

Exercise

Q: A Company has 2 production facilities (منشآت الإنتاج) S1 and S2 with production capacity (إنتاجية) of 100 and 110 units per week of a product, respectively. These units are to be shipped to 3 warehouses (مستودعات) D1, D2 and D3 with requirement (الطلب) of 80, 70 and 60 units per week, respectively. The transportation costs (in \$) per unit between factories to warehouses are given in the table below.

شركة لديها ٢ منشآت الإنتاج S1 و S2 مع قدرة إنتاج ١٠٠ و ١١٠ وحدة في الأسبوع للمنتج. يتم شحن الوحدات إلى ٣ مستودعات D1 و D2 و D3 مع الطلب ٨٠، ٧٠ و ٦٠ وحدة في الأسبوع. تكاليف النقل (بالدولار) لكل وحدة بين المصانع و المستودعات في الجدول التالي:

A)

Destination \ Sources	D ₁	D ₂	D ₃	Supply
S ₁	1	2	3	100
S ₂	4	1	5	110
Demand	80	70	60	

Find initial basic feasible solution (IBFS) (حل أساسي ممكن مبدئي) to the following transportation problem using NWCM (طريقة الركن الشمالي الغربي), then optimize the solution using MODI method (Modified Distribution Method) (طريقة التوزيع المعدل) or (u - v) method).

Answer:

$$\begin{aligned}
 \text{Min } Z &= x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + x_{22} + 5x_{23} \\
 x_{11} + x_{12} + x_{13} &\leq 100 \\
 x_{21} + x_{22} + x_{23} &\leq 110 \\
 x_{11} + x_{21} &\geq 80 \\
 x_{12} + x_{22} &\geq 70 \\
 x_{13} + x_{23} &\geq 60
 \end{aligned}$$

$$\begin{aligned}
 \text{Min } Z &= \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij} \\
 \text{s.t} \\
 \sum_{j=1}^m x_{ij} &\leq s_i \\
 \sum_{i=1}^n x_{ij} &\geq d_j
 \end{aligned}$$

$\sum_{i=1}^m s_i = \sum_{j=1}^n d_j = 210$, so we don't need dummy demand or dummy supply.

$\min(S_1 = 100, D_1 = 80) = 80$, This satisfies the complete demand of D₁ and leaves $100 - 80 = 20$ units with S₁.

$\min(S_1 = 20, D_1 = 70) = 20$, This exhausts the capacity of S₁ and leaves $70 - 20 = 50$ units with D₂.

$\min(S_2 = 110, D_2 = 50) = 50$, This satisfies the complete demand of D₂ and leaves $110 - 50 = 60$ units with S₂.

$\min(S_2 = 60, D_3 = 60) = 60$, This satisfies S₂ and D₃.

Destination \ Sources	D ₁	D ₂	D ₃	Supply		
S ₁	1 80	2 20	3	100	20	0
S ₂	4	1 50	5 60	110	60	0
Demand	80	70	60			
	0	50 0	0			

Initial feasible solution (IBFS) (حل اساسي ممكن مبدئي) is:

$$X_{11} = 80, X_{12} = 20, X_{22} = 50, X_{23} = 60$$

The minimum total transportation cost:

$$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 = 470\$$$

Here, the number of allocated cells = 4 is equal to $m + n - 1 = 3 + 2 - 1 = 4$

Optimality test using MODI method...

$$\delta_{kj} = v_j + u_i - C_{ij},$$

1. Find u_i and v_j for all occupied cells (i, j) , where $v_j + u_i = C_{ij}$
 - Substituting, $u_1=0$, we get
 - $c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 1 - 0 \Rightarrow v_1 = 1$
 - $c_{12} = u_1 + v_2 \Rightarrow v_2 = c_{12} - u_1 \Rightarrow v_2 = 2 - 0 \Rightarrow v_2 = 2$
 - $c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 1 - 2 \Rightarrow u_2 = -1$
 - $c_{23} = u_2 + v_3 \Rightarrow v_3 = c_{23} - u_2 \Rightarrow v_3 = 5 + 1 \Rightarrow v_3 = 6$
2. Find $\delta_{kl} = v_l + u_k - C_{kl}$ for all unoccupied cells (k, l) . If all $\delta_{kl} \leq 0$, the solution is optimal solution.
3. Now choose the maximum positive value from all δ_{kj} (opportunity cost) = $\delta_{13} = 3$ and draw a closed path $S1D3 \rightarrow S1D2 \rightarrow S2D2 \rightarrow S2D3$ with plus/minus sign allocation.

Minimum allocated value among all negative position (-) on closed path $\theta = 20$ Subtract 20 from all (-) and Add it to all (+).

		V ₁ =1	V ₂ =2	V ₃ =6		
		D ₁	D ₂	D ₃	Supply	
Sources	Destination					
	U ₁ =0	S ₁	1 80	-2 20	3 + δ ₁₃ = 3	100
U ₂ =-1	S ₂	4 δ ₂₁ = -4	+50	1 -60	5 -5	110
	Demand	80	70	60		

4. Repeat the step 1 to 4, until an optimal solution is obtained.

		V ₁ = 1	V ₂ = -1	V ₃ = 3	
		D ₁	D ₂	D ₃	Supply
Sources	Destination				
	U ₁ = 0	S ₁	1 80	2 δ ₁₂ =-3	3 20
U ₂ = 2	S ₂	4 δ ₂₁ = -1	1 70	5 40	110
	Demand	80	70	60	

We note that all $\delta_{kj} \leq 0$, so final optimal solution is arrived

Therefore, the optimal solution $X_{11} = 80, X_{13} = 20, X_{22} = 70, X_{23} = 40$

$$\text{And } Z = 80 * 1 + 20 * 3 + 70 * 1 + 40 * 5 = 410\$$$

B) same previous example (A) but change S2 to 130 rather than 110.

Answer:

Sources \ Destination	D ₁	D ₂	D ₃	Supply
Sources				
S ₁	1	2	3	100
S ₂	4	1	5	130
Demand	80	70	60	230

Here Total Demand = 210 is less than Total Supply = 230. So, we add a dummy demand constraint with 0 unit cost and with allocation 20.

Sources \ Destination	D ₁	D ₂	D ₃	D ₄ (Dummy)	Supply
Sources					
S ₁	1	2	3	0	100
S ₂	4	1	5	0	130
Demand	80	70	60	20	230=230

		V ₁ =1	V ₂ =2	V ₃ =6	V ₄ =-1	
Sources \ Destination		D ₁	D ₂	D ₃	D ₄ (Dummy)	Supply
Sources						
U ₁ =0	S ₁	1 80	-2 20	+3 δ ₁₃ =3	0 δ ₁₄ =1	100 20 0
U ₂ =-1	S ₂	4 δ ₂₁ =-4	+1 50	-5 60	0 20	130 80 20 0
Demand		80	70	60	20	
		0	50	0	0	

We note that not all $\delta_{kj} \leq 0$, so we don't reach to optimal solution yet.

Initial feasible solution (IBFS) is:

$$X_{11} = 80, X_{12} = 20, X_{22} = 50, X_{23} = 60, X_{24} = 20$$

The minimum total transportation cost:

$$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 + 20 * 0 = 470$$

Here, the number of allocated cells = 5 is equal to $m + n - 1 = 2 + 4 - 1 = 5$

		$V_1 = 1$	$V_2 = -1$	$V_3 = 3$	$V_4 = -2$	
Destination Sources		D_1	D_2	D_3	$D_4(\text{Dummy})$	Supply
$U_1 = 0$	S_1	1 80	2 $\delta_{12} = -3$	3 20	0 $\delta_{14} = -2$	100
$U_2 = 2$	S_2	4 $\delta_{21} = -1$	1 70	5 40	0 20	110
	Demand	80	70	60	20	

We note that all $\delta_{kj} \leq 0$, so final optimal solution is arrived

C) same previous example in part (B) but change D_1 , D_2 and D_3 to 90, 80 and 100 units per week, respectively.

Answer:

Destination Sources		D_1	D_2	D_3	Supply
S_1		1	2	3	100
S_2		4	1	5	130
Demand		90	80	100	270

Here Total Demand = 270 is greater than Total Supply = 230. So, we add a dummy supply constraint with 0 unit cost and with allocation 40.

		$V_1 = 1$	$V_2 = 2$	$V_3 = 6$	
Destination Sources		D_1	D_2	D_3	Supply
$U_1 = 0$	S_1	1 90	-2 10 ↑ 10 ↓ 10 $\delta_{13} = 3$	+3 60 ↑ 60 ↓ 60 $\delta_{12} = -4$	100 0
$U_2 = -1$	S_2	4 $\delta_{21} = -4$	+1 70 ↑ 70 ↓ 70 $\delta_{12} = -4$	-5 60 ↑ 60 ↓ 60 $\delta_{13} = 3$	130 0
$U_3 = -6$	$S_3(\text{Dummy})$	0 $\delta_{12} = -5$	0 $\delta_{12} = -4$	0 40 ↑ 40 ↓ 40 $\delta_{13} = 3$	40 0
	Demand	90	80	100	270
		0	70 0	40 0	270

H.W Example: The ICARE Company has three factors located throughout a state with production capacity 40, 15 and 40 gallons. Each day the firm must furnish its four retail shops D₁, D₂, D₃ with at least 25, 55, and 20 gallons respectively. The transportation costs (in \$.) are given below.

Destination \ Sources	D ₁	D ₂	D ₃	Supply
Sources				
S ₁	10	7	8	40
S ₂	15	12	9	15
S ₃	7	8	12	40
Demand	25	55	20	95 100

Q: Find the **optimum** transportation schedule and minimum total cost of transportation.

Answer:

The minimum total transportation cost = $7 \times 40 + 9 \times 15 + 7 \times 25 + 8 \times 15 + 0 \times 5 = 710$

		V ₁ = 3	V ₂ = 0	V ₃ =4		
Destination \ Sources		D ₁	D ₂	D ₃	Supply	
U ₁ = 7	S ₁	10 25	7 15	8 $\delta_{13}= 3$	40	15 0
U ₂ = 12	S ₂	15 $\delta_{21}= 0$	- 12 15	+ 9 $\delta_{23}= 7$	15	0
U ₃ =8	S ₃	7 $\delta_{31}= 4$	+ 8 25	- 12 15	40	15 0
U ₄ = -4	S ₄ (Dummy)	- $\delta_{41}= -1$	- $\delta_{42}= -4$	- 5	- 0	0
	Demand	25	55	20	...	100
		0	40 25 0	5 0		

$\theta = 15$ Subtract 15 from all (-) and Add it to all (+).

Sources \ Destination	$V_1=10$	$V_2=7$	$V_3=4$	Supply	
U ₁ =0	S_1	- 10 25	+ 7 15	8 $\delta_{13}=-4$	40
U ₂ =5	S_2	15 $\delta_{21}=0$	- 12 0	+ 9 15	15
U ₃ =1	S_3	7 $\delta_{31}=4$	8 40	12 $\delta_{31}=-7$	40
U ₄ =-4	S_4 (Dummy)	+ - $\delta_{41}=6$	- ·	- 5	·
Demand	25	55	20	100	

Here, the number of allocated cells = 6 is equal to $m + n - 1 = 3 + 4 - 1 = 6$

$\theta = 0$ Subtract 15 from all (-) and Add it to all (+).

Sources \ Destination	$V_1=0$	$V_2=-3$	$V_3=0$	Supply	
U ₁ =10	S_1	10 25	7 15	8 $\delta_{13}=2$	40
U ₂ =9	S_2	15 $\delta_{21}=-6$	12 $\delta_{22}=-6$	9 15	15
U ₃ =11	S_3	7 $\delta_{31}=4$	8 40	12 $\delta_{31}=-1$	40
U ₄ =0	S_4 (Dummy)	· 0	· $\delta_{42}=-3$	· 5	·
Demand	25	55	20	100	

$\theta = 25$ Subtract 15 from all (-) and Add it to all (+).

		$V_1=0$	$V_2=1$	$V_3=0$	
		D₁	D₂	D₃	Supply
Destination					
Sources					
$U_1=6$	S_1	10 $\delta_{11}=-4$	7 40	8 $\delta_{13}=-2$	40
$U_2=9$	S_2	15 $\delta_{21}=-6$	12 $\delta_{22}=-2$	9 15	15
$U_3=7$	S_3	7 25	8 15	12 $\delta_{31}=-5$	40
$U_4=0$	S_4 (Dummy)	· 0	· $\delta_{42}=1$	· 5	·
	Demand	25	55	20	100