part 1

Chapter 1 Fundamentals

(Real Numbers, Exponents and Radicals, Algebraic expressions, Rational Expressions, Equations, complex numbers, Inequalities, The coordinate Plane: graphs of equations, circles, lines) chap 1 (textbook 1)

(Definition of a function, linear functions and models (chap2 Textbook 1) Quadratic functions and models, polynomial functions and their graphs, polynomial division, real zeros of polynomials, complex zeros, rational functions (chap3 Tex 1)

1.1 Real Numbers:

$\mathbb{N} = \{1, 2, 3, \ldots\}$ natural numbers set

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$ integers set

$\mathbb{Q} = \{\frac{a}{b} \,;\, a \in \mathbb{Z},\, b \in \mathbb{Z}^*\}$ rational numbers set

$\mathbb{R} = \mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$

irrational numbers $\{\sqrt{2}, \sqrt{3}, \pi, \ldots\}$

- Properties of real numbers

① $a + b = b + a$
$a \cdot b = b \cdot a$.

② $(a+b)+c = a+(b+c)$
$(ab) \cdot c = a \cdot (b \cdot c)$

③ $a(b+c) = ab + ac$
$(b+c)a = ba + ca$.

expl 1: $2(x+3) =$
$(a+b)(x+y) =$

- Properties of negatives:

① $(-1)a = -a$
② $-(-a) = a$
③ $(-a)b = -(ab)$
④ $(-a)(-b) = ab$
⑤ $-(a+b) = -a-b$
⑥ $-(a-b) = -a+b$

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expl 2: Let x, y and z be real numbers. Then

$$-(x+z) =$$
$$-(x+y-z) =$$

- ## Properties of fractions

① $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

② $\frac{a}{b} \div \frac{c}{d} = \frac{a/b}{c/d} = \frac{ad}{bc}$

③ $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

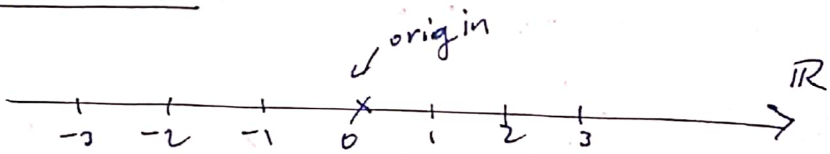
④ $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

⑤ $\frac{ac}{bc} = \frac{a}{b}$

⑥ if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$ (cross multiply)

expl 3 Evaluate $\frac{5}{36} + \frac{7}{120} = ?$

- ## The real line



$-\pi < -3$
$\sqrt{2} > 1$
$2 \leq 2$

- ## Sets and Intervals

A set is a collection of objects and these objects are called the element of the set.

If S is a set, the notation $a \in S$ means that a is an element of S and $b \notin S$ means that b is not an element of S.

Ex $A = \{1, 2, 3, 4, 5, 6\} = \{x \in \mathbb{N} \,/\, 0 < x < 7\}$

- Let S & T are sets their union $S \cup T$ is the set that consists of all elements that are in S or T.
The intersection of S and T is the set $S \cap T$ consisting of all elements that are in both S and T.
- The empty set, denoted by ϕ, is the set that contains no element.

expl 4 Let $S = \{1, 2, 3, 4, 5\}$ & $T = \{4, 5, 6, 7\}$
$V = \{6, 7, 8\}$

Then $S \cup T$, $S \cap T$; $S \cap V = ?$

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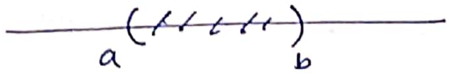
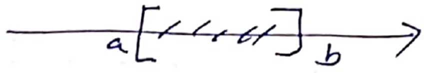
Certain sets of real numbers, called Intervals.

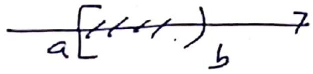
- If $a<b$ then the open interval from a to b consists of all numbers between a and b and is denoted (a,b)
- The closed interval from a to b includes the endpoints is denoted $[a,b]$

$(a,b) = \{ x \in \mathbb{R} / a < x < b \}$; $[a,b] = \{ x \in \mathbb{R} / a \leq x \leq b \}$

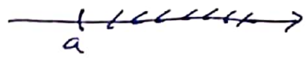




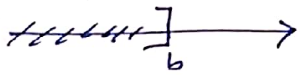
⚠ $[a,b) = \{ x \in \mathbb{R} / a \leq x < b \}$



$(a,\infty) = \{ x \in \mathbb{R} / x > a \}$



$(-\infty, b] = \{ x \in \mathbb{R} / x \leq b \}$



$\mathbb{R} = (-\infty, +\infty)$

<u>exple</u> Express each interval in terms of inequalities and then graph the interval:

(a) $[-1,2) =$ (c) $(-3,\infty) =$

(b) $[3/2, 4] =$

<u>exple</u> Graph each set:

(a) $(1,3) \cap [2,7] =$

$(1,3) \cup [2,7] =$

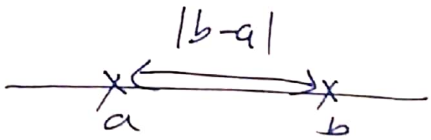
* Absolute value and distance

<u>Def</u> If a is a real number, then the absolute value of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

<u>exple</u> $|3| = 3$, $|-3| = -(-3) = 3$; $|0| = 0$; $|3-\pi| = -(3-\pi) = \pi - 3$

- Properties of absolute value:



① $|a| \geq 0$

② $|a| = |-a|$

③ $|ab| = |a|\,|b|$

④ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

⑤ $|a+b| \leq |a| + |b|$ (Triangle Inequality)

⚠ If a & b are real numbers, then the distance between the points a & b is $d(a,b) = |a-b|$

exp $a(-8)$ and $b(2)$ then $d(a,b) = 10$

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§ 1.2 : Exponents & Radicals

- **Exponential notation**

If a is any real number and n is a positive integer, then the nth power of a is $a^n = \underbrace{a \cdot \ldots a}_{n \text{ times}}$

The number a is called the base and n is called the exponent.

expl : ① $\left(\frac{1}{2}\right)^5$

② $(-3)^4$

③ -3^4

Zero and negative exponents

If $a \neq 0$ be a real number and n is a positive integer then

$$a^0 = 1 \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

exple $\left(\frac{4}{7}\right)^0 =$ __ $\quad x^{-1} =$ -- $\quad (-2)^{-3} =$ ..

Laws of exponents

$a^m . a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{a^m}{a^n} = a^{m-n}$ $\quad (ab)^n = a^n b^n$ $\quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

expl ① $x^4 . x^7 =$ $\quad y^4 . y^{-7} =$ $\quad \frac{c^9}{c^5} =$ $\quad (b^4)^5 =$

$(3x)^3 =$ $\quad \left(\frac{x}{2}\right)^5$

② Simplify : (a) $(2a^3 b^2)(3ab^4)^3$

(b) $\left(\frac{x}{y}\right)^3 \left(\frac{y^2 x}{z}\right)^4$

③ Eliminate negative exponents and simplify each expression

(a) $\frac{6st^{-4}}{2s^{-2}t^2}$ $\quad$ (b) $\left(\frac{y}{3z^3}\right)^{-2}$

- **Radicals** :

The symbol $\sqrt{\ }$ means " the positive square root of "

$\sqrt{a} = b \Rightarrow b^2 = a$ and $b \geq 0$

$\sqrt{9} = 3$

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If n is any positive integer, then the principal nth root of a is defined as follows:

$$\sqrt[n]{a} = b \quad \text{means} \quad a = b^n$$

If n is even, we must have $a \geq 0$ and $b \geq 0$.

For example $\sqrt[4]{81} = 3$, $\sqrt[3]{-8} = -2$

⚠ $\sqrt{a^2} = |a|$.

· Properties of n^{th} roots

① $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

② $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

③ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

④ $\sqrt[n]{a^n} = a$ if n is odd

⑤ $\sqrt[n]{a^n} = |a|$ if n is even.

exple ① $\sqrt[3]{x^4} = x\sqrt[3]{x}$

$\sqrt[4]{81x^8y^4} = 3x^2|y|$.

⚠ $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

② simplify : $\sqrt{32} + \sqrt{200} = 14\sqrt{2}$

if $b > 0$; $\sqrt{25b} - \sqrt{b^3} = (5-b)\sqrt{b}$

$\sqrt{49x^2 + 49} = 7\sqrt{x^2+1}$

Def (Rational exponents)

For any rational exponent $\frac{m}{n}$ in lowest terms, where m, n are integers and $n > 0$, we define

$$a^{m/n} = (\sqrt[n]{a})^m \iff a^{m/n} = \sqrt[n]{a^m}$$

if n is even then $a \geq 0$

examples ① $4^{1/2} = 2$ $\quad 125^{-1/3} = \frac{1}{5}$

$8^{2/3} = 4$

② $a^{1/3} \cdot a^{7/3} = a^{8/3}$

③ $\frac{a^{2/5} \cdot a^{7/5}}{a^{3/5}}$

④ $\left(2\frac{x^{3/4}}{y^{1/3}}\right)^3 \left(\frac{y^4}{x^{-1/2}}\right) = \ldots$

⑤ $\frac{1}{\sqrt[3]{x^4}} = x^{-4/3}$

⑥ $2\sqrt{x}\ 3\sqrt[3]{x} =$

⑦ $\sqrt{x\sqrt{x}} = \ldots$

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⑤ Put each fractional expression into standard form by rationalizing the denominator.

(a) $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ (b) $\frac{1}{\sqrt[3]{5}} = \frac{\sqrt[3]{25}}{5}$

(c) $\sqrt[7]{\frac{1}{a^2}} = \frac{\sqrt[7]{a^5}}{a}$

1.3 Algebraic Expressions (polynomials)

Def : A polynomial in the variable x is an expression of the form : $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$

where $a_0, \dots, a_n$ are real numbers and n is nonnegative integer.

If $a_n \neq 0$, then the polynomial has degree n. The monomials $a_k x^k$ that make up the polynomial are called the terms of the polynomial.

exple ① Find the sum $(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x)$

② Find the difference $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$

③ Mulipy $(2x+1)(3x-5) = \dots$

④ Find the product : $(2x+3)(x^2 - 5x + 4) = \dots$

Special Product Formulas

Let a & b are any real numbers or algebraic expressions

$(a+b)(a-b) = a^2 - b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

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