

$$a + \infty = \infty + a = \infty, \quad a \in \mathbb{R}$$

$$a - \infty = \infty - a = \infty, \quad a \in \mathbb{R}$$

$$a \cdot \infty = \infty \cdot a = \infty, \quad a \in \mathbb{R}, a \neq 0$$

$$\infty \cdot \infty = \infty$$

$$\frac{a}{\infty} = 0, \quad a \in \mathbb{R}$$

$$\frac{\infty}{\infty} = \infty, \quad a \in \mathbb{R}$$

$$\frac{0}{\infty} = \infty, \quad a \in \mathbb{R}, a \neq 0$$

## Arithmetic of Infinity

### Sum

1.  $\infty + \infty = \infty$

2.  $n + \infty = \infty$

### Difference

1.  $\infty - \infty = \text{Indeterminate}$

2.  $n - \infty = -\infty$

3.  $\infty - n = \infty$

4.  $n^- - n = 0^- = -\frac{1}{\infty}$

5.  $n^+ - n = 0^+ = \frac{1}{\infty}$

### Product

1.  $\infty \cdot \infty = \infty$

2.  $n \cdot \infty = \infty \cdot n = \infty$

3.  $0 \cdot \infty = \infty \cdot 0 = \text{Indeterminate}$

### Quotient

1.  $\frac{\infty}{\infty} = \text{Indeterminate}$

2.  $\frac{\infty}{n} = \infty$

3.  $\frac{n}{\infty} = 0$

4.  $\frac{n}{-\infty} = 0$

5.  $\frac{n}{0} = \pm\infty$

6.  $\frac{n}{0^-} = -\infty$

7.  $\frac{n}{0^+} = \infty$

### Power

1.  $\infty^\infty = \infty$  2.  $\infty^n = \infty$  3.  $n^\infty = \infty$

4.  $\infty^0 = 1$  5.  $0^\infty = 0$

### Trig

1.  $-1 \leq \sin(\pm\infty) \leq 1$

2.  $-1 \leq \cos(\pm\infty) \leq 1$

### Basic Limit Evaluations at $\pm \infty$

Note :  $\text{sgn}(a) = 1$  if  $a > 0$  and  $\text{sgn}(a) = -1$  if  $a < 0$ .

- $\lim_{x \rightarrow \infty} e^x = \infty$  &  $\lim_{x \rightarrow -\infty} e^x = 0$
- $\lim_{x \rightarrow \infty} \ln(x) = \infty$  &  $\lim_{x \rightarrow 0^+} \ln(x) = -\infty$
- If  $r > 0$  then  $\lim_{x \rightarrow \infty} \frac{b}{x^r} = 0$
- If  $r > 0$  and  $x^r$  is real for negative  $x$   
then  $\lim_{x \rightarrow -\infty} \frac{b}{x^r} = 0$
- $n$  even :  $\lim_{x \rightarrow \pm \infty} x^n = \infty$
- $n$  odd :  $\lim_{x \rightarrow \infty} x^n = \infty$  &  $\lim_{x \rightarrow -\infty} x^n = -\infty$
- $n$  even :  $\lim_{x \rightarrow \pm \infty} ax^n + \dots + bx + c = \text{sgn}(a)\infty$
- $n$  odd :  $\lim_{x \rightarrow \infty} ax^n + \dots + bx + c = \text{sgn}(a)\infty$
- $n$  odd :  $\lim_{x \rightarrow -\infty} ax^n + \dots + bx + c = -\text{sgn}(a)\infty$

$$\lim_{x \rightarrow \pm\infty} \cos x = \text{D.N.E.}$$

$$\lim_{x \rightarrow \pm\infty} \sin x = \text{D.N.E.}$$

$$\lim_{x \rightarrow \pm\infty} \tan x = \text{D.N.E.}$$

$$\lim_{x \rightarrow \pm\infty} \sec x = \text{D.N.E.}$$

$$\lim_{x \rightarrow \pm\infty} \csc x = \text{D.N.E.}$$

$$\lim_{x \rightarrow \pm\infty} \cot x = \text{D.N.E.}$$