King Saud University
College of Sciences
Department of Mathematics

Mid-Term Exam, S1 1439/1440
M 507 -Operation Research
Time: 90 minutes

Answer the following questions:

Q1: [5+5+5+5]

a) Prove that: the average failure rate over the interval (t_1, t_2) is given by

$\bar{\lambda} = \frac{\Lambda(t_2) - \Lambda(t_1)}{t_2 - t_1}$, and then derive the average failure rate for the two parameter Weibull distribution.

b) The life of a product follows a Weibull distribution with a shape parameter of 1.5 and a scale parameter of 1000 hours. Compute the instantaneous failure rate at its characteristic value and the average failure rate over the time interval $(1000, 2000)$ in hours.

c) Suppose that a number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10000 miles. If a person desires to take a 5000-mile trip, what is the probability that he will be able to complete his trip without having to replace the car battery?

d) Given the joint probability mass functions of two random variables X and Y as in the following table:

X \ Y	1	2	3
0	1/8	0	0
1	0	1/4	1/8
2	0	1/4	1/8
3	1/8	0	0

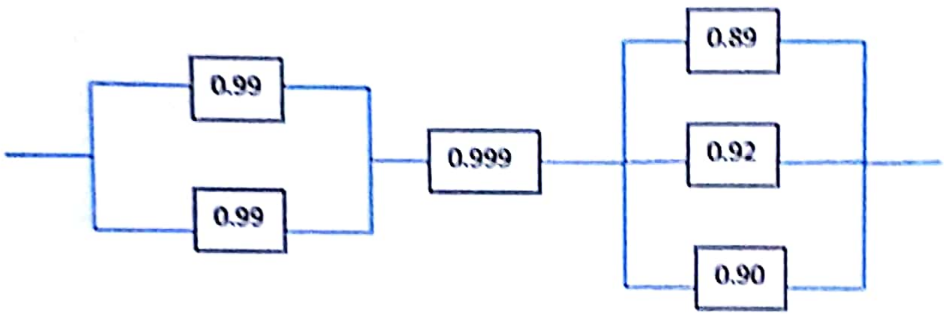
i) Find $\rho(X,Y)$

ii) Determine whether X and Y are two independent random variables or not? Justify your answer.

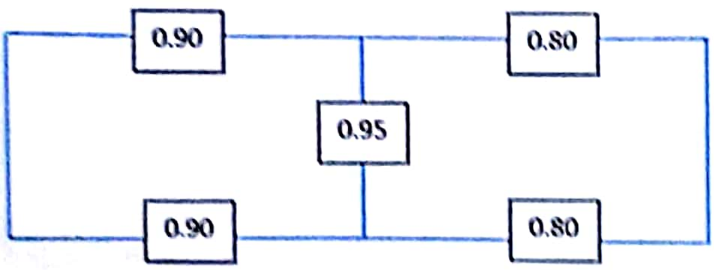
Q2: [5+5]

Compute the system reliability for the following configuration diagram where each component has the indicated reliability

a)



b)



1

Model answer of Mid-Term Exam

for M507

30

Q1

a) The hazard fn is given by

5

$$\lambda(x) = \frac{f(x)}{R(x)}$$

$$= \frac{1}{R(x)}\left[-\frac{dR(x)}{dx}\right]$$

$$\therefore \lambda(x)\,dx = \frac{-dR(x)}{R(x)}$$

$$\Rightarrow -\int_0^x \lambda(t)\,dt = \int_0^x \frac{dR(t)}{R(t)}$$

$$\Rightarrow \left[\ln R(t)\right]_0^x = -\int_0^x \lambda(t)\,dt$$

$$\therefore \ln R(x) - \underbrace{\cancel{\ln R(0)}}_{0} = -\int_0^x \lambda(t)\,dt$$

$$\therefore R(x) = e^{-\int_0^x \lambda(t)\,dt}$$

$$\therefore R(x) = e^{-\Lambda(x)}, \quad \Lambda(x) = \int_0^x \lambda(t)\,dt \text{ (Cumulative hazard fn)}$$

(Reliability fn)

$\therefore$ The average failure rate over the interval (t_1, t_2) is given by

$$\bar{\lambda} = \frac{\int_{t_1}^{t_2} \lambda(x)\,dx}{t_2 - t_1} = \frac{\int_0^{t_2} \lambda(x)\,dx - \int_0^{t_1} \lambda(x)\,dx}{t_2 - t_1}$$

$$\therefore \bar{\lambda} = \frac{\Lambda(t_2) - \Lambda(t_1)}{t_2 - t_1} \quad (1)$$

for 2p Weibull $R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$, $R(t) = e^{-\Lambda(t)}$

$$\therefore \Lambda(t) = \left(\frac{t}{\eta}\right)^{\beta} \quad (2)$$

$$(1),(2) \Rightarrow \therefore \bar{\lambda} = \frac{t_2^{\beta} - t_1^{\beta}}{\eta^{\beta}(t_2 - t_1)}$$

#

$\underline{\underline{2}}$

b) $\beta = 1.5$

$\eta = 1000$ hrs

$$\lambda(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1}$$

$$\Rightarrow \therefore \lambda(1000) = \frac{1.5(1000)^{0.5}}{(1000)^{1.5}}$$

$$= \frac{1.5}{1000} = 1.5\times10^{-3}$$

$\boxed{5}$

The average failure rate over the interval (1000, 2000)

is $$\bar{\lambda} = \frac{t_2^{\beta} - t_1^{\beta}}{\eta^{\beta}(t_2-t_1)}$$

$$= \frac{(2000)^{1.5} - (1000)^{1.5}}{(1000)^{1.5}(1000)} \simeq 1.8\times10^{-3}$$

c) $X \sim \exp(1/10000)$ where $\lambda = \frac{1}{\mu} = \frac{1}{10000}$

$$pr(X > 5000) = e^{-\frac{5000}{10000}} = e^{-0.5}$$

$$\simeq 0.6065$$

$\boxed{5}$

d) $$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

$\mu_X = E(X) = 1.5$, $\mu_Y = E(Y) = 2$

$E(X^2) = 3$, $E(Y^2) = 4.5$

$\sigma_X^2 = 0.75$, $\sigma_Y^2 = 0.5$

$\sigma_X = 0.866$, $\sigma_Y = 0.7071$

3/

$Cov(X,Y) = E(XY) - E(X)E(Y)$

$, E(XY) = 3$

$, Cov(X,Y) = 3 - 1.5(2) = 0$ $\boxed{5}$

$\therefore \rho(X,Y) = \frac{0}{0.866(0.7071)} = 0$

It's not necessary for $\rho(X,Y) = 0$ implies that X and Y are independent r.v's

e.g $p(0,1) = 1/8$ but $p(0)p(1) = 1/8(1/4) = \frac{1}{32}$

thus means that X and Y are not independent r.v's.

Q2

a)

$R_{sys} = [1-(1-0.99)^2](0.999)[1-(1-0.89)(1-0.92)(1-0.90)]$ $\boxed{5}$

$= 0.998$

b) $R^+ = (1-0.01)(1-0.04) = 0.99(0.96) = \boxed{0.9504}$

$R^- = 1 - [1-0.9(0.8)][1-0.9(0.8)]$ $\boxed{5}$

$R^- = 1 - (1-0.72)^2 = \boxed{0.9216}$

$R_{sys} = R_3R^+ + (1-R_3)R^-$

$= 0.95(0.9504) + 0.05(0.9216)$

$\therefore R_{sys} = \boxed{0.94896}$

Note: We use the decomposition method and we take the component 3 as a pivot element.

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