King Saud University

Faculty of Sciences

Department of Mathematics

First Examination	Math 106	Semester I	1439-1440
	Time: 1H30		

Exercise 1 : (3+3+2)

a) If $F(x) = \ln(2x) \int_1^{4x^2} (1+t^2)^{10} dt$. Find $F'(\frac{1}{2})$.

b) Use Riemann sums to find the value of $\int_0^2 3x^2 dx$.

c) Use Trapezoid rule with $n = 4$ to approximate the integral $\int_0^{\pi} \sin^4(x) dx$.

Exercise 2 : (2+3+3)

a) If $f(x) = \log_2(\sin^{-1}(x))$, $x > 0$. Find $f'(x)$.

b) Compute the integral $\int \frac{4 - \ln(x)}{x} dx$.

c) If $y = x^{2x^2}(x-1)^{\frac{3}{2}}$, $x > 1$. Find y'.

Exercise 3 : (3+3+3)

a) Evaluate the integral $\int \frac{2x+3}{\sqrt{4-x^2}} dx$.

b) Find $\int \frac{e^{\frac{x}{2}}}{7+e^x} dx$.

c) Compute the integral $\int \frac{\sin(x)}{\sqrt{e^{\cos(x)} - 1}} dx$.

Solution of the first examination   Math 106   Semester 1
1439-1440

Exercise 1 :

a) If $f = \ln(2x)$ and $g(x) = \int_1^{4x^2} (1+t^2)^{10} dt$. Since $f(\frac{1}{2}) = 0$ and $g(\frac{1}{2}) = 0$, then $F'(\frac{1}{2}) = 0$.   1

$$F'(x) = \frac{1}{x}\int_1^{4x^2} (1+t^2)^{10} dt + 8x\ln(2x)(1+16x^4)^{10}$$ 2

b)

$$\int_0^2 3x^2 dx = \lim_{n\to+\infty} 3\frac{2}{n}\sum_{k=1}^{n}\frac{4k^2}{n^2}$$ 2

$$= \lim_{n\to+\infty} 24\left(\frac{n(n+1)(2n+1)}{6n^3}\right) = 8.$$ 1

c) $f(x) = \sin^4(x)$

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	0	1	0
1	$\frac{\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
2	$\frac{\pi}{2}$	1	2	2
3	$\frac{3\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
4	π	0	1	0
				3

1,5

$$\int_0^{\pi} \sin^4(x)dx \approx \frac{3\pi}{8}$$ 0,5

Exercise 2 :

8

a) $f'(x) = \dfrac{1}{(\ln 2)(\sin^{-1}(x))\sqrt{1-x^2}}$. 2

b) $\displaystyle\int \frac{4^{-\ln(x)}}{x}dx \overset{t=-\ln(x)}{=} -\int 4^t dt = -\frac{4^{-\ln(x)}}{\ln 4} + c.$ 2 + 1

c) $y = e^{2x^2 \ln(x)}(x-1)^{\frac{3}{2}}$, $\ln(y) = 2x^2 \ln(x) + \frac{3}{2}\ln(x-1)$. 1

$\dfrac{y'}{y} = 4x\ln(x) + 2x + \dfrac{3}{2(x-1)}$ and $y' = \left(4x\ln(x) + 2x + \dfrac{3}{2(x-1)}\right)y$.

1,5 0,5

Exercise 3 :

a)

$$\int \frac{2x+3}{\sqrt{4-x^2}}dx = \int \frac{2x}{\sqrt{4-x^2}}dx + 3\int \frac{dx}{\sqrt{4-x^2}} \quad 1$$
$$= -2\sqrt{4-x^2} + 3\sin^{-1}(\frac{x}{2}) + c. \quad 2$$

9

b) $\int \frac{e^{\frac{x}{2}}}{7+e^x}dx \overset{t=e^{\frac{x}{2}}}{=} 2\int \frac{dt}{7+t^2} = \frac{2}{\sqrt{7}}\tan^{-1}(e^{\frac{x}{2}}) + c$ $\quad 2+1$

c)

$$\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}}dx \overset{t=\cos(x)}{=} -\int \frac{dt}{\sqrt{e^t-1}} \quad 1,5$$
$$\overset{u^2=e^t-1}{=} -\int \frac{2du}{1+u^2} = -2\tan^{-1}(\sqrt{e^{\cos(x)}-1}). \quad 1,5$$

Or

$t^2 = e^{\cos x}$

$$\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}}dx \overset{t=e^{\frac{1}{2}\cos(x)}}{=} -2\int \frac{dt}{t\sqrt{t^2-1}} \quad 2$$
$$= -2\sec^{-1}(e^{\frac{1}{2}\cos(x)}) + c. \quad 1$$