Consider the reduction formula $\int x^m e^x \, dx = x^m e^x - m\int x^{m-1} e^x \, dx$. The objective of the problem is to use integration by parts to derive the given reduction formula.

If u and v are differentiable functions, then the formula for the integration by parts is

$$\int u \, dv = uv - \int v \, du$$

where dv equal the most complicated part of the integrand that can be readily integrated.

Here, the possible choices for dv are $dx, x^m \, dx, e^x \, dx, x^m e^x \, dx$. The most complicated of these expressions that can be readily integrated is $e^x \, dx$. So, let $dv = e^x \, dx$ so that

$$v = \int e^x \, dx$$

$$= e^x \quad \text{Integrate}$$

Now, let $u = x^m$ so that

$$du = mx^{m-1} \, dx \quad \text{Find the derivative}$$

Substituting these values in the integration by parts formula implies

$$\int x^m e^x \, dx = x^m e^x - \int e^x m x^{m-1} \, dx$$

$$= x^m e^x - m\int e^x x^{m-1} \, dx \qquad \text{Use the rule: } \int cf(x)dx = c\int f(x)dx$$

Thus, $\int x^m e^x \, dx = x^m e^x - m\int x^{m-1} e^x \, dx$.

Hence, the reduction formula is derived.

Consider the reduction formula $\int \sec^m x\,dx = \frac{\sec^{m-2} x \tan x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\,dx$ for $m \neq 1$.

The objective of the problem is to use integration by parts to derive the given reduction formula.

First, rewrite the given integral as

$$\int \sec^m x\,dx = \int \sec^{m-2} x \sec^2 x\,dx$$

If u and v are differentiable functions, then the formula for the integration by parts is

$$\int u\,dv = uv - \int v\,du$$

where dv equal the most complicated part of the integrand that can be readily integrated.

Here, the possible choices for dv are dx, $\sec^{m-2} x\,dx$, $\sec^2 x\,dx$ and $\sec^{m-2} x \sec^2 x\,dx$. The most complicated of these expressions that can be readily integrated is $\sec^2 x\,dx$.

So, let $dv = \sec^2 x\,dx$ so that

$$v = \int \sec^2 x\,dx$$

$$= \tan x \quad \text{Integrate}$$

Now, let $u = \sec^{m-2} x$ so that

$$du = (m-2)\sec^{m-3} x \sec x \tan x\,dx \quad \text{Find the derivative}$$

Substituting these values in the integration by parts formula implies

$$\int \sec^m x\, dx = \sec^{m-2} x \tan x - \int \tan x (m-2) \sec^{m-3} x \sec x \tan x\, dx$$

$$= \sec^{m-2} x \tan x - (m-2) \int \tan^2 x \sec^{m-2} x\, dx \qquad \text{Simplify}$$

Now, use the trigonometric identity, $\tan^2 x = \sec^2 x - 1$ and simplify further.

$$\int \sec^m x\, dx = \sec^{m-2} x \tan x - (m-2) \int \left(\sec^2 x - 1\right) \sec^{m-2} x\, dx$$

$$= \sec^{m-2} x \tan x - (m-2) \int \left(\sec^m x - \sec^{m-2} x\right) dx \qquad \text{Simplify}$$

$$= \sec^{m-2} x \tan x - (m-2) \int \sec^m x\, dx + (m-2) \int \sec^{m-2} x\, dx \qquad \text{Simplify}$$

Consequently,

$$\int \sec^m x\,dx + (m-2)\int \sec^m x\,dx = \sec^{m-2} x \tan x - (m-2)\int \sec^m x\,dx + (m-2)\int \sec^{m-2} x\,dx$$

$$(m-2+1)\int \sec^m x\,dx = \sec^{m-2} x \tan x + (m-2)\int \sec^{m-2} x\,dx$$

$$(m-1)\int \sec^m x\,dx = \sec^{m-2} x \tan x + (m-2)\int \sec^{m-2} x\,dx \quad \text{Simplify}$$

Finally, dividing both sides by $m-1$ implies

$$\int \sec^m x\,dx = \frac{\sec^{m-2} x \tan x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\,dx$$

Thus, $\int \sec^m x\,dx = \frac{\sec^{m-2} x \tan x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\,dx$.

Hence, the reduction formula is derived.

Consider the integral $\int x^5 e^x\, dx$. The objective of the problem is to find the value of the integral using the reduction formula $\int x^m e^x\, dx = x^m e^x - m\int x^{m-1} e^x\, dx$.

To find the value of the given integral, substitute 5 for m in the reduction formula $\int x^m e^x\, dx = x^m e^x - m\int x^{m-1} e^x\, dx$ and simplify further.

$$\begin{aligned}\int x^5 e^x\, dx &= x^5 e^x - 5\int x^{5-1} e^x\, dx \\ &= x^5 e^x - 5\int x^4 e^x\, dx \quad \text{Simplify}\end{aligned}$$

Now, to find $\int x^4 e^x\, dx$, apply the reduction formula $\int x^m e^x\, dx = x^m e^x - m\int x^{m-1} e^x\, dx$ with $m = 4$. This gives

$$\begin{aligned}\int x^4 e^x\, dx &= x^4 e^x - 4\int x^{4-1} e^x\, dx \\ &= x^4 e^x - 4\int x^3 e^x\, dx \quad \text{Simplify}\end{aligned}$$

To find $\int x^3 e^x \, dx$, again use the reduction formula $\int x^m e^x \, dx = x^m e^x - m\int x^{m-1} e^x \, dx$ with $m = 3$.
This gives

$$\int x^3 e^x \, dx = x^3 e^x - 3\int x^{3-1} e^x \, dx$$

$$= x^3 e^x - 3\int x^2 e^x \, dx \quad \text{Simplify}$$

To find $\int x^2 e^x \, dx$, again use the reduction formula $\int x^m e^x \, dx = x^m e^x - m\int x^{m-1} e^x \, dx$ with $m = 2$.
This gives

$$\int x^2 e^x \, dx = x^2 e^x - 2\int x^{2-1} e^x \, dx$$

$$= x^2 e^x - 2\int x e^x \, dx \quad \text{Simplify}$$

Finally, to find $\int xe^x\,dx$, again use the reduction formula $\int x^m e^x\,dx = x^m e^x - m\int x^{m-1}e^x\,dx$ with $m=1$. This gives

$$\int xe^x\,dx = xe^x - 1\int x^{1-1}e^x\,dx$$

$$= xe^x - \int e^x\,dx \quad \text{Simplify}$$

$$= xe^x - e^x + c_1 \quad \text{Find the integral}$$

Consequently,

$$\int x^5 e^x\,dx = x^5 e^x - 5\int x^{5-1}e^x\,dx$$

$$= x^5 e^x - 5\left[x^4 e^x - 4\left(x^3 e^x - 3\left(x^2 e^x - 2\left(xe^x - e^x + c_1\right)\right)\right)\right]$$

$$= e^x\left(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120\right) + C \quad \text{Simplify}$$

where $C = 120c_1$, is a constant.

Hence, $\int x^5 e^x\,dx = \boxed{e^x\left(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120\right) + C}$.