Journal of Petroleum Science and Engineering 122 (2014) 280–289

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

New Inflow Performance Relationship for solution-gas drive oil reservoirs

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ARTICLE INFO

Article history:
Received 22 October 2011
Accepted 21 July 2014
Available online 31 July 2014

Keywords:
Inflow Performance Relationship
oil mobility
solution-gas drive oil reservoir
empirically IPR correlations
analytically IPR correlations

ABSTRACT

The Inflow Performance Relationship (IPR) describes the behavior of the well's flowing pressure and production rate, which is an important tool in understanding the reservoir/well behavior and quantifying the production rate. The IPR is often required for designing well completion, optimizing well production, nodal analysis calculations, and designing artificial lift. Different IPR correlations exist today in the petroleum industry with the most commonly used models are that of Vogel's and Fetkovitch's, in addition to a few analytical correlations, that usually suffer from limited applicability.

In this work, a new IPR model was developed based on actual field data for almost 50 field cases in addition to several simulated tests. This method is simple, covers wide range of reservoir parameters, easy to apply, and requires only one test point. Therefore, it provides a considerable advantage compared to the multipoint test method of Fetkovich. It also provides significant advantages over the widely used method of Vogel. (Comparison between the new method and Vogel shows that the new method is almost three times more accurate in our field tests database.)

After the development of the new model, its accuracy was compared with the most common IPR models that can be computed with the readily available test and field data (Vogel, Fetkovitch, Wiggins, and Sukarno). Twelve field cases were used for comparison. The best two methods were our new model and that of Fetkovich's with average absolute error of 6.6% and 7%, respectively. However, the new method has the advantage of requiring only one test point as opposed to Fetknovich's which requires a multi-rate test. A method to compute future IPR performance was also developed and tested against field data with superior results to other available models. Finally, the application of the new model is illustrated with field examples for current and future IPR computations.

1. Introduction

For slightly compressible fluids, the productivity index is given by

$$J = \frac{0.00708kh}{\ln(r_e/r_w) - 0.75 + S}\left[\frac{k_{ro}}{\mu_o B_o}\right]. \tag{1}$$

Therefore, the variables that affect the productivity index and in turn the inflow performance are the pressure dependent parameters (μ_o, B_o, and k_{ro}). Fig. 1 schematically illustrates the behavior of those variables as a function of p_r. Above p_b, k_{ro} equals unity and the term $(k_{ro}/\mu_o B_o)$ is almost constant. As the pressure declines below p_b, the gas is released from solution which can cause a large decrease in both k_{ro} and $(k_{ro}/\mu_o B_o)$.

If PI is the constant, the oil flow rate can be calculated as

$$q_o = J(p_r - p_{wf}) \tag{2}$$

Eq. (2) suggests that the inflow into a well is directly proportional to Δp. Evinger and Muskat (1942) observed that as the pressure drops below p_b the inflow performance curves deviate from the simple straight-line relationship as shown in Fig. 2. Therefore, the above relationship is not valid for two-phase flow or in case of solution gas drive reservoirs.

Many IPR correlations addressed the curvature in Fig. 2 of the inflow performance curves in case of solution gas drive oil reservoirs in which p_b is the initial reservoir pressure. Based on the literature survey, the most known IPR correlations can be

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http://dx.doi.org/10.1016/j.petrol.2014.07.021
0920-4105/

Nomenclature

A	drainage area of well, sq ft
a_0,a_1,a_2,a_3	constants for Sukarno and Wisnogroho, dimensionless
API	stock tank oil liquid gravity in, ^{O} API
b_0,b_1,b_2,b_3	constants for Sukarno and Wisnogroho, dimensionless
B_o	oil formation volume factor, bbl/STB
B_g	gas formation volume factor, bbl/SCF
C_A	shape constant or factor, dimensionless
C_1, C_2, C_3, C_4, D	Wiggins's constants, dimensionless
d, e	Del Castillo and Yanil's constants, dimensionless
h	formation thickness, ft
J	productivity index of the reservoir (PI), STB/psi
n	deliverability exponent for Fetkovich, dimensionless
N_1	oil IPR parameter for Klins's equation, dimensionless
p_b	bubble Point Pressure, psia
p_D	dimensionless pressure
p_e	pressure at the outer boundary, psia
PI_α	productivity index from the new IPR model, STB/psi
p_r	average reservoir pressure, psia
p_{wf}	bottom hole flowing pressure, psia
q_o	oil flow rate, STB/D
$q_{o,\ max}$	maximum oil flow rate, STB/D
r_e	drainage radius, ft
R_s	solution gas–oil ratio, scf/STB
r_w	well radius, ft
S	radial flow skin factor, dimensionless
S_o	oil saturation, fraction
T	reservoir temperature, ^{o}F
x	reciprocal model constant, dimensionless
y	reciprocal model constant, dimensionless
$\left(\frac{K_{ro}}{\mu_o\beta_o}\right)_{p_D=0}$	mobility ratio at zero dimensionless pressure
$\left(\frac{K_{ro}}{\mu_o\beta_o}\right)^{\backslash}_{p_D=0}$	mobility ratio first derivative at zero dimensionless pressure
$\left(\frac{K_{ro}}{\mu_o\beta_o}\right)^{\backslash\backslash}_{p_D=0}$	mobility ratio second derivative at zero dimensionless pressure
$\left(\frac{K_{ro}}{\mu_o\beta_o}\right)^{\backslash\backslash\backslash}_{p_D=0}$	mobility ratio third derivative at zero dimensionless pressure
α	oil IPR parameter for the new IPR model, dimensionless
γ	Euler's constant (0.577216)
γ_g	gas gravity, fraction
γ_o	oil gravity, fraction
μ_o	oil viscosity, cp
Δp	pressure drawdown, psi

subdivided into empirically and analytically derived correlations. Some of the most known empirical correlations are Vogel (1968), Fetkovich (1973), Klins and Majher (1992), Wiggins (1993), and Sukarno et al. (1995). Some of the most known analytical correlations are Wiggins et al. (1991, 1992), Archer et al. (2003) and Del Castillo (2003).

1.1. The empirical derived correlations

1.1.1. Vogel's method

Vogel used a computer program based on Weller's (1966) assumptions and 21 reservoir data sets to develop an IPR as

$$\frac{q_o}{q_{o,\ \max}}=1-0.2\left[\frac{p_{wf}}{p_r}\right]-0.8\left[\frac{p_{wf}}{p_r}\right]^2. \quad (3)$$

Vogel's correlation gave a good match with the actual well inflow performance at early stages but deviates at later stages of the reservoir life. Therefore, this will affect the prediction of inflow performance curves in case of solution gas drive reservoirs, because at later stages of production the amount of the free gas that comes out of the oil will be greater than the amount at the early stages of production.

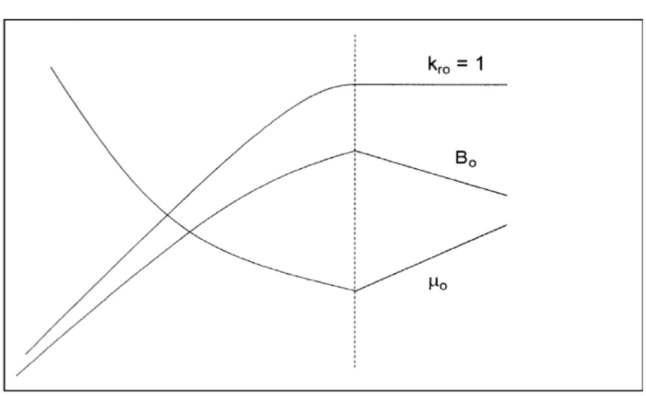

Fig. 1. Effect of p_r on B_o, μ_o, and k_{ro} (after Ahmed,T.).

1.1.2. Fetkovich's method

Fetkovich developed an IPR based on multi-rate tests "40 different oil wells from six fields" and the general treatment of the inflow performance provided by Raghavan (1993) under pseudo-steady state conditions. Eq. (4) gives the oil flow rate as introduced by Raghavan (1993):

$$q_o=\bar{J}\int_{p_{wf}}^{p_r}\frac{k_{ro}(S_o)}{\mu_o B_o}dp \quad (4)$$

where $\bar{J}$ is defined by

$$\bar{J}=\frac{1}{141.2}\frac{kh}{\ln(r_e/r_w)-0.75+S} \quad (5)$$

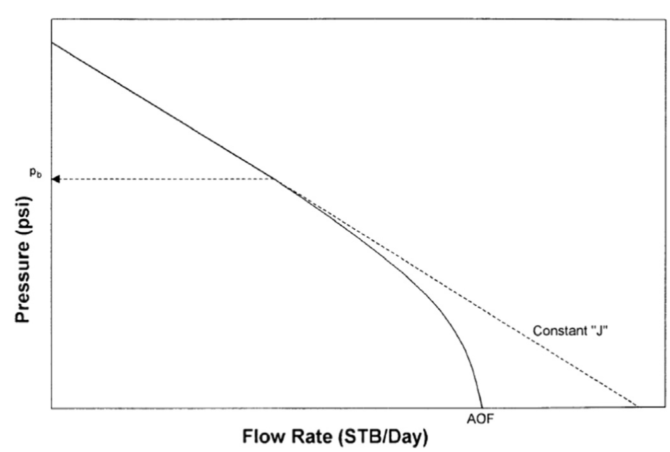

Fig. 2. The inflow performance curve below the bubble-point pressure (after Ahmed, T.).

Eq. (4) is not useful in a practical sense; therefore, Fetkovich proposed the following relationship between the oil mobility function and p_r:

$$\frac{k_{ro}(S_o)}{\mu_o B_o} = x p_r \tag{6}$$

where x is a constant. Finally, the "Fetkovich form" of the IPR equation is given as the "backpressure" modification form, which is written as

$$\frac{q_o}{q_{o,\ max}} = \left[1 - \frac{p_{wf}^2}{p_r^2}\right]^n. \tag{7a}$$

Eq. (7a) requires a multi-rate test to determine the value of n. As indicated, the main parameter that affects Fetkovich's model is the oil mobility as a function of p_r, which is assumed to be in a linear relationship as illustrated in Fig. 3.

1.1.3. Klins and Majcher's method

Based on Vogel's work, Klins and Majher (1992) developed the following IPR that takes into account the change in bubble-point pressure and reservoir pressure:

$$\frac{q_o}{q_{o,\ max}|_{S=0}} = \left[1.0 - 0.295\left(\frac{p_{wf}}{p_r}\right) - 0.705\left(\frac{p_{wf}}{p_r}\right)^{N_1}\right] \tag{7b}$$

where

$$N_1 = \left(0.28 + 0.72\frac{p_r}{p_b}\right)(1.235 + 0.001 p_b) \tag{8}$$

1.1.4. Wiggins's method

Wiggins (1993) developed the following generalized empirical three phase IPR similar to Vogel's correlation based on his developed analytical model in 1991.

$$\frac{q_o}{q_{o,\ max}} = 1 - 0.519167\left(\frac{p_{wf}}{p_r}\right) - 0.481092\left(\frac{p_{wf}}{p_r}\right)^2 \tag{9}$$

1.1.5. Sukarno and Wisnogroho's method

Sukarno and Wisnogroho (1995) developed an IPR (Eq. (10)) based on simulation results that attempts to account for the flow-efficiency variation caused by rate-dependent skin.

$$\frac{q_o}{q_{o,\ max}|_{S=0}} = FE\left[1 - 0.1489\frac{p_{wf}}{p_r} - 0.4416\left(\frac{p_{wf}}{p_r}\right)^2 - 0.4093\left(\frac{p_{wf}}{p_r}\right)^3\right] \tag{10}$$

where

$$FE = a_o + a_1\left(\frac{p_{wf}}{p_r}\right) + a_2\left(\frac{p_{wf}}{p_r}\right)^2 + a_3\left(\frac{p_{wf}}{p_r}\right)^3 \tag{11}$$

$$a_i = b_{oi} + b_{1i}S + b_{2i}S^2 + b_{3i}S^3. \tag{12}$$

In Eq. (12), a_i, b_{oi}, b_{1i}, b_{2i}, and b_{3i} are the fitting coefficients.

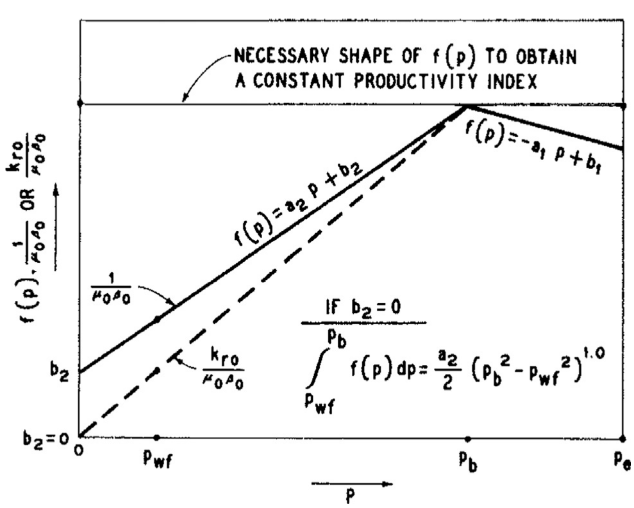

Fig. 3. Mobility–pressure behavior for a solution gas drive reservoir (after Fetkovich).

1.2. The analytical derived correlations

1.2.1. Wiggins's method

Wiggins (1991) and Wiggins et al. (1992) studied the three-phase (oil, water, and gas) inflow performance for oil wells in a homogeneous, bounded reservoir. They started from the basic principle of mass balance with the pseudo-steady state solution to develop the following analytically IPR:

$$\frac{q_o}{q_{o,\ max}} = 1 + \frac{C_1}{D}\left(\frac{p_{wf}}{p_r}\right) + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4 \tag{13}$$

where C_1, C_2, C_3,…,C_n, and D coefficients are determined based on the oil mobility function and its derivatives taken at p_r.

Wiggins et al. found that the main reservoir parameter that plays a major role in the inflow performance curve is the oil mobility function. The major problem in applying this IPR is its requirement for the mobility derivatives as a function of p_r, which is very difficult in practice. Therefore, in 1993 Wiggins developed an empirical IPR (i.e., Eq. (9)) from this analytical IPR model by assuming a third degree polynomial relationship between the oil mobility function and p_r. Wiggins et al. also presented plots of the oil mobility as a function of p_r taken at various flow rates. An example of the oil mobility–pressure profile that is presented by Wiggins et al. is shown in Fig. 4.

1.2.2. Del Castillo's method

Del Castillo (2003) developed a theoretical attempt to relate the IPR with the fundamental flow theories. In this model, a second-degree polynomial IPR is obtained with a variable coefficient (ν), or the oil IPR parameter that in fact is a strong function of pressure and saturation. The starting point for this development is the pseudo-pressure formulation for the oil phase, which is given as

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n} \int_{p_{base}}^{p} \frac{k_{ro}}{\mu_o B_o} dp \tag{14}$$

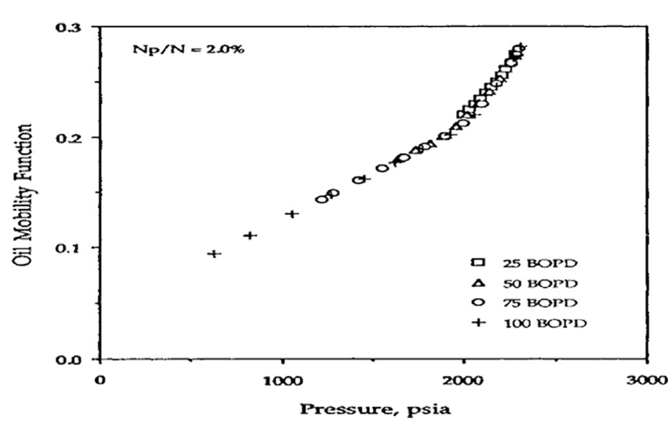

Fig. 4. The oil mobility profiles as a function of pressure – various flow rates (Case 2, after Wiggins, et al.).

In that work, the authors presumed that the oil mobility function has a linear relationship with the average reservoir pressure as given below:

$$\left.\frac{k_{ro}}{\mu_o B_o}\right|_{p_r} = f(p_r) = e + 2dp_r. \tag{15}$$

where e and d are constants established from the presumed behavior of the oil mobility profile. Fig. 3 refers to the physical interpretation of Eq. (15). Substituting Eq. (15) into Eq. (14) and manipulating, the following equation could be presented:

$$\frac{q_o}{q_{o,\,\max}} = 1 - v\left[\frac{p_{wf}}{p_r}\right] - (1-v)\left[\frac{p_{wf}}{p_r}\right]^2. \tag{16}$$

Specifically, the ν-parameter is given as

$$v = \frac{1}{(1+(d/e)p_r)} \tag{17}$$

Wiggins (1991) and Del Castillo (2003) relationships can only be applied indirectly or inferred, by estimating the oil mobility as a function of p_r to construct the IPR curve.

1.3. Summary of literature survey

As indicated, the empirical IPRs suffer from the limitation of their application range as they depend on the data used in their generation, and lack of accuracy. In addition, they do not explicitly function on reservoir rock and fluid data, which are different from one reservoir to another. On the other hand, the analytical IPRs suffer from their difficulty to be applied due to its requirement to the oil mobility profiles with its derivatives, and the assumptions used in their development. As discussed, the main parameter that affects the PI and in turn the IPR curves is the oil mobility as a function of p_r. Therefore, the relationship between the oil mobility and p_r should be accurately determined. In addition, the most common equation that represents a basic start point for the development of any IPR in case of solution gas drive reservoirs is Eq. (14), which mainly is a function of the oil mobility ($k_{ro}/\mu_o B_o$).

Most of the empirical IPRs did not take into their consideration the whole effect of the oil mobility function; this in turn largely reduces the accuracy and utility of these IPRs. Even though the models that considered this effect, such as the models of Fetkovich (1973) and Wiggins (1993), assumed the relationships between this function and p_r, as a linear form and a third polynomial form for Fetkovich and Wiggins, respectively. In fact, these linear and polynomial forms do not accurately describe the behavior of the oil mobility as a function of p_r with an accurate manner. On the other hand, some of analytical IPRs did not consider the effect of oil mobility, except the models of Wiggins et al. (1991, 1992) and Del Castillo (2003). Wiggins's model is so complicated because it requires the oil mobility to be represented in its derivates as a function of p_r; this is greatly difficult in application. Del Castillo's model is not accurate; this is because Del Castillo assumed a liner relationship between the oil mobility function and p_r, which in turn reduces the accuracy of this model.

Another parameter that should be considered in the selecting of the IPR method is the aspect of conducting the flow tests. It is evident that test costs have to be taken into consideration. Finally, the range of applicability will also affect the selecting of the IPRs to predict the well performance.

Accordingly based on the literature survey in this work, it is necessary to

(1) develop a new, more general, simple, and consistent method to correlate inflow performance trends for solution gas drive oil reservoirs. This new method takes into consideration the behavior of the oil mobility function with the average reservoir pressure without the direct knowledge of this behavior;
(2) determine the applicability and accuracy of the proposed new model by applying it on different field cases with a comparison of some of the most known and used IPR equations, considering a wide range of fluid, rock, and reservoir characteristics;
(3) test some of the available IPR methods on field data;
(4) address the prediction of future performance from current test information.

2. The new developed IPR model

In this work, a single well 3D radial reservoir model using MORE (2006) reservoir simulator was built. The reservoir simulation was used to investigate the shape and in turn the relationship between the oil mobility function and p_r. Then, a new IPR was derived based on the resulted oil mobility–pressure profile; this new IPR is mainly a function of the relationship between the oil mobility and p_r. Then, 47 field cases (published cases) were used to develop an empirical relationship between the oil mobility and p_r. From this relationship, a new IPR model that is explicitly functioning of the oil mobility is obtained. The new IPR model is highly affected by the oil mobility.

2.1. Mobility–reservoir pressure relationship

Production rate and pressure results from six simulation cases were used to develop the inflow performance curves. Table 1 presents the ranges of reservoir, rock, and fluid parameters used in the six simulation cases. The saturation and pressure information was also used to develop the mobility function profiles. The general simulation assumptions that were used in building the reservoir model can be summarized as follows:

(1) 3D radial flow into the well bore;
(2) the reservoir initially at the bubble point pressure;
(3) vertical well at the center of the formation;
(4) the well is completed through the whole formation thickness;
(5) homogeneous, bounded reservoir;
(6) isothermal conditions exist;
(7) no initial O.W.C. exist;
(8) capillary pressure is neglected; and
(9) interfacial tension effects and non-Darcy flow effects are not considered.

Table 1
Ranges of reservoir parameters – simulation cases.

Rock/fluid property	Range	Units
Average reservoir pressure, p_r	2000–5000	psia
Bubble point pressure, p_b	2000–5000	psia
Reservoir temperature, T	100–300	°F
Oil specific gravity relative to water, γ_o	0.7–0.85	Dimensionless
Gas specific gravity relative to air, γ_g	0.5–1.2	Dimensionless
Water specific gravity, γ_w	1.0–1.25	Dimensionless
Water viscosity, μ_w	0.1–1.0	cp
Initial solution gas oil ratio, R_{soi}	0.47–2.16	Mcf/STB
Initial oil formation volume factor, B_{oi}	1.12–2.52	bbl/STB
Initial oil viscosity, μ_{oi}	0.09–0.44	cp
Z-factor	0.7–1.042	Dimensionless
k_{rw} @ (S_{or})	0.1–0.4	Fraction
k_{ro} @ (S_{wc})	0.2–1.0	Fraction
k_{rg} @ ($1-S_{wc}-S_{or}$)	0.3–1.0	Fraction
Reservoir radius, r_e	100–10,000	ft
Formation thickness, h	10–1000	ft
Absolute permeability, k	0.5–500	md

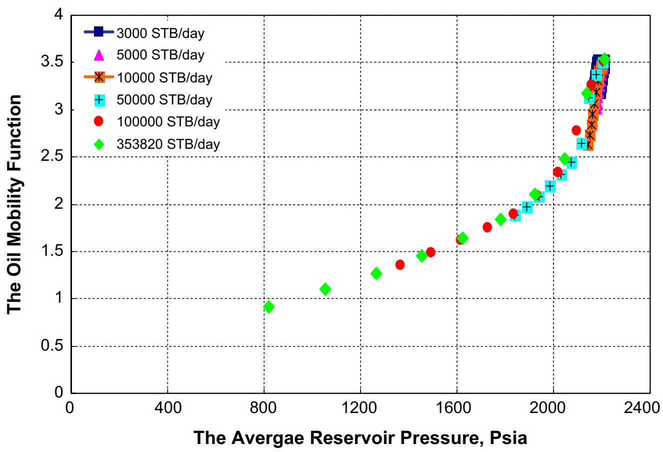

Fig. 5. Mobility–pressure behavior for solution gas drive oil reservoir – Case S1.

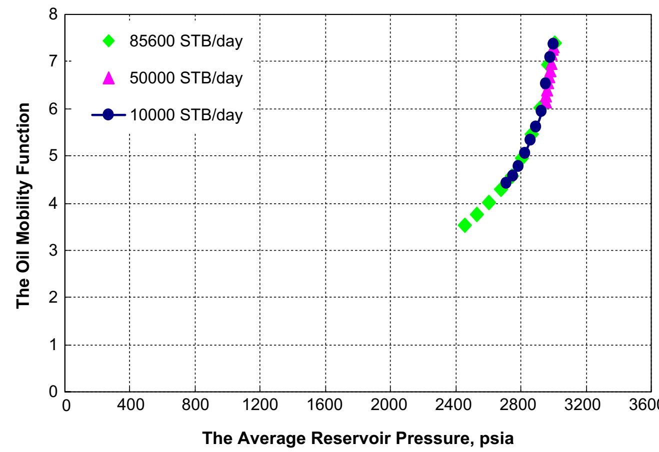

Fig. 5.3. Mobility–pressure behavior for solution gas drive oil reservoir – Case S4.

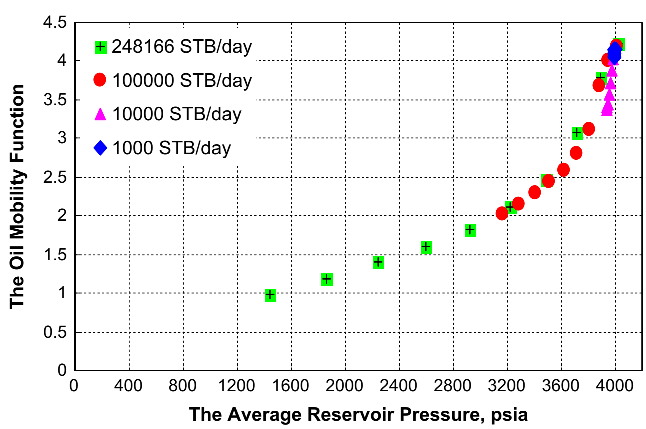

Fig. 5.1. Mobility–pressure behavior for solution gas drive oil reservoir – Case S2.

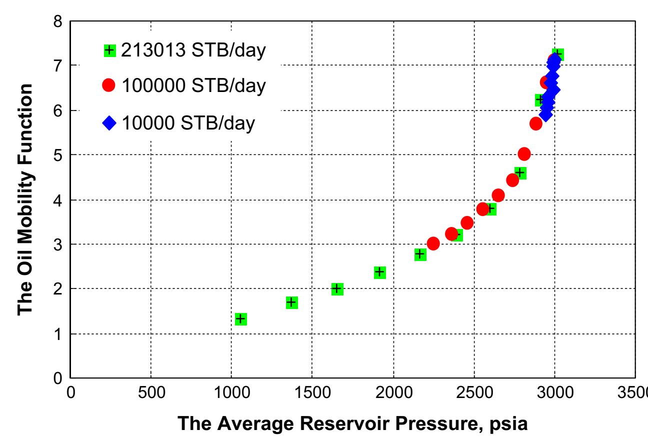

Fig. 5.4. Mobility–pressure behavior for solution gas drive oil reservoir – Case S5.

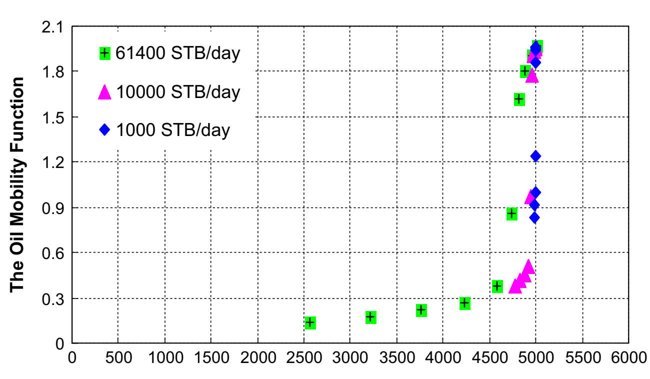

Fig. 5.2. Mobility–pressure behavior for solution gas drive oil reservoir – Case S3.

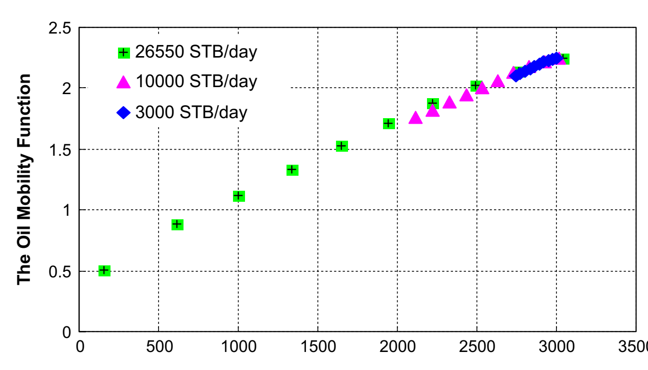

Fig. 5.5. Mobility–pressure behavior for solution gas drive oil reservoir – Case S6.

2.2. *Results of the simulator*

Fig. 5 shows the behavior of the mobility as a function of the pressure at different values of the flow rate during the two-phase flow for simulation Case No. 1 (Case-S1). The other five cases are shown in Figs. 5.1–5.5. The curves are typical of trend in addition to Fig. 5. Therefore, based on the six simulation cases, a reciprocal relationship between the oil mobility function and the average reservoir pressure was assumed and gives an acceptable and good match with the calculated simulator data as shown in Fig. 6.

2.3. *Derivation of the new IPR equation*

The starting point for the derivation is the definition of the oil-phase pseudopressure for a single well in a solution gas drive reservoir and the pseudo-steady state flow equation for the oil-phase.

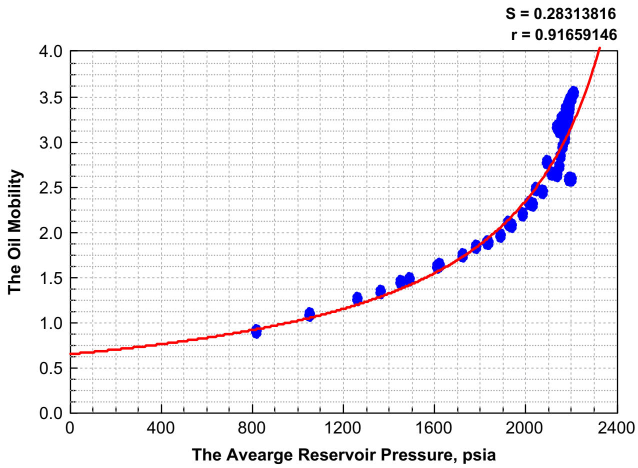

Fig. 6. Reciprocal model – Case S1.

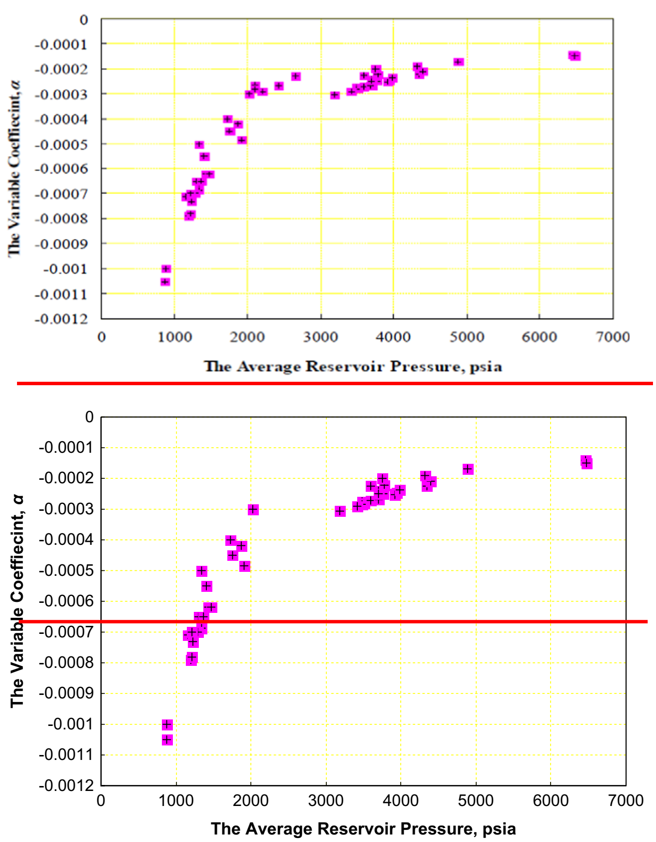

Fig. 7. The variable coefficient α and the average reservoir pressure chart.

In this work, a new form for the oil mobility function at different values of the average reservoir pressure (i.e., the reciprocal relationship) is obtained from the simulation study that is performed on MORE simulators using the six simulation cases. This reciprocal relationship was used as

$$\left[\frac{k_{ro}}{\mu_o B_o}\right]_{p_r} = \frac{1}{xp_r + y} \tag{18}$$

where x and y are the constants established from the presumed behavior of the mobility profile.

Substituting Eq .(18) in Eq. (14) and manipulating (the details are provided in Appendix A), the following new IPR equation will be introduced:

$$\frac{q_o}{q_{o,\ max}} = 1 - \frac{\ln(\alpha \cdot p_{wf} + 1)}{\ln(\alpha \cdot p_r + 1)} \tag{19}$$

where α is the oil IPR parameter for the new IPR model and at the same time it represents the two constants a and b that are shown in Eq. (18) $(\alpha = x/y)$.

Eq. (19) is the proposed new IPR model. As recognized, the α-parameter is not "constant"; therefore 47 field cases (published cases) were used to develop a generalized chart between the α-parameter and p_r (i.e. Fig. 7). Table 3 introduces the ranges of data used in the development of these two equations.

Fig. 8 shows the fitting results with the used 47 field cases. As we can see, the fitting line can be divided into two fitting trends to obtain the best accuracy, and we found by trial and error that p_r threshold is 1600 psi.

Based on Figs. 7 and 8, two empirical relationships of the variable coefficient α as function of p_r were developed as

When p_r-range is less than or equal to 1600 psia the following relationship was developed:

$$\alpha = \frac{1}{ap_r + b} \tag{20}$$

where $a = -0.981$ and $b = -152.585$.

When p_r-range is greater than or equal to 1600 psia the following relationship was developed:

$$\alpha = c + dp_r + ep_r^2 + fp_r^3 + gp_r^4 + hp_r^5 \tag{21}$$

Table 2
Ranges of data used in the development α–p_r relationship.

Properties	Data range
1. Fluid properties data:	
Gas specific gravity	0.60–0.8
API gravity of oil	20–60
Water specific gravity	1.04–1.074
Initial oil formation volume factor (bbl/STB)	1.3–1.94
Initial oil viscosity (cp)	0.27–0.99
Initial solution gas oil ratio (Mscf/STB)	0.132–4.607
Bubble point pressure (psia)	Up to 7000
2. Rock properties data	
Porosity	0.1–0.35
Absolute permeability (md)	2.5–2469
Irreducible water saturation	0.1–0.32
Residual oil saturation (W/O)	0.08–0.17
Residual oil saturation (G/O)	0.07–0.14
Critical gas saturation	0.02- 0.17
Total compressibility (psi^{-1})	0.33×10^{-3}–30×10^{-6}
Oil relative permeability @ 0.02 and 0.1S_{gc}	0.444–0.52
3. Reservoir and well dimension:	
Average reservoir pressure (psia)	Up to 7000
Drainage area (acres)	20–80
Formation thickness (ft)	10–182
Reservoir radius (ft)	250–1053
Well bore radius (ft)	0.33–0.35
Reservoir temperature (°F)	156–238

Table 3
Constants of Eq. (21).

Constant	Value
$c=$	-0.0043065
$d=$	$4.98E-06$
$e=$	$-2.41E-09$
$f=$	$5.69E-13$
$g=$	$-6.48E-17$
$h=$	$2.85E-21$

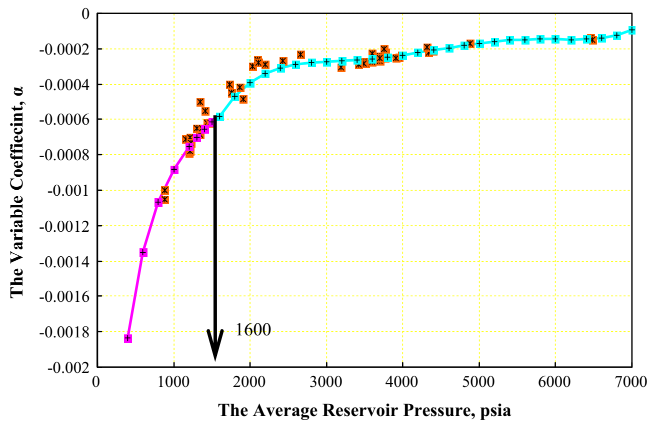

Fig. 8. The variable coefficient α and p_r fitting chart.

As an important note, if we calculate the variable coefficient α at 1600 psi, it must be the same from Eqs. (20) and (21) which increase the power and utility of the two developed α-empirical relationships. Table 2 shows the constants of Eq. (21).

Table 3 introduces the ranges of data used in the development of Eqs. (20) and (21). Finally, Eqs. (19)–(21) represent the new developed IPR model.

3. Methodology to use the new IPR model

Step 1. If p_r is less than or equal to 1600 psia, the oil IPR parameter (α) is calculated using Eq. (20) as follows:

$$\alpha = \frac{1}{-0.981 \times p_r - 152.585}$$

If p_r is greater than or equal to 1600 psia, the oil IPR parameter (α) is calculated using Eq. (21) as follows:

$$\alpha = -0.0043065 + 4.98E-06p_r - 2.41E-09p_r^2 + 5.69E-13p_r^3 - 6.48E-17p_r^4 + 2.85E-21p_r^5$$

Step 2. Calculate $q_{o,max}$ using Eq. (19) at any given test point:

$$q_{o,\ max} = q_o(test) \Big/ \left[1 - \frac{\ln(-\alpha \times p_{wf(test)} + 1)}{\ln(-\alpha \times p_r + 1)}\right] \text{STB/day}$$

Step 3. Assume several values for p_{wf} and calculate the corresponding q_o using Eq. (19):

$$q_o = q_{o,\ max} \times \left[1 - \frac{\ln(\alpha \times p_{wf} + 1)}{\ln(-\alpha \times p_r + 1)}\right] \text{STB/day}$$

Step 4. For future IPR, calculate α_f using the future value of $p_r)_f$ using Eqs. (20) and (21) according to the value of $p_r)_f$.

Step 5. Solve for $q_{o,max,}$ at future conditions using Fetkovich's equation as follows:

$$q_{o,\ max})_f = q_{o,\ max})_p \times \left[\frac{p_r)_f}{P_r)_p}\right]^{3.0} \text{STB/day}$$

Step 6. Generate the future inflow performance-curve by applying Eq. (19) as follows:

$$q_o = q_{o,\ max})_f \times \left[1 - \frac{\ln(\alpha_f \times p_{wf} + 1)}{\ln\ (\alpha_f \times p_r)_f + 1)}\right] \text{STB/day}$$

4. Validation of the new IPR model

To verify and validate the new developed IPR model, information from 12 field cases were collected and analyzed to get the present inflow performance, and two field cases were collected and used to predict the future IPR curve. The ranges of fluid properties, rock properties, and reservoir data of these cases are included in Table 4. Each field case uses actual field data which represent different producing conditions. In order to test the accuracy and reliability of the new developed IPR model, which is the single point method, it will be compared to some of the other two-phase IPR methods currently available in the industry. These methods are those of Vogel (single point method), Fetkovich (multi-point method), Wiggins (single point method), and Sukarno (single point method) for the present inflow performance and Vogel, Fetkovich, Wiggins for the future inflow performance

Elias (2009) presents the complete details of the comparison analysis while the cases analyzed for present and future performances are summarized in Tables 4 and 5, respectively.

4.1. Field Case No. 1: Carry City Well

Gallice (1997) presented multirate-test data for a well producing from the Hunton Lime in the Carry City Field, Oklahoma. The test was conducted in approximately 2 weeks during which the well was producing at random rates, rather than in an increasing or decreasing rate sequence. The average reservoir pressure was 1600 psia, with an estimated bubble-point pressure of 2530 psia and an assumed skin value of zero. The multi-rate test of this well is summarized in Table 6.

Table 6 presents the predictions of the well's performance for the test information at a flowing bottomhole pressure of 1194 psia, which represents a 25% of the pressure drawdown. As can be observed, the maximum well deliverability varies from 2550 to 4265 STB/D. The largest flow rate was calculated with Wiggins's IPR, while the smallest rate was obtained using Fetkovich model. Fig. 9 shows the resultant IPR curves for the different methods of calculations such as Vogel, Fetkovich, Wiggins, and Sukarno in

Table 4
Validation field cases analyzed for the present performance.

Case	Case name	Case type	p_r (psia)
1	Carry City Well	Vertical well	1600
2	Well M110-1979	Vertical well	2320
3	Well M200	Vertical well	3263
4	Case X1B	Horizontal well	2580
5	Well 1, Gulf of Suez, Egypt	Vertical well	2020
6	Well 3-Field C	Vertical well	3926
7	Well 4	Vertical well, layered reservoir	5801
8	Well E, Keokuk Field	Vertical well	1710
9	Well A, Keokuk Field	Vertical well	1734
10	Well TMT-27	Vertical well	868
11	Well A	Vertical well	1785
12	Well 8, West Texas	Vertical well	640

Table 5
Validation field cases analyzed for the future performance.

Case	Test chronology	Case name	p_r (psia)
1	Present	Well M110-1979	2321
1	Future	Well M110-1987	2067
2	Present	Well A, Keokuk Field-1934	1734
2	Future	Well A, Keokuk Field-1935	1609

Table 6
Prediction of the performance of Case No. 1 at 25% of the pressure drawdown.

Field data		The new IPR method	Vogel method	Fetkovich method	Wiggins method	Sukarno method
p_{wf}, psia	q_o, STB/D	q_o, STB/D	q_o, STB/D	q_o, STB/D	q_o, STB/D	q_o, STB/D
1600	0	0	0	0	0	0
1558	235	297	169	213	164	177
1497	565	614	408	444	398	423
1476	610	703	489	516	477	505
1470	720	728	511	536	499	528
1342	1045	1140	977	920	965	995
1267	1260	1321	1233	1115	1225	1244
1194	1470	1470	1470	1288	1470	1470
1066	1625	1688	1856	1559	1879	1828
996	1765	1790	2051	1690	2091	2003
867	1895	1954	2382	1905	2462	2290
787	1965	2044	2569	2021	2679	2445
534	2260	2284	3062	2309	3297	2830
351	2353	2428	3329	2447	3680	3018
183	2435	2544	3507	2522	3985	3133
166	2450	2555	3521	2527	4013	3142
0		2657	3627	2550	4265	3205

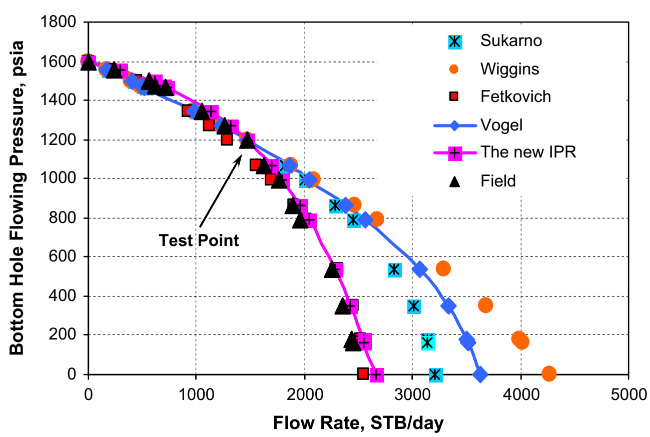

Fig. 9. The predicted inflow curves by the different used methods in comparison to the actual field data for Case No. 1.

comparison with the actual field data and the new developed IPR model. It is clear from this figure that the method of the new developed IPR model is successful to estimate the actual well performance. In addition, it can be clearly concluded from this figure that the methods of the new developed IPR model and Fetkovich's model nearly estimate the maximum oil flow rate for this well more accurately than the other models, and as indicated, the other methods overestimate the actual performance.

The average absolute errors percent between the actual flow-rate data and the calculated rate for the five IPR methods that are used in this study are shown in Fig. 10 for comparison. It is clear from this figure that the new developed IPR model has the lowest average absolute error percent that is 6.47%, while the average absolute error percent for Fetkovich's method is 8.56%. The other single-point methods have average absolute errors percent ranging from 20.1% to 32.3% for Sukarno and Wiggins, respectively.

In summary, the new model provided the best estimates of well performance for this case's entire range of interest. The multipoint method of Fetkovich tends to do a better job of predicting well performance than the other three single-point methods. Overall, the single-point methods of Vogel, Wiggins, and Sukarno provided similar great average differences in this case. As indicated in this work, the more important relationship to evaluate well performance is the relationship between the oil mobility function and the average reservoir pressure, and this was clearly demonstrated from the value of the average absolute error percent that resulted from using the new developed IPR model (Table 7).

4.2. Field cases summary for the present inflow performance

The additional cases and their analysis are presented in detail in Del Castillo (2003). Table 7 presents a summary of the average absolute errors percent that was obtained for each method in each one of the twelve case studies that were examined. As indicated, the method of the new developed IPR model always provided the most reliable estimates of the actual well data analyzed. It has the lowest value of the total average absolute error percent, which is 6.6% in comparison with that of Fetkovich's method, which has a reasonable average absolute error percent of 7% but is still higher

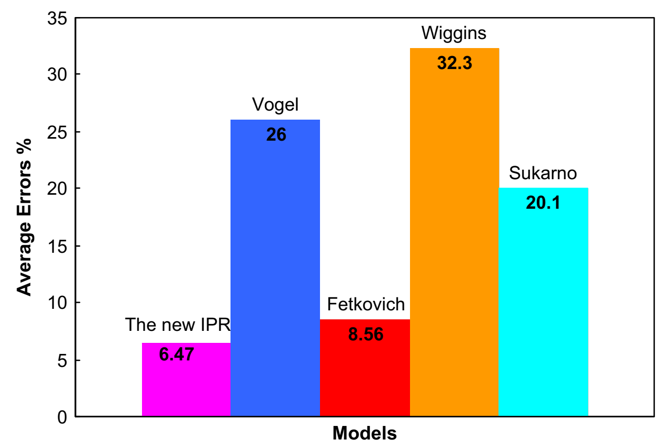

Fig. 10. The average absolute errors percent at 25% drawdown for Case No. 1.

Table 7
Summary of the average absolute error percent – 12 cases.

Case	Total average errors %				
	The new IPR	Vogel	Fetkovich	Wiggins	Sukarno
1	6.47	26	8.56	32.3	20.1
2	1	5.1	6.4	6.8	3.5
3	7.4	15.1	7.7	15.8	14.2
4	19.6	3	6.3	3.7	10.5
5	3.4	7	4.15	7.7	5.9
6	5.7	14	9.61	17.2	9.7
7	3.1	13	6.72	15.1	11.1
8	8	32	6	33.3	31
9	5.6	14.7	6	17.0	11.6
10	5.9	13.7	9.7	16.0	10.6
11	5.4	6	7.6	8.7	3.4
12	7.2	14	4.7	14.3	13.1
Average (%)	**6.6**	**13.7**	**7**	**15.7**	**12.1**

Table 8
Summary of the average absolute errors percent – future field cases.

Future field case	Total average errors (%)			
	The new IPR	Vogel	Fetkovich	Wiggins
1	21	28	36	23
2	9	35	15	38
Average (%)	**15**	**31.5**	**25.5**	**30.5**

than the method of the new developed IPR model. The other methods always provided less accurate values for the pressure-rate estimates of the actual well data that used in this analysis.

The method of the new developed IPR model tends to do a better job of predicting well performance than the other methods, and this it may be because it assumes an accurate relationship between the oil mobility function and the average reservoir pressure (i.e., the reciprocal relationship). Overall, the single-point methods of Vogel, Wiggins, and Sukarno provided great average absolute errors percent in the cases examined – 12.1–15.7%.

However, the following comment should be introduced based on the above table Case Gallice and Wiggins (1999), Vogel's model provided the best estimates of well performance for this case. The single point method of Wiggins tends to do a good job of predicting well performance in this case. Finally, it can be concluded from this case that the new IPR model has some limitations in case of low-pressure reservoirs, which has a reservoir pressure less than 1000 psia. This is because there were no sufficient data below 1000 psia in case of the development of the new IPR model; therefore it is recommended in this case to use Vogel's model.

4.3. Field case summary for the future performance

The analyses of the two future field cases are presented in detail in Elias (2009). Table 8 presents a summary of the average absolute errors percent that was obtained for each method in each one of the two future case studies that were examined. As indicated, the method of the new developed IPR model always provided the most reliable estimates of the actual well data analyzed. It has the lowest value of the total average absolute error percent, which is 15%. The other methods always provided less accurate values for the pressure-rate estimates of the actual future well data that was used in this analysis. Overall, the other methods of Fetkovich, Vogel, and Wiggins provided great average absolute errors percent in the cases examined ranging from 25.5% to 31.5% for Fetkovich and Vogel, respectively.

However, general comment can be presented based on Table 8 and the future cases are analyzed in this work; the average absolute error percent is almost as great for Fetkovich's method as compared to the method of the newly developed IPR model which is −25.5% compared to 15%. As indicated, the main reasons for that are

(1) Fetkovich considered that the relationship between the mobility function and the average reservoir pressure is a linear relationship (see Fig. 3), but the new IPR model considered this relation is a reciprocal relationship (see Figs. 5 and 6) which covers the entire range of interest more accurately.
(2) The backpressure equation parameter (n) of Fetkovich IPR equation does not take into consideration the change in the average reservoir pressure.
(3) The new model IPR parameter (α) of Eq. (19) takes into consideration the change in the average reservoir pressure (see Eqs. (20) and (21)).

5. Conclusions

In this work, we reviewed the most commonly used IPR models; also, we developed a new IPR model. The new IPR was compared to the most commonly used models using field data (12 field cases). Based on this work, we can conclude the following:

(1) A general correlation for α-parameter that represents the oil mobility as a function of p_r was developed by using 47 field cases. A new method to construct and predict the IPR curve for solution gas drive reservoirs was developed by using this general correlation of α-parameter.
(2) The validity of the new IPR model was tested through its application on 12 field cases in comparison with the behavior of the most common methods that are used in the industry. The results of this validation showed that the new IPR model ranked the first model that succeeded to predict the behavior of the IPR curve for the 12 examined field cases, while the other models of Fetkovich, Sukarno, Vogel, and Wiggins ranked the second, the third, the forth, and the fifth, respectively.
(3) The new IPR model requires one test point and is as accurate or more than Fetkovich's model which requires three test points.
(4) The new developed IPR outperformed all available IPR models except at low pressures (less than 1000 psia). At these low pressures Vogel's correlation was found to be the most accurate model.
(5) The range of applicability of alpha-pressure relationships (Eqs. (20) and (21)) is 860–7000 psi.

Appendix A

In this Appendix, the derivation of the new IPR equation is based on the pseudo-steady state flow equation for a single well in a solution gas drive reservoir system (pseudopressure formulation). In addition the relation between the mobility of the oil phase and p_r (i.e., reciprocal relationship – $Mo = 1.0/(ap_r + b)$ is used, where a and b are the two equation variables). The definition of the oil-phase pseudopressure for a single well in a solution gas drive reservoir is given as

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n} \int_{p_{base}}^{p} \frac{k_{ro}}{\mu_o B_o} dp. \tag{A.1}$$

The pseudo-steady state flow equation for the oil-phase in a solution gas drive reservoir is given by

$$p_{po}(p_r) = p_{po}(p_{wf}) + q_o b_{ss} \tag{A.2}$$

where

$$b_{ss} = 141.2 \left.\frac{\mu_o B_o}{k_{ro}}\right|_{p_n} \left[\frac{1}{h}\left(\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + S\right)\right] \tag{A.3}$$

In this work, a new form for the oil mobility function at different values of the average reservoir pressure (i.e., the reciprocal relationship) is obtained from the result of the simulation study that performed on MORE simulators using the six simulation cases as follows:

$$\left[\frac{k_{ro}}{\mu_o B_o}\right]_{p_r} = \frac{1}{xp_r + y} \tag{A.4}$$

where x and y are two variables established from the presumed behavior of the mobility profile.

Solving Eq. (A.2) for the oil rate, q_o, the following equation for the oil flow rate can be presented:

$$q_o = \frac{1}{b_{ss}}[p_{p_o}(p_r) - p_{p_o}(p_{wf})] \tag{A.5}$$

Solving Eq. (A.5) for the maximum oil rate, $q_{o,max}$ (i.e., at $p_{wf} = 0$ or p_{po} $(p_{wf}) = 0$):

$$q_{o,\max} = \frac{1}{b_{ss}}\left[p_{p_o}(p_r) - p_{p_o}(p_{wf} = 0)\right] \tag{A.6}$$

Dividing Eq. (A.5) by Eq. (A.6) gives the "IPR" form (i.e., $q_o/q_{o,\,max}$) in terms of the pseudopressure functions, which yields

$$\frac{q_o}{q_{o,\,max}}=\frac{p_{p_o}(p_r)-p_{p_o}(p_{wf})}{p_{p_o}(p_r)-p_{p_o}(p_{wf}=0)} \tag{A.7}$$

At this point, it should be noted that, *it is not the goal* to proceed with the development of an IPR model in terms of the pseudopressure functions, p_{po} (p)-rather, the goal is to develop a simplified IPR model using Eqs. (A.4) and (A.7) as the base relations. Substituting Eq. (A.4) into Eq. (A.1) yields

$$p_{p_o}(p)=\left[\frac{\mu_oB_o}{k_{ro}}\right]_{p_n}\int_{p_{base}}^{p}\left[\frac{1}{xp+y}\right]dp=\left[\frac{\mu_oB_o}{k_{ro}}\right]_{p_n}\frac{1}{a}[\,\ln(xp+y)]_{p_{base}}^{p}$$

or,

$$p_{p_o}(p)=\left[\frac{\mu_oB_o}{k_{ro}}\right]_{p_n}\frac{1}{a}[\,\ln(xp+y)-\,\ln(xp_{base}+y)] \tag{A.8}$$

Substituting Eq. (A.8) into Eq. (A.7) gives

$$\frac{q_o}{q_{o,\,max}}=\frac{[\,\ln(xp_r+y)-\,\ln(xp_{base}+y)]-[\,\ln(xp_{wf}+y)-\,\ln(xp_{base}+y)]}{[\,\ln(xp_r+y)-\,\ln(xp_{base}+y)]-[\,\ln(y)-\,\ln(xp_{base}+y)]}$$

or,

$$\frac{q_o}{q_{o,\,max}}=\frac{\ln(x\cdot p_r+y)-\,\ln(x\cdot p_{wf}+y)}{\ln(x\cdot p_r+y)-\,\ln(y)} \tag{A.9}$$

Rearranging Eq. (A.9) gives the following form:

$$\frac{q_o}{q_{o,\,max}}=\frac{\ln(xp_r+y)}{\ln(xp_r+y)-\,\ln(y)}-\frac{\ln(xp_{wf}+y)}{\ln(xp_r+y)-\,\ln(y)} \tag{A.10}$$

or

$$\frac{q_o}{q_{o,\,max}}=\frac{\ln(xp_r+y)}{\ln((xp_r+y)/y)}-\frac{\ln(xp_{wf}+y)}{\ln(xp_r+y/y)} \tag{A.11}$$

Dividing the right term through Eq. (A.11) by the term b gives the following form:

$$\frac{q_o}{q_{o,\,max}}=\frac{\ln((x/y)p_r+1)}{\ln((x/y)p_r+1)}-\frac{\ln((x/y)p_{wf}+1)}{\ln((x/y)p_r+1)} \tag{A.12}$$

And then, replace x/y by α and substituting this definition into Eq. (A.12) yields the following IPR form:

$$\frac{q_o}{q_{o,\,\max}}=1-\frac{\ln(\alpha p_{wf}+1)}{\ln(\alpha p_r+1)} \tag{A.13}$$

where α is the oil IPR parameter for the new IPR model.

It is suggested that Eq. (A.13) serves as an equation of the proposed new IPR model.

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