$f_x = x^3$ on $[1, 3]$

31) $\Delta x = \frac{3-1}{n} = \frac{2}{n}$ Right $x_k = a + k\Delta x = 1 + \frac{2k}{n}$

$$f_x(x_k) = f\left(1 + \frac{2k}{n}\right) \Rightarrow \sum_{k=1}^{n} f_x(x_k)\Delta x = \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^3 \frac{2}{n}$$

$$= \sum_{k=1}^{n} \left(1 + \frac{6k}{n} + \frac{12k^2}{n^2} + \frac{8k^3}{n^3}\right)\left(\frac{2}{n}\right) = \sum_{k=1}^{n} \frac{2}{n} + \frac{12k^2}{n^2} + \frac{24k^2}{n^3} + \frac{16k^3}{n^4}$$

$$\lim_{n\to\infty} \left(2 + \frac{12}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\right)$$

$$\lim_{n\to\infty} \left(2 + 6 + \frac{6}{n} + 8 + \frac{4}{n} + \frac{8}{n} + \frac{4}{n^2} + \frac{8}{n} + 4 + \frac{4}{n^2}\right)$$

$$= 2 + 6 + 0 + 8 + 0 + 0 + 0 + 0 + 4 + 0 = 20$$

32) $f_x = x^3 + 2$ a) $[1, 3]$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$ Right $x_k = a + k\Delta x = 1 + \frac{2k}{n}$

$$\sum_{k=1}^{n} f_x\left(\left(1 + \frac{2k}{n}\right)^3 + 2\right)\left(\frac{2}{n}\right) = \sum_{k=1}^{n}\left(1 + \frac{6k}{n} + \frac{12k^2}{n^2} + \frac{8k^3}{n^3} + 2\right)\frac{2}{n}$$

$$= \sum_{k=1}^{n} \frac{6}{n} + \frac{12k^2}{n^2} + \frac{24k^2}{n^3} + \frac{16k^3}{n^4} =$$

$$\lim_{n\to\infty}\left(6 + \frac{12}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{16}{n^4}\left(\frac{n^2(n^2+2n+1)}{4}\right)\right)$$

$$= 6 + 6 + 8 + 4 = 24$$