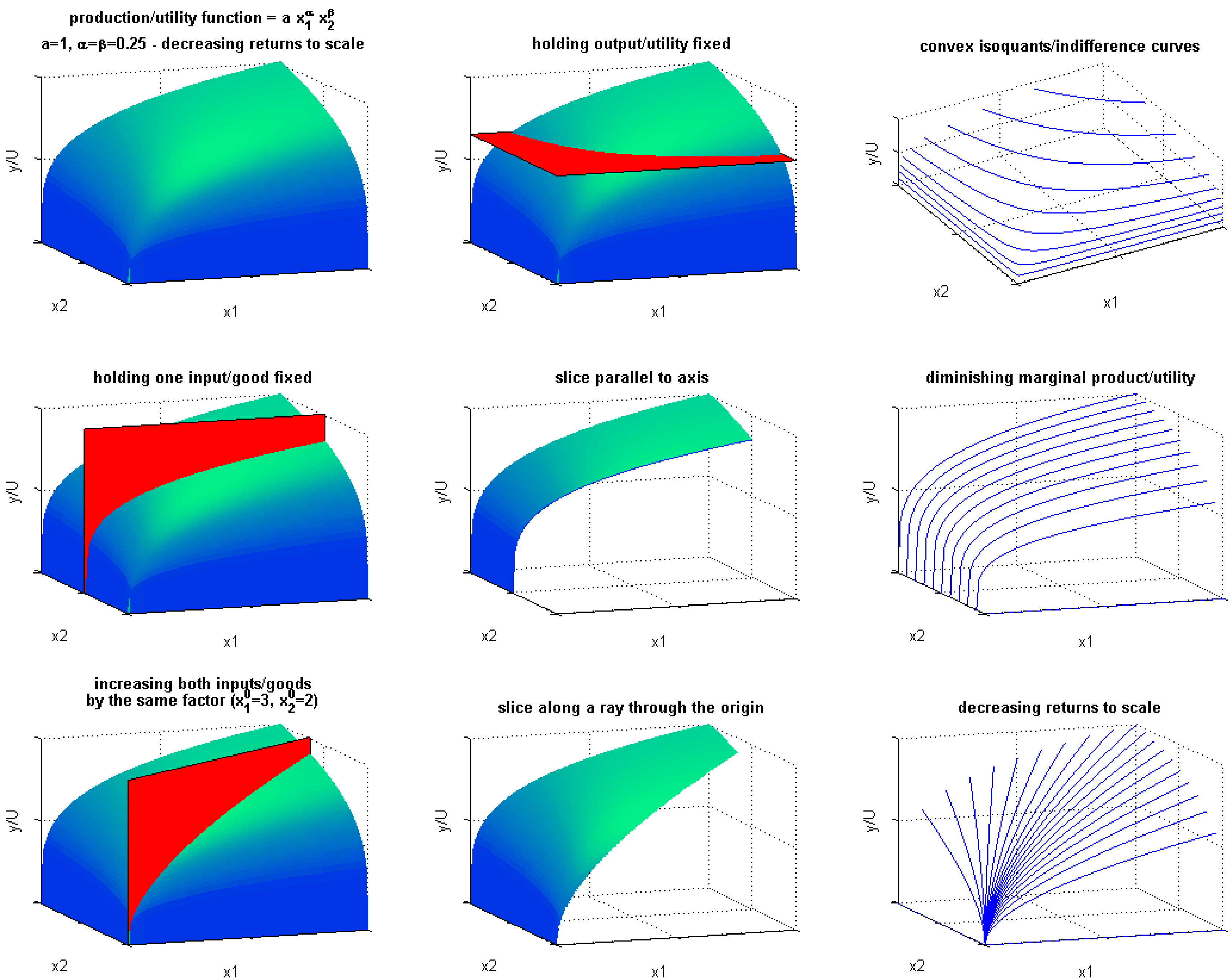
Decreasing returns to scale (Strongly concave y/U)



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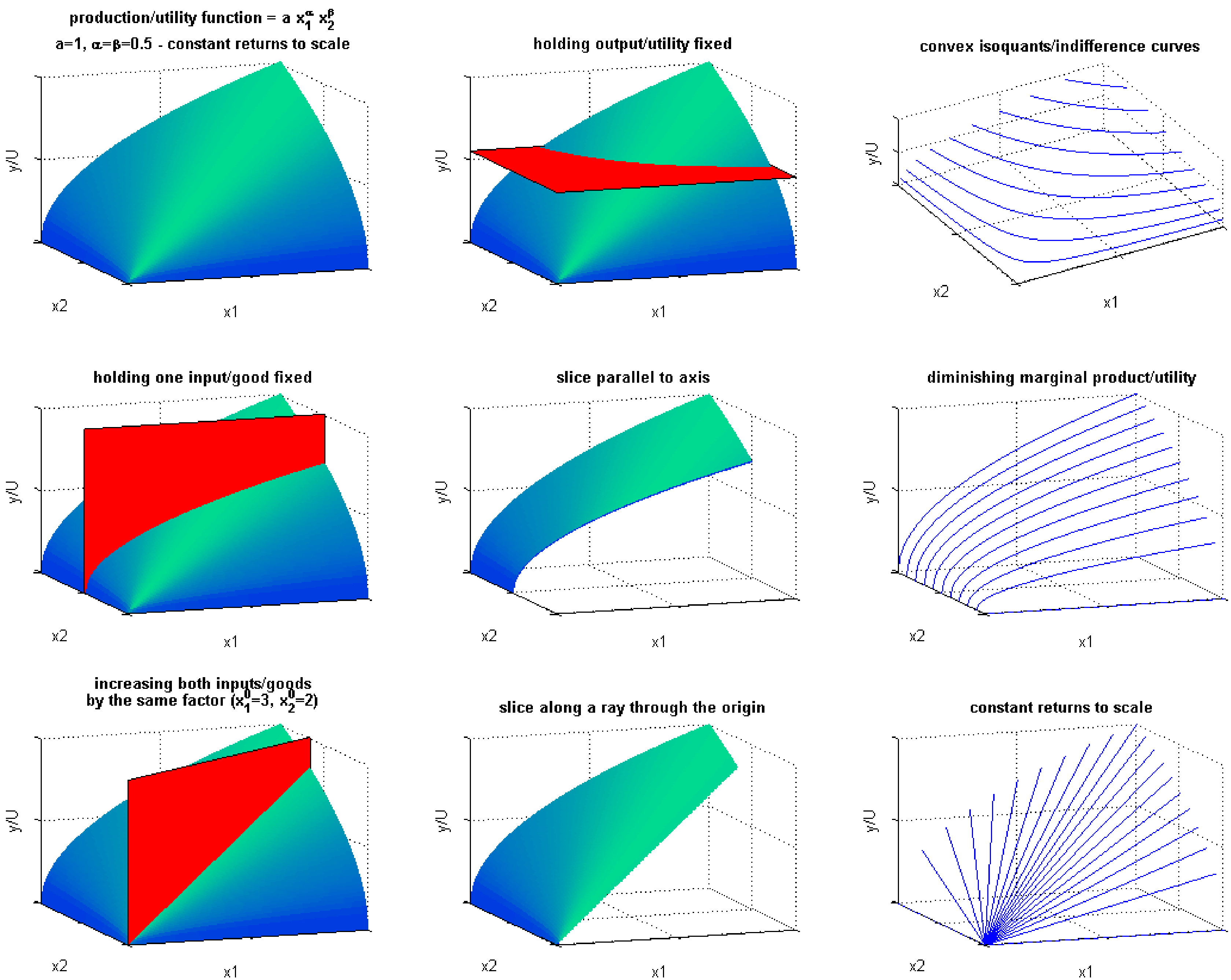
Constant returns to scale (Weakly concave y/U)



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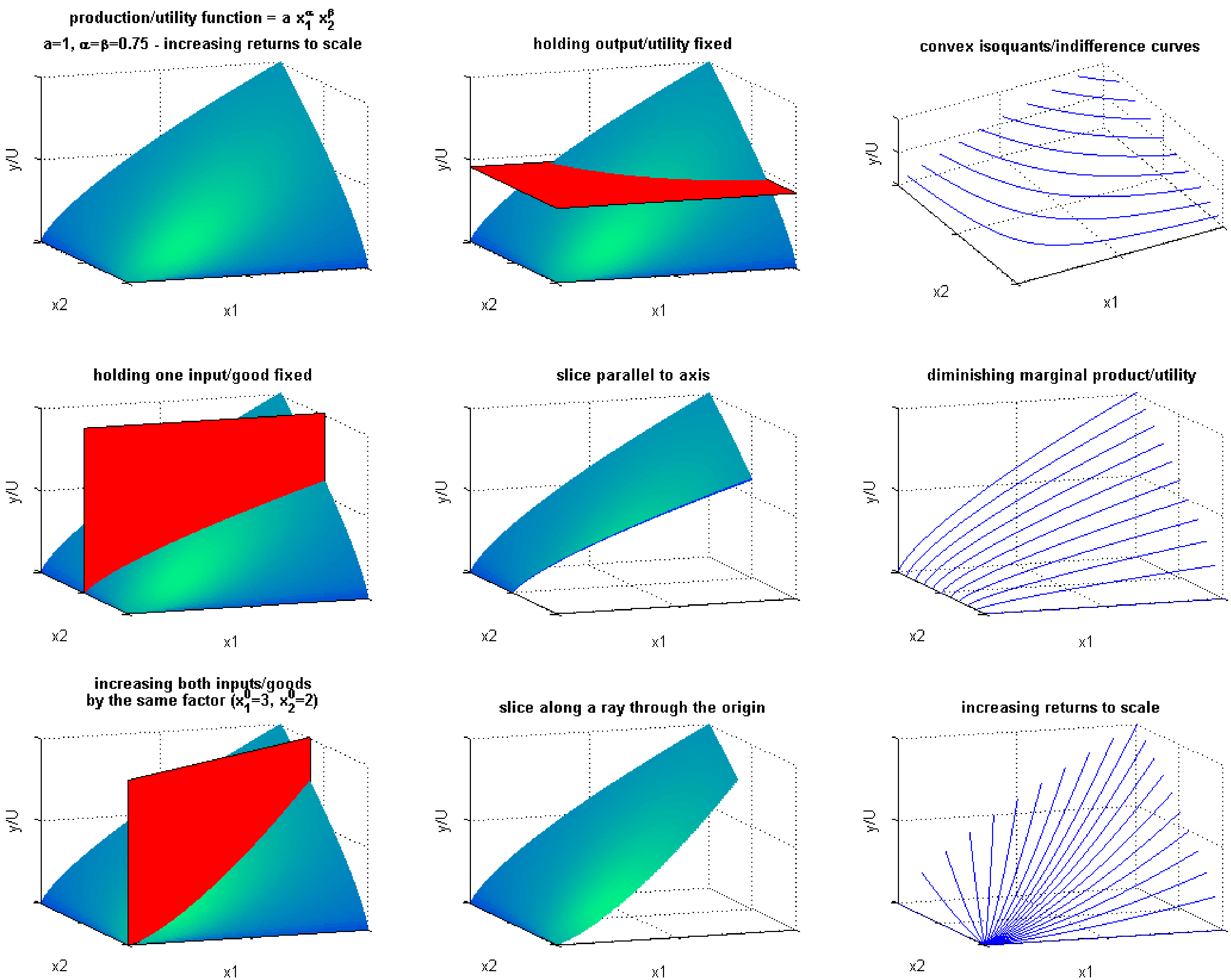
Increasing returns to scale with diminishing marginal product/utility (Quasiconcave y/U)



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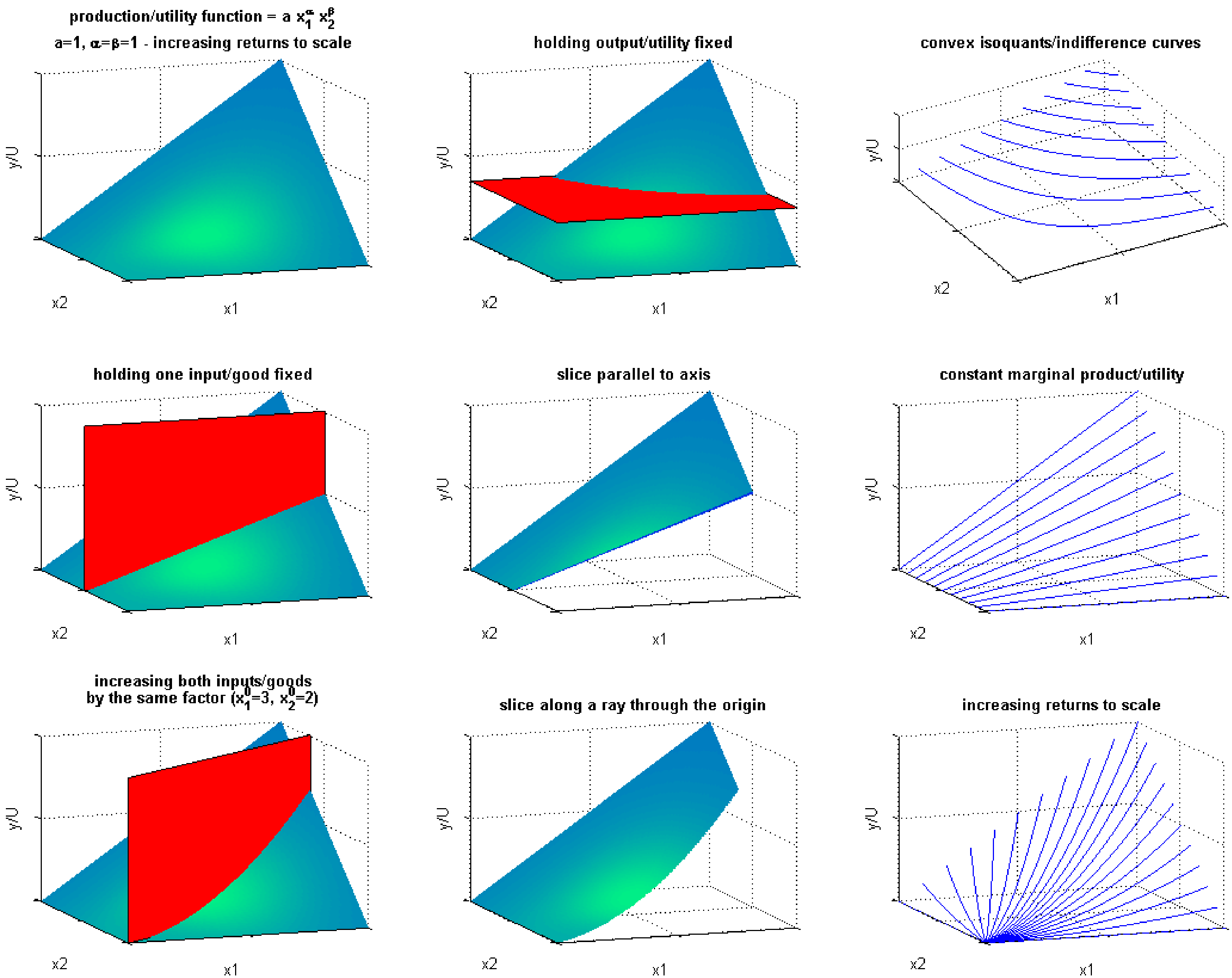
Increasing returns to scale with linear marginal product/utility (Quasiconcave y/U)



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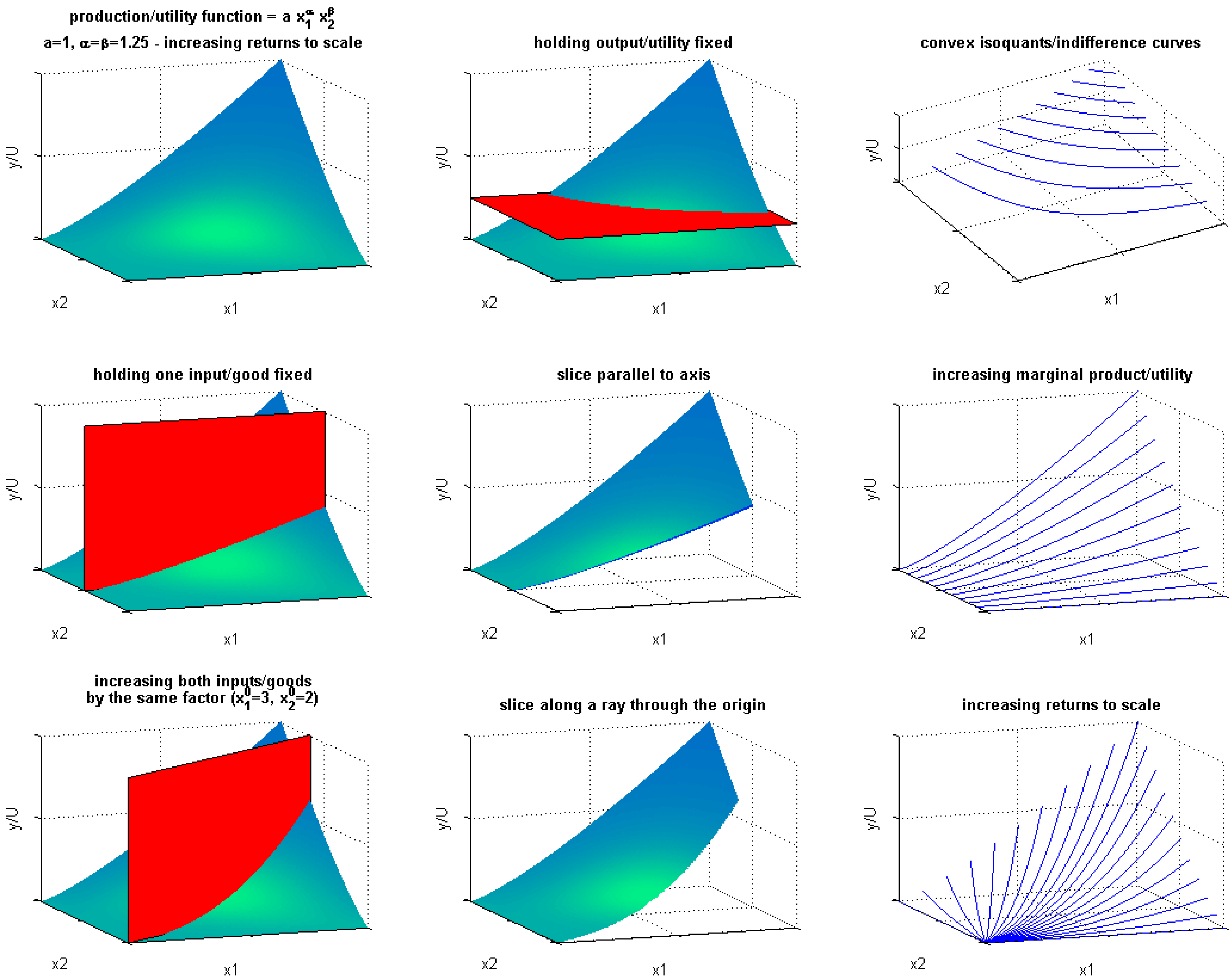
Increasing returns to scale with increasing marginal product/utility (Quasiconcave y/U)



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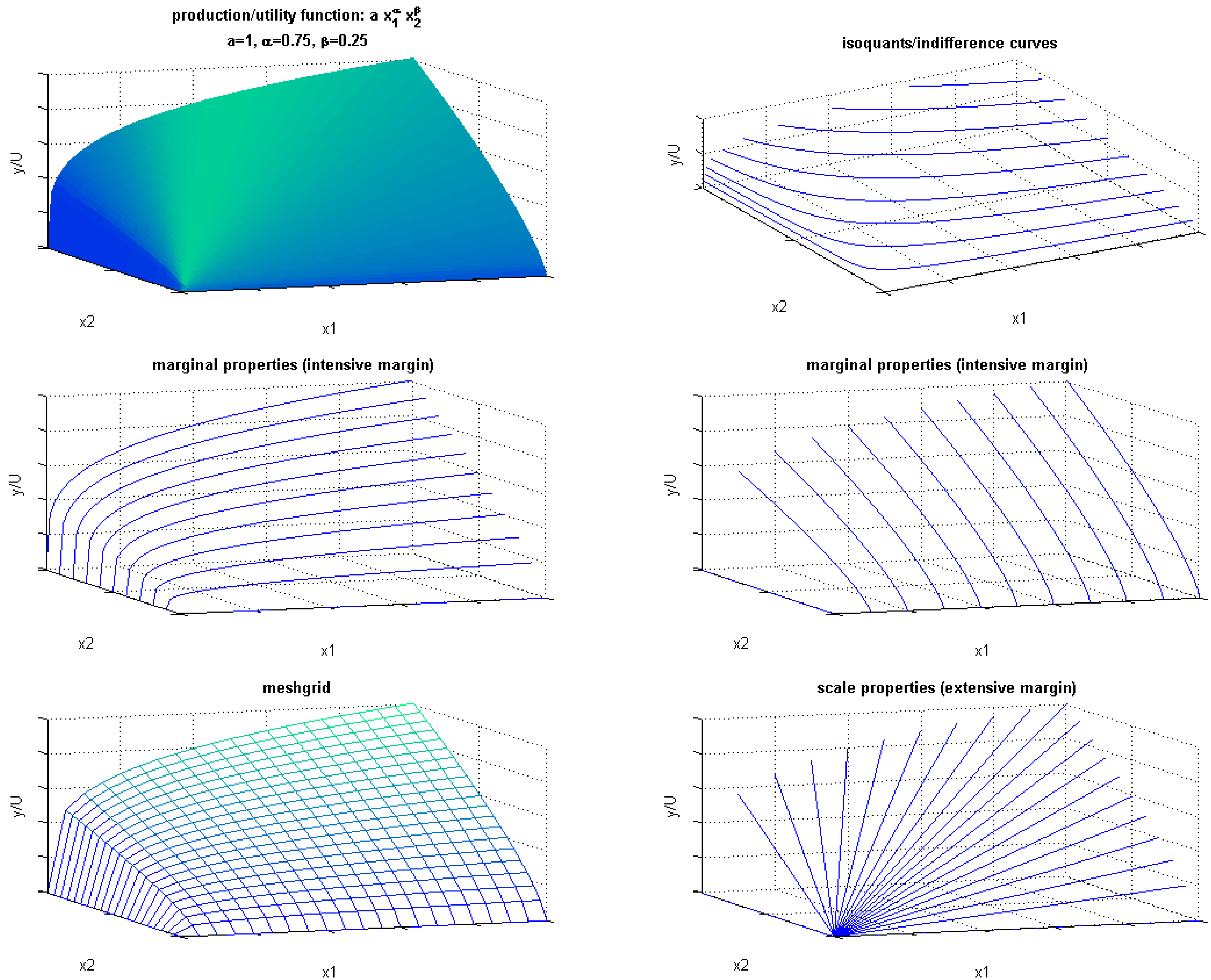
Non-symmetric production/utility function with constant returns to scale



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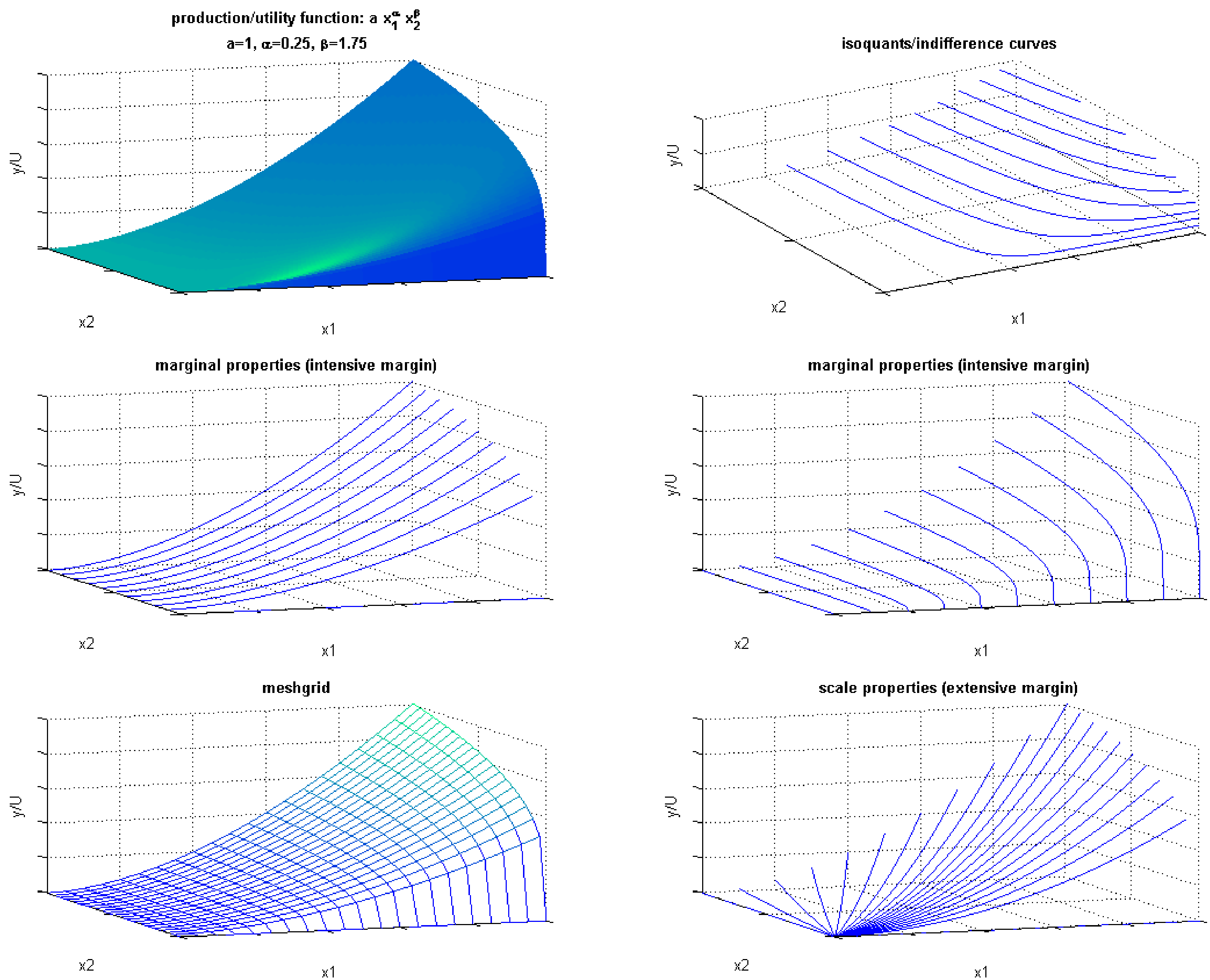
Non-symmetric production/utility function with increasing returns to scale



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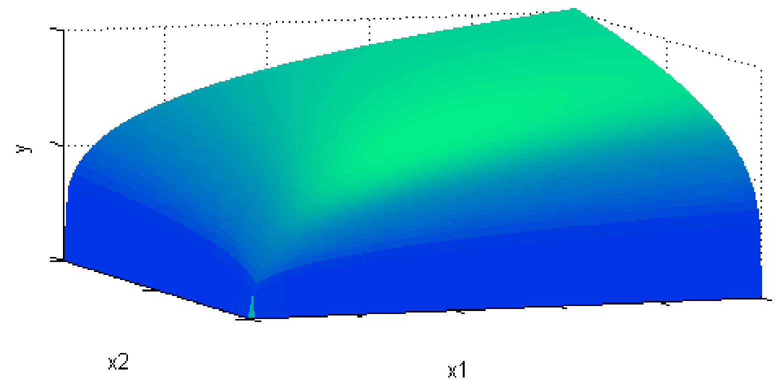
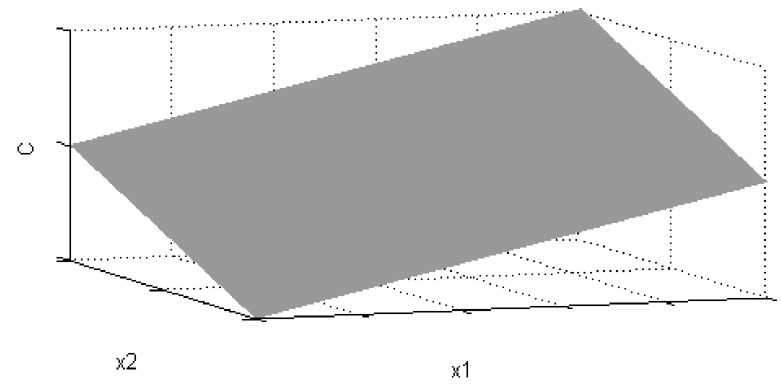
Profit maximization: production function with decreasing returns to scale

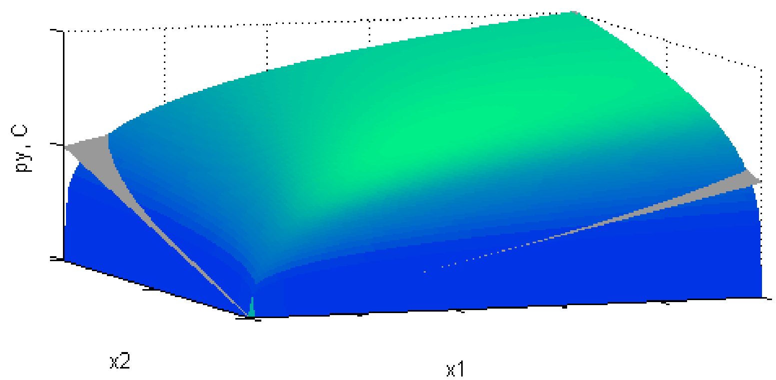
production function: $y = a\, x_1^{\alpha}\, x_2^{\beta}$
$a=1$, $\alpha=\beta=0.25$ - decreasing returns to scale



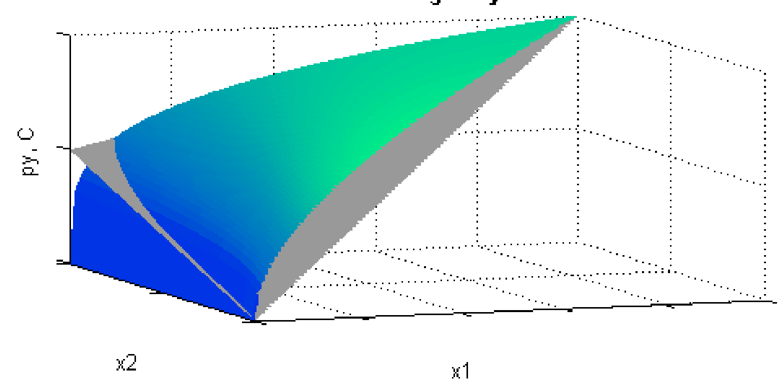
cost function: $C = w_1 x_1 + w_2 x_2$
$w_1=w_2=1$



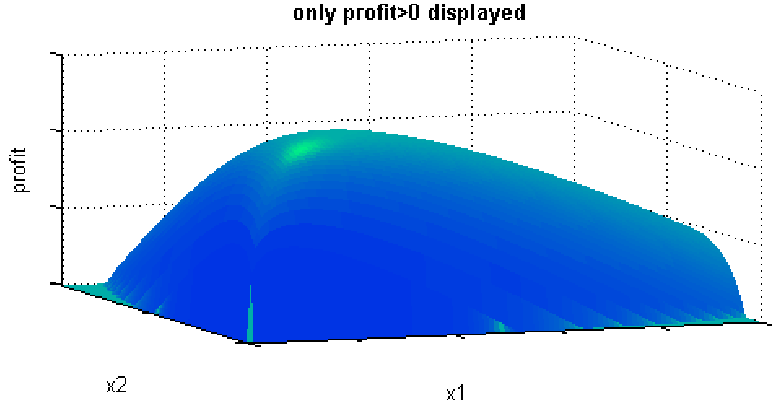
the two functions combined



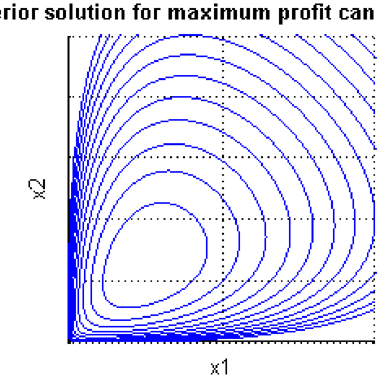
slice along a ray



the vertical difference: value of output - cost = profit
only profit>0 displayed



with decreasing returns to scale,
an interior solution for maximum profit can be achieved



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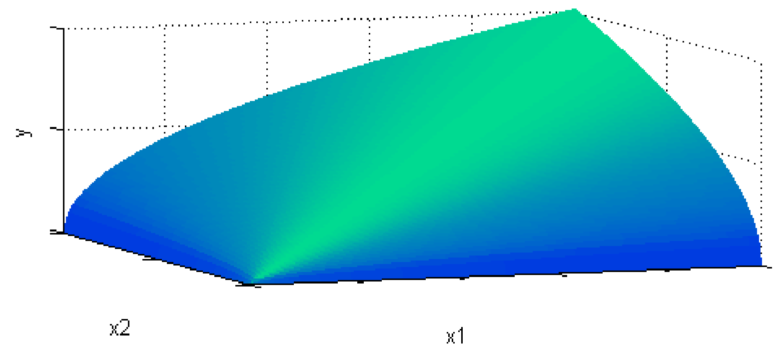
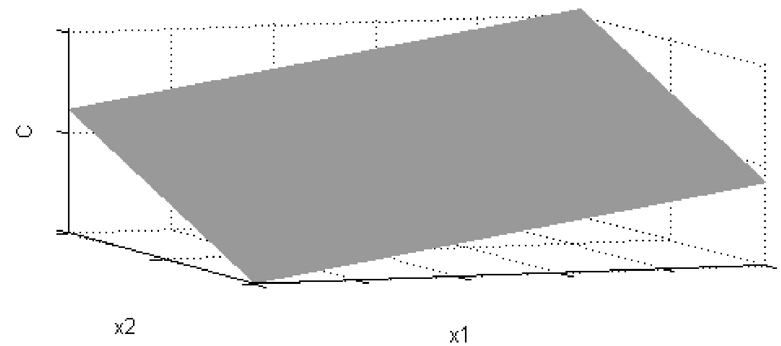
Profit maximization: production function with constant returns to scale

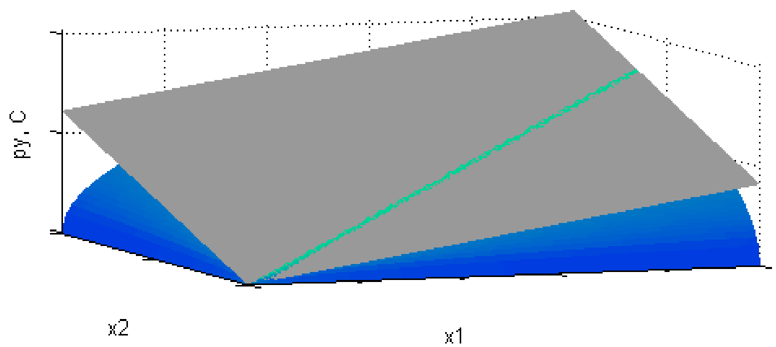
production function: $y = a\, x_1^{\alpha}\, x_2^{\beta}$
$a=1$, $\alpha=\beta=0.5$ - constant returns to scale



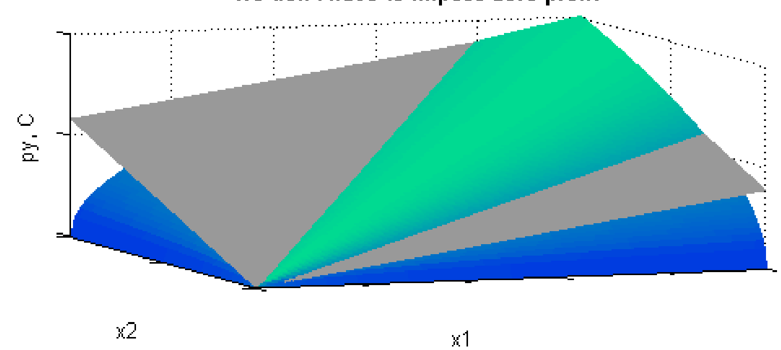
cost function: $C = w_1\, x_1 + w_2\, x_2$
$w_1=2$, $w_2=3$



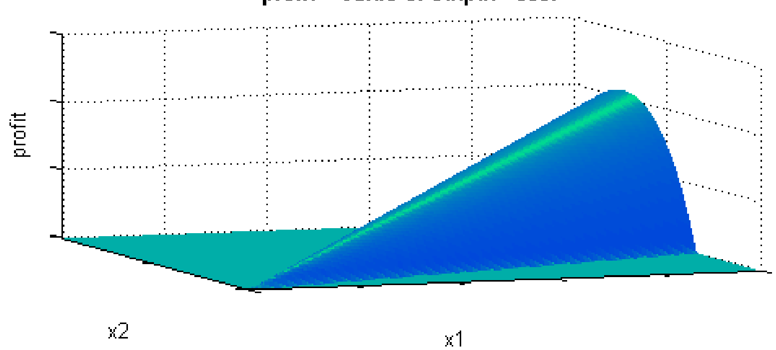
in a competitive market, price and wage adjust:
$p = 2\ \mathrm{sqr}\ (w_1\ w_2) = 2\ \mathrm{sqr}\ (6)$
profit = value of output - cost = zero



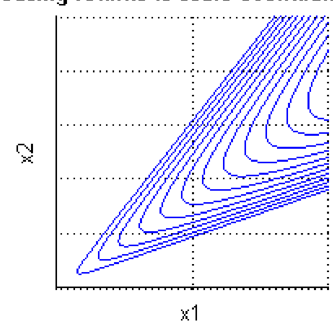
in a non-competitive market,
we don't have to impose zero profit



profit in a non-competitive market
profit = value of output - cost



in a non-comp. mkt., profit could be infinite
(but with increasing production,
decreasing returns to scale eventually kick in)



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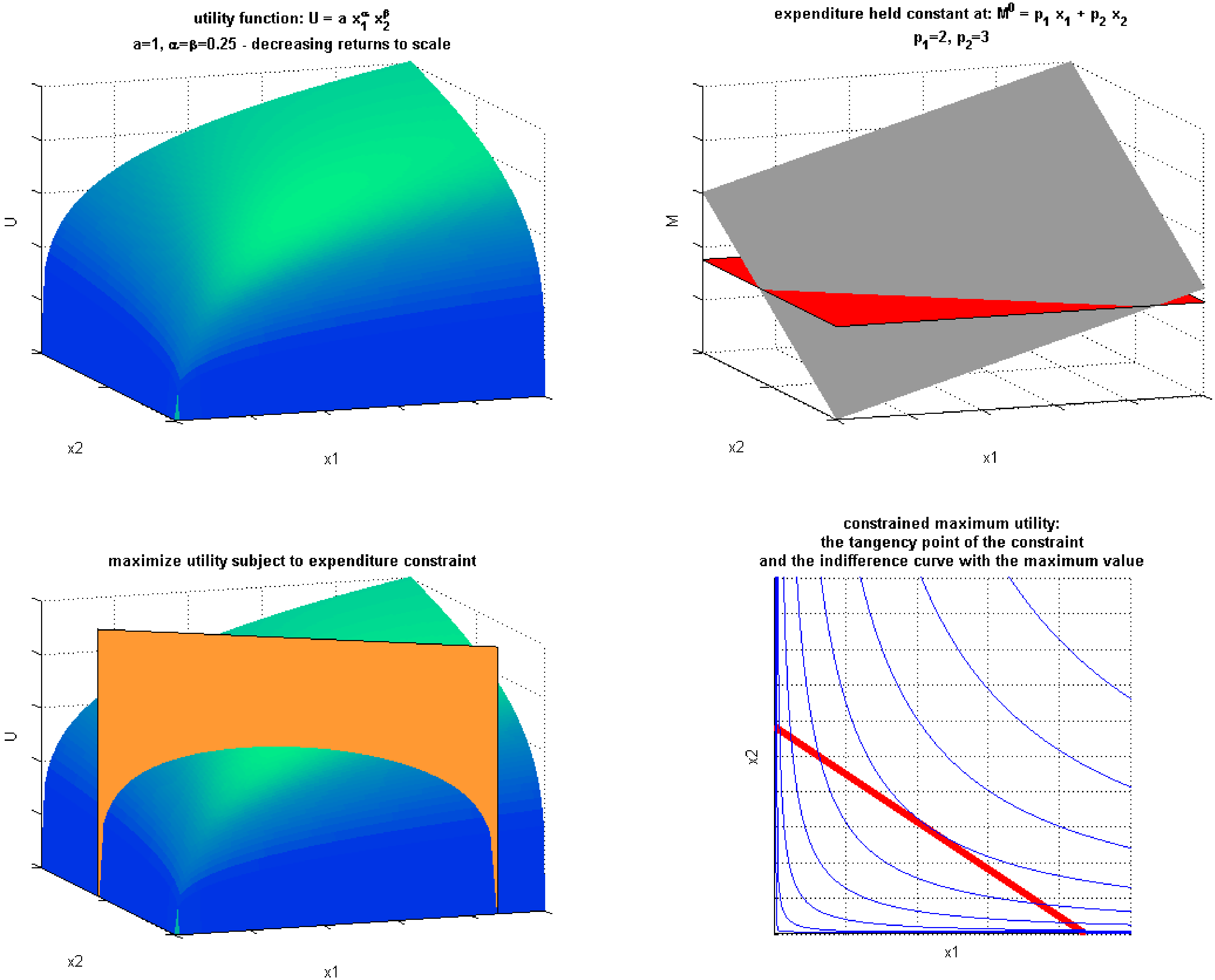
Utility maximization: utility function with decreasing returns to scale



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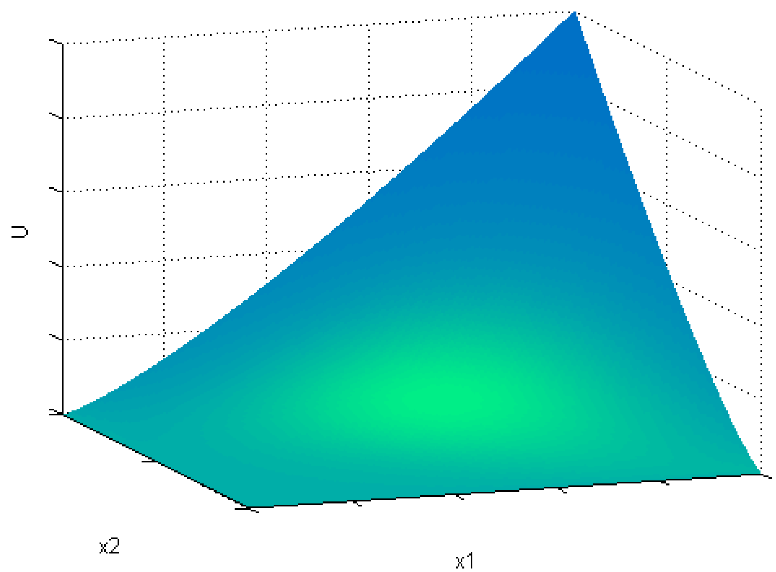
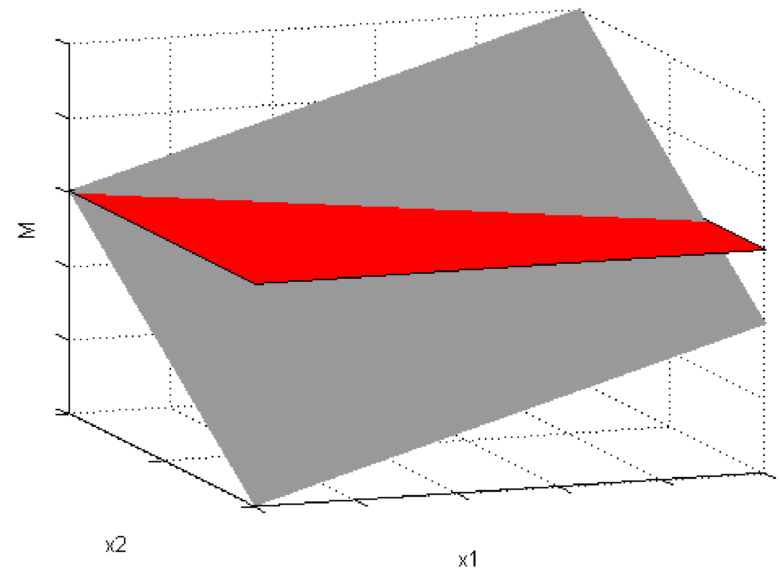
Utility maximization: utility function with increasing returns to scale

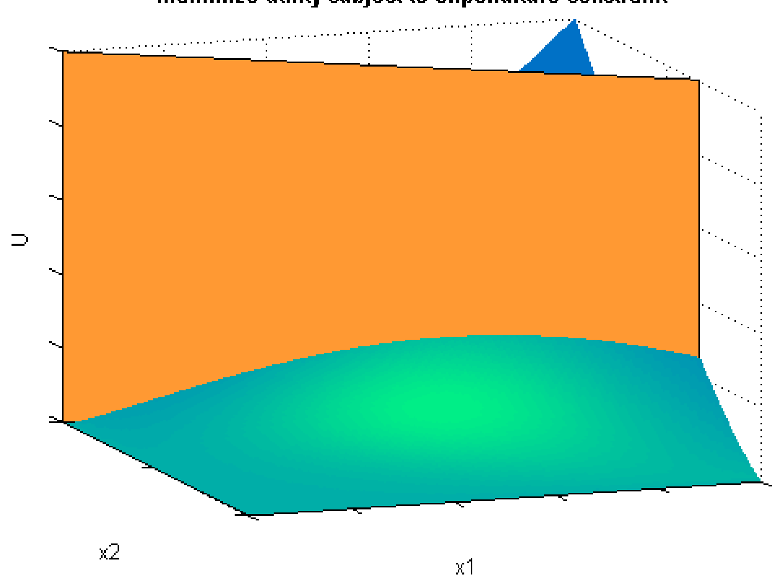
utility function: $U = a\, x_1^{\alpha}\, x_2^{\beta}$
$a=1$, $\alpha=\beta=1.25$ - increasing returns to scale



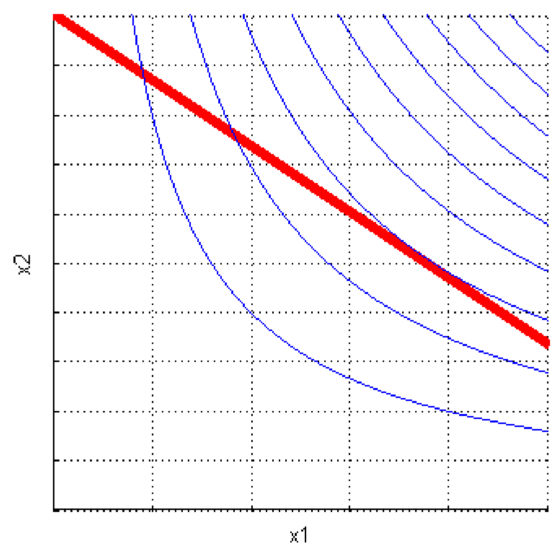
expenditure held constant: $M^0 = p_1\, x_1 + p_2\, x_2$
$p_1=2$, $p_2=3$



maximize utility subject to expenditure constraint



constrained maximum utility:
the tangency point of the constraint
and the indifference curve with the maximum value



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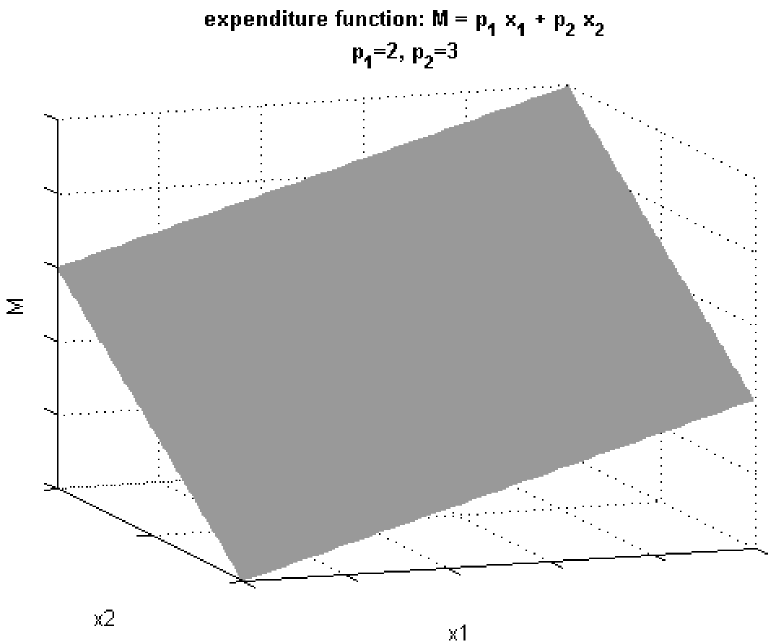
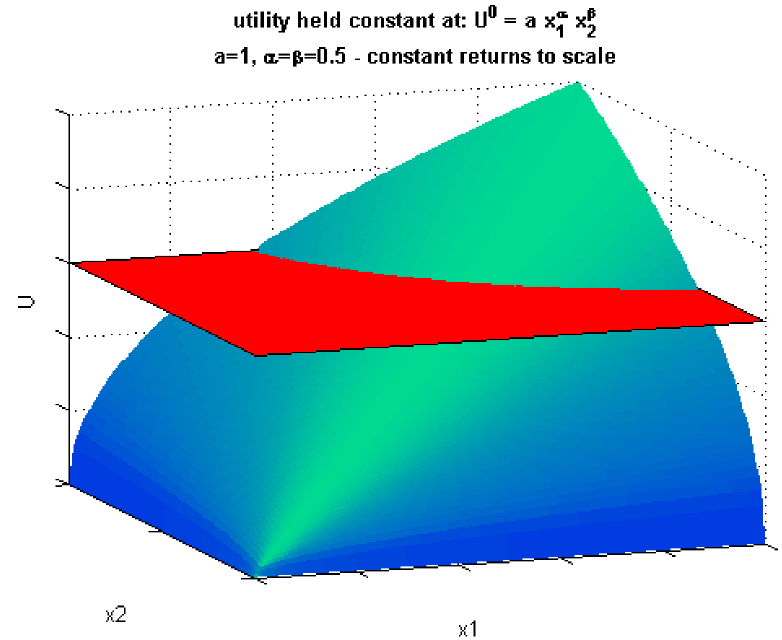
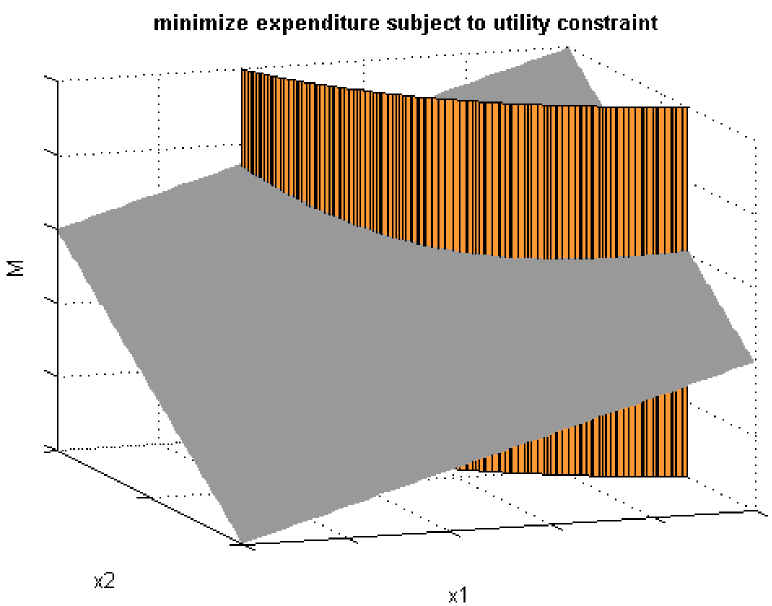
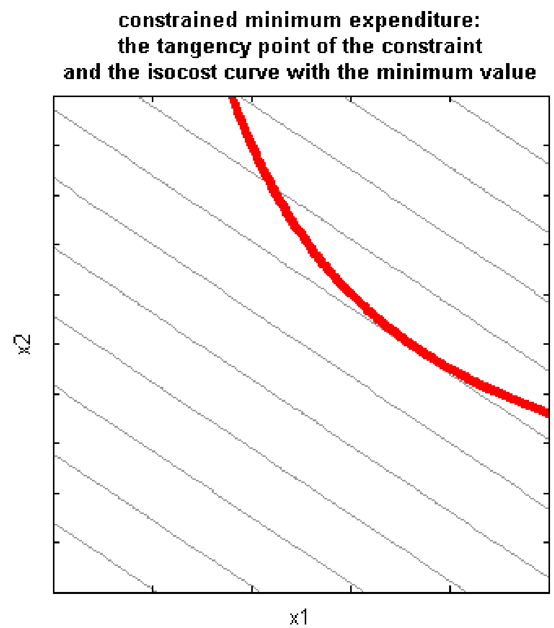
Expenditure minimization: utility function with constant returns to scale









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Literature and further reading:

The Structure of Economics, 3rd ed., Eugene Silberberg
MATLAB Documentation, MathWorks