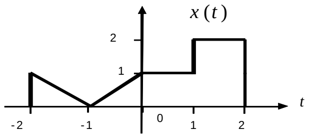
❑ Student Name: **SOLUTION**	**Student ID:**	❑ Section#

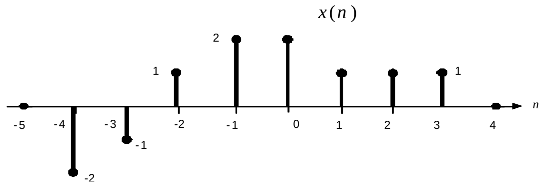
EE 301- Signals and Systems
Midterm Exam # 1

Date: Monday, 24 Oct 2011, 6:00-7:15pm

Question 1 (30 *marks*)**:**

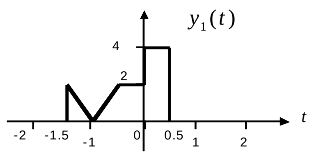
Given the continuous-time (CT) and discrete-time (DT) signals $x(t)$ and $x(n)$ shown below, sketch and label carefully the signals in (a) and (b). [Note: $u(t)$ and $u(n)$ denote the CT and DT unit step signals respectively]

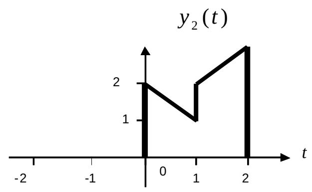




(a) (i) $y_1(t) = 2x(2t+1)$, (ii) $y_2(t) = [x(t) + x(-t)]u(t)$.

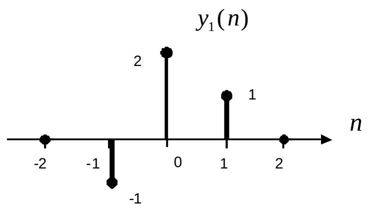
Ans: (i) $y_1(t) = 2x[2(t+0.5)]$, (ii) Note that: $y_2(t) = [x(t)u(t) + x(-t)u(t)]$

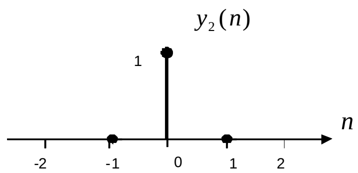




(b) (i) $y_1[n] = x[3n]$, (ii) $y_2[n] = x[3n+1]u[n]$.

Ans: (i) $y_1(n) = x[3(n)]$, (ii) Note that: $y_2(n) = x[3n+1] \cdot u[n]$





(c) Evaluate the integrals: (i) $\int_0^{\pi/2} \cos(t)\,\delta(t-\pi)\,dt$, (ii) $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt$.

Ans: (i) $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt = 0$, (ii) $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt = \int_{-\infty}^{\infty} e^{-t}\,\delta[2(t-1)]\,dt = \int_{-\infty}^{\infty} e^{-t}\,\frac{1}{|2|}\delta(t-1)\,dt = 0.5e^{-1}$

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Question 2 (30 *marks*)**:**

(a) Classify the following signal as periodic or aperiodic and find the fundamental period, if periodic:

(i) $x(t) = [\cos(2t + \frac{2\pi}{3})]^2$, (ii) $x(n) = \cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - \cos(\frac{\pi}{2}n + \frac{\pi}{3})$.

<u>Ans</u>: (i) Periodic, since $x(t) = \frac{1}{2}\{1 + \cos(4t - 4\pi/3)\}$; $\omega_0 = 4$, $T_0 = \frac{\pi}{2}$.

(ii) Periodic, since $\omega_0 = \gcd(\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{2}) = \frac{\pi}{8}$; thus $N = \frac{2\pi}{\omega_0} = 16$.

(b) (i) Determine the total energy or average power for the signal $x(t) = 0.5e^{-2t}u(t)$, and classify it as energy or power signal. (ii) State (without proof), whether or not the system with input-output relation $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, possess the properties: invertibility, linearity, time invariance, memoryless, causality.

<u>Ans</u>: (i) $E_{tot} = \lim_{T \to \infty} \int_{-T}^{T} |0.5e^{-2t}u(t)|^2 \, dt = \frac{0.25}{4}$, $P_{ave} = 0$.

(ii): I. invertible, since $\frac{dy(t)}{dt} = x(t)$

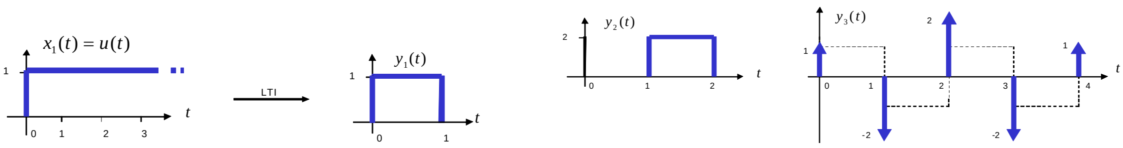
II. linear, since additivity & homogeneity properties would be satisfied.

III. not time-invariant, since $y(t_1) \neq y(t_2)$ for same input $x(t)$, if $t_1 \neq t_2$.

IV. not memoryless, since past inputs must be kept in order to compute $y(t)$.

V. Causal, since future inputs are not required in order to compute $y(t)$.

(c) An LTI system has input-output pairs $x_1(t)$ and $y_1(t)$ as shown below. Compute and sketch the response of the system to the input signals: (i) $x_2(t) = 2u(t-1)$, (ii) $x_3(t) = \delta(t) - \delta(t-1) + \delta(t-2) - \delta(t-3)$.



<u>Ans</u>: (i) $x_2(t) = 2x_1(t-1) \xrightarrow{\text{LTI}} y_2(t) = 2y_1(t-1)$. Note that: $y_1(t) = \{u(t) - u(t-1)\}$

(ii) $x_3(t) = \frac{d}{dt}\{x_1(t) - x_1(t-1) + x_1(t-2) - x_1(t-3)\} \xrightarrow{\text{LTI}} y_3(t) = \frac{d}{dt}\{y_1(t) - y_1(t-1) + y_1(t-2) - y_1(t-3)\}$

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Question 3 (40 *marks*)**:**

(a) The output of a CT LTI system with input $x(t)$ and impulse response $h(t)$ is given by the convolution expression: $y(t) = x(t) * h(t)$. Show that: $x(t-t_0) * h(t-t_0) = y(t-2t_0)$.

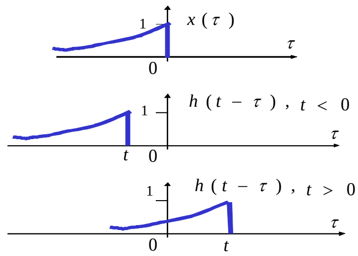
(b) Given that $h(t) = e^{-2t}u(t)$ and $x(t) = e^{2t}u(-t)$, determine and sketch $y(t)$ for the system in part (a).

(c) Calculate the output of a DT LTI system with impulse response $h(n) = 0.5\delta(n) + 2\delta(n-1)$, given that the input signal is $x(n) = \delta(n) + 0.5\delta(n-1) + 0.25\delta(n-2)$.

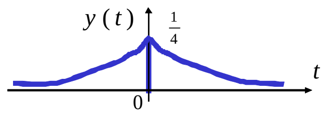
(d) Assess the stability of the LTI systems with impulse response: (i) $h(n) = u(n)$, (ii) $h(t) = \sum_{k=0}^{\infty}(0.5)^k \delta(t-k)$.

<u>Ans</u>: (a) $x(t-t_0) * h(t-t_0) = \int_0^{\infty} x(\tau - t_0)h(t-\tau-t_0)d\tau$; let $\nu = \tau - t_0$, then this convolution integral becomes:

$\int_0^{\infty} x(\nu)h(t-\nu-2t_0)d\nu = y(t-2t_0)$, since $y(t') = \int_0^{\infty} x(\nu)h(t'-\nu)d\nu$ equals integral above if we let $t' = t - 2t_0$.

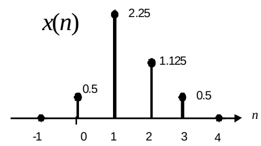


(b) For $t < 0$, $y(t) = \int_{-\infty}^{t} e^{2\tau}e^{-2(t-\tau)}d\tau = \dfrac{e^{2t}}{4}$



For $t > 0$, $y(t) = \int_{-\infty}^{0} e^{2\tau}e^{-2(t-\tau)}d\tau = \dfrac{e^{-2t}}{4}$

(c) $y(n) = h(n) * x(n) = [0.5\delta(n) + 2\delta(n-1)] * x(n) = 0.5x(n) + 2x(n-1)$
$= 0.5\delta(n) + 2.25\delta(n-1) + 1.125\delta(n-2) + 0.5\delta(n-3)$



[note: graphical convolution gives same result]

(d) (i) $\sum_{k=0}^{\infty}|h(n)| = \sum_{k=0}^{\infty}|u(n)| = \infty$: unstable, (ii) $\int_0^{\infty}|h(t)|\,dt = \int_0^{\infty}\left|\sum_{k=0}^{\infty}(0.5)^k\delta(t-k)\right|dt = \dfrac{1}{1-0.5}$: stable